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The Study of a Fractional Model of Diffusion Equation and its Analytical Solution using Laplace Transforms

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ABSTRACT

In this article, we will look at some of the results related to fractional Laplace variation with an exaggerated ability to cope with the specific conditions of a fractional differential. Mittag-Leffler's work suggests a significant role in Laplace's fractional changes to discover the structure of the differential conditions of the fractional query. HPTM is a combination of Laplace's transformation process and homeopathic parteratin. Numerical solutions understood by the Recommended solutions point out that the method is simple to realize and is being improved. Here we solve several problems related to the differential condition of a fractional query using the fractional Laplace transform with initial conditions.

1. Introduction

Partial-request common differential conditions, as speculations of classical whole-number request customary differential conditions, are increasingly used to demonstrate issues in fluid flow, mechanics, viscosity, science, physics, and designing, and various applications. Fractional derivatives give excellent material in drawings of various materials and cycle memories and inherent characteristics. Quasi-request derivatives and integrals are more valuable for the definition of some electrochemical issues than the classical model. It is regularly realized that the positions of gas elements are numerical expressions of the conservation laws that exist in designing practices for the protection of mass, the protection of force, the conservation of energy, and so on. of nonlinear waves such as unconscious fronts, rarefaction, and contact discontinuities. In 1981, Steiger and Warming point out that the conservation law type of the inverse gas dynamical condition has an amazing property by morality, in that the nonlinear flux vectors are homogeneous elements of degree one that allow the flux vectors to be sub-fields. Equality allows for division by change, and thus, new express and deemed disruptive 'night-distinction' schemes have been created to deal with the previously-requested exaggerated structure of terms. The request is described for description and classification in science, physics, and morphology, in general condition, peculiar type of verbal verb, tail flow, mechanical, speedy. Different derivatives are wrong-requested to calculate the sound as well as incorrectly to calculate the other type of sound. ; In order to feel like such an environment it is necessary that as the state of behavior is in the like state, as in the word like behavior protect, energy conserve, energy conserve, and as in like condition Will happen. ofretractables such as stun pecking, tearing, and contact discreteness. In 1981, the substance was turned to and as an incredible property in the type of chemical dynamical control protection, quickly becoming a powerful group member group member of the flux group into sub-regions used to convert. uniformly designed to develop symmetry.

2. Fundamental concepts

In this paper, the Laplace homotopy perturbation technique basically shows how the Laplace transformation can be used to thicken the arrangement of straight and non-linear differential positions by controlling the homotopy annoyance strategy. Annoyance techniques are used for the most part to take care of non-linear issues, for example, the general arrangement involves arranging small bounds which represents a difficulty since most nonlinear issues have no problem. There is no small limit. Means. Although reasonable decisions of small limits repeatedly lead to an ideal system, most of the time, improper decisions have real effects in systems. The homotopy burning technique was presented and implemented by him. Recently, many analysts have derived the system of integral position and fractional differential position usinghomotopy burning technique. The proposed strategy is mainly a coupling of the Laplace transform, the homotopy burning technique, and that polynomial due to Ghearbani. Recently, a number of constructs have focused on considering the arrangement of direct and non-linear incomplete differential conditions using various strategies involving the Laplace transform. These include the Laplace dissociation strategy and the Laplace homotopy burning technique.

3. Methodology

In this paper , the homotopy perturbation strategy is used to refer the state of time-fractional gas elements with the continuation of the Laplace transform and the polynomial. Using the implied position, rough practical expressions of u(x, t) are obtained for various fractional Brownian movements and additionally for standard movement. The approximate arrangement is obtained mathematically and depicted graphically. The lusciousness of this article can be attributed to the overly simplification method in search of a precise scientific arrangement of the issue. In this work we are concerned with the mathematical arrangement of the time-varying diffusion condition. The free speed of the molecule is represented by the old-style diffusion condition. The condition of time-partial diffusion under external power is addressed as follows:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = D\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial}{\partial x}(F(x)u(x,t)),$$
$$0 < \alpha \leq 1,\ D > 0, \qquad (1.1)$$

where u(x, t) addresses the probability thickness of finding a molecule at point x in time moment t, the positive constant D depends on the temperature, the erosion coefficient, the all-inclusive gas that eventually ends up on the Avogordo number, F. is compatible. (x) is external force. In the present paper, we have thought of two models. In the main model, we think of Eq. (1.1) for d = 1 and f(x) = -x. In the latter model, we consider two-dimensional example situations with non-local boundary conditions, which emerge in many important issues of numerical physics, for example, thermo-versatility, back issues, hot burdens. Hypothesis, clinical design, control hypothesis, synthetic diffusion, heat conduction measures, population elements, vibration issues, nuclear reactor elements, clinical science, natural chemistry and some organic cycles.

4. Review of Literature

Padlubani (1999) discusses that mathematical research applies to the selection of data, electrical wiring, electrical science, arithmetic, brains, and biology (Padlubani, 1999).). Mathematical fraction is also involved in the problems: polymer science, polymer science, science science, biophysics, rheology, and thermodynamics (Hilfer, 2000). In addition, it is necessary for the following studies: Electrochemical Processes (Miller & Ross, 1993; Oldham, Spanier, 1974; Podlubani, 1999), Methodology (David, Linearis, Palon , 2011; Podlubeni, 1999), Physics (Subtier), Agarwal, and Machado, 2007), Science and Technology (Kumar, Saxena, 2016), Transport in a Semi-Infinite Environment (Oldham, Spanier, 1974). , Industry (Sheng, Chen, & Q, 2011), Food (Rahimi, 2010), Food Digestion (David, Katayama, 2013), Fractional Dynamics (Tarsov, 2011; Jaslavsky, 200).), Musculoskeletal Disorders (Magin, 2006), David, Lee Narasimha and Palon of Nutrition and Econophysics, 2011), Organic Compounds of Tissues (Surface, 2010), viscosity (Dalir, Bashur , 2010; Mainardi, 2010; Podluboni, 1999; Rakhineimi, 2010; Sabatiar, Agarwal and Machado, 200), displaying the swing rating (Gomez -Aguilar, -Martinez, Calderon-Raman, Cruz-Ordu). na, Escobargimenez and Olivers-Peregrino, 2012 5). Some of the applications have been tested in the following. In the study of science and technology, special applications of impact analysis have been developed over the last twenty years. For example, fractional arithmetic has been used in graphic design, measurement, corrosion, biological research, the use of technology, fractional oscillation, sound and intelligence (Kumar and Saxena, 2016). . It uses the conventional acytihh model as the basis of the fractional material (Das and Roy, 2014). In the field of electrochemical cycles, subordinates and mixes to the queries in question become better models for the development of electrochemical problems, especially those using technology (Miller & Ross, 1993; Oldham). &Spanier, 1974; Podlebony, 1999). In the case of viscoelasticity it is necessary to write using fractional calculus to place the viscoelastic data. For viscoelastic materials, the main stress factor is more likely to be due to the presence of separation factors (Carpentry, Corneti, Sapora, 2014; Dalir, Bashour, 2010; Duan, 201; Koeler, 1984; Mainardi, 2010; Podlubani). May, 1999). Numerical fractions, which are one of the models of mathematical fractions, are used to create subordinates in decision making (Podlubani, 1999). Recently, some of the following staff members have used (models) to describe the real problem.

5. Analysis

Basic definitions of fractional calculus

In this section, we give some basic definitions and properties of the fractional calculus theory which shall be used in this paper

Definition 1. A real function f (t), t> 0, is called located in the space Cμ, μ ∈ R, if there exists a real number p> μ such that f (t) = tp f1 (t) where f1 (t) ∈ C (0, ∞), and it is said to be in the space Cn if and only if f (n) ∈ Cμ, n ∈ N.

Definition 2. The left sided Riemann–Liouvillefractional integral order operator μ ≥ 0, of function f ∈ Cα, α ≥ −1 that is defined by:

$$I^\mu f(t) = \begin{cases} \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} f(\tau)d\tau, & \mu > 0,\ t > 0, \\ f(t), & \mu = 0 \end{cases} \quad (2.1)$$

where 0(·) is the well known Gamma function.

Definition 3. The left fractional Caputo derivative of functions f, f ∈ C m −1, m ∈ N {0} that is defined by Podlubnyy (7) and Samko et al. (12) by:

$$D_*^{\mu} f(t) = \frac{\partial^{\mu} f(t)}{\partial t^{\mu}} = \begin{cases} I^{m-\mu}\left[\frac{\partial^m f(t)}{\partial t^m}\right], \\ \quad m-1 < \mu < m,\ m \in N, \\ \frac{\partial^m f(t)}{\partial t^m}, \quad \mu = m. \end{cases} \qquad (2.2)$$

Note that by [3,30],

(i) $I_t^{\mu} f(x,t) = \frac{1}{\Gamma(\mu)} \int_0^t \frac{f(x,t)}{(t-s)^{1-\mu}} dt,\ \mu > 0,\ t > 0,$

(ii) $D_*^{\mu} f(x,t) = I_t^{m-\mu} \frac{\partial^m f(x,t)}{\partial t^m}\ m-1 < \mu \le m.$

Definition 4. The Mittag-Leffler function, Eα (z), for α>0, that is defined by the following series representation acting in the entire complex Mainardi plane (11):

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}. \qquad (2.3)$$

Definition 5. The Laplace transform of a continuous (or almost piecewise continuous) function f (t) to [0, ∞) that is defined by:

$$F(s) = L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt. \qquad (2.4)$$

Definition 6. The Laplace transform L [f (t)] of the Riemann - Liouville fractional integral thatis defined by Miller and Ross (6) as:

$$L[I^{\alpha} f(t)] = s^{-\alpha} F(s). \qquad (2.5)$$

Definition.7. The Laplace transform, L[f(t)], of the Caputo fractional derivative is defined by Miller and Ross (6) as:

$$L[D^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{(\alpha-k-1)} f^{(k)}(0),$$
$$n-1 < \alpha \le n. \qquad (2.6)$$

Examples

In this part, two instances of time fractional diffusion conditions are settled to exhibit the presentation and proficiency of the HPM with the coupling of the Laplace change

Example. In this example, we consider the following time fractional diffusion equation as follows:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial}{\partial x}(x\,u(x,t)),$$
$$u(x,0) = f(x).$$

The methodology consists of applying the Laplace transform first on both sides of Thus, we get

$$L[D_t^{\alpha} u(x,t)] = L[u_{xx} + (x\,u)_x].$$

Using the differentiation property of the Laplace transform, we get:

$$L[u(x,t)] = s^{-1} f(x) + s^{-\alpha} L[u_{xx} + (x\,u)_x].$$

Applying the inverse Laplace transform on both sides, we get:

$$u(x,t) = f(x) + L^{-1}\left(s^{-\alpha} L[u_{xx} + (x\,u)_x]\right).$$

By applying the aforesaid homotopy perturbation method, we have:

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = f(x) + p\left(L^{-1}\left(s^{-\alpha} L[u_{xx} + (x\,u)_x]\right)\right).$$

Equating the coefficient of the like power of p on both sides, we get:

$$p^0 : u_0(x,t) = f(x),$$
$$p^n : u_n(x,t) = L^{-1}\left(s^{-\alpha} L[u_{nxx} + (x\,u_n)_x]\right),$$
$$n \ge 0,\ n \in N.$$

6. Results

In this study, the basic idea was generated about an iterative fractional Laplace change strategy for solving (n + 1) fractional time conditions. Iterative fractional Laplace modification method was applied to three (n + 1) spatial time fractional diffusion conditions with initial conditions to obtain their enclosed devices in the form of an infinite arrangement of fractional forces and in terms of Mittag-Leffler capabilities at one boundary, which quickly combine for correct location. Locking devices, such as the limitless arrangement of fractional forces and the Mittag-Leffler capabilities at a single boundary, which are quickly combined to correct agreements, have been effectively identified using the iterative fractional Laplace modificationmethod. The results, judged interestingly, the fractional diffusion conditions are in decent agreement with the pre-existing spelling. Precisely, iterative fractional Laplace modification method efficiently handles fractional time diffusion conditions with initial conditions to obtain their closed mechanisms in the form of infinite fractional forces and, in terms of Mittag-Leffler functioning at the boundary, exactly the same basic amount of computation.

7. Conclusion

Fractional calculus emerged from a study presented by L'Hospital and Lebniz in 1965. This is an assumption about mathematics of the whole order. By evaluating the set of experiences, we find that fractional calculus has been a truly intriguing topic for mathematicians for a long time, regardless of the lack of reason to use it. In the coming years, an increasing number of specialists are paying attention to the fact that fractional calculus is used in true issues such as viscoelastic structure, dielectric polarization, electromagnetic waves, etc. Thanks to the extraordinary efforts of specialists, there have been rapid improvements in the hypothesis of fractional analytics and its applications. The purpose of this article is to characterize the fractional Laplace transform using the work of Mittag-Leffler. The work of Mittag-Leffler is characterized, and with its help the fractional Laplace transform is additionally characterized. Here we draw a conclusion about some properties of the fractional Laplace transform, depending on our definition, which is more useful for finding a combination of a fractional differential condition with initial conditions.

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