

An Investigation On String Cosmological Model With And Without Magnetic Field in Axial Symmetry

Vinay Kumar Chaudhary

Asst. Professor, Department of Physics, Nalanda College of Engineering, Chandi (Nalanda)

ARTICLE DETAILS

Article History

Published Online: 19 June 2018

Keywords

model, axial symmetry space time, electromagnetic field, magnetic field, shear, expansion.

ABSTRACT

In this paper we have obtained a string cosmological model in axially symmetric space time with and without magnetic field using different assumption. To get a deterministic model we have assumed the condition $\lambda = \ell + \mathbf{m}\mathbf{U}$ (Where ℓ and m are constant). Under the different cases $\ell = 0$ (for magnetic field) & $K = 0$ (for absence of magnetic field).

1. Introduction

In recent years many relativists have focussed their mind in string cosmology [1, 28, and 23]. In fact at the early stages of the universe a phase transition occurs as the temperature lowers below some critical temperature and this can give rise to the various topologically stable defects of which strings are the most important whose world sheets are two dimensional time-like surfaces (Kibble [15], [20]) studied a gauge invariant model of a cloud formed by a geometric strings and used this model as a source of the gravitational field. In particular, he solved the Einstein equations for a plane-symmetric, a spherically symmetric and particular case of cylindrically symmetric space-time [18, 19]. There were two main reasons to study the above mentioned model. First as a test of consistency, for some particular field theories model. First as a test of consistency, for some particular field theories based on string models and other models that use strings as basic elements we must have a reasonable behaviour of the gravitational field produced by these strings. Second, we point out that the universe can be represented by a collection of extended objects (galaxies). So a "string-dust" cosmology gives us a model to investigate this fact. Also, the existence of strings in the early universe can be used to introduce density fluctuations that might shed some light on the problem of galaxy formation [35, 36].

The presence of the strings in the early universe can be explained using grand unified theories [15, 35] (GUTs). In spontaneously broken gauge theories the spontaneously broken symmetry can be restored at a temperature T greater than some critical temperature T_c . In standard "big bang" cosmological models T_c will be exceeded in the very earliest stages of the evolution of the universe. A phase transition will occur, as the universe cools below T_c , in which a multiplet value of scalar-fields develops a vacuum expectation value $\langle \phi \rangle = \eta$. Cosmic strings are capable of producing observable effects such as the production of double images of quasars. Moreover, cosmic strings may play a vital role in the formation of the large scale structure of the universe. Investigations regarding different aspects of cosmic strings include the gravitational fields produced by them. The gravitational field of a static infinitely long gauge string produced by the breaking of a $U(1)$ gauge symmetry has been studied in great detail. Vilenkin [31] used the linearized approximation of the Einstein field equations for this purpose. Gregory [12] analyzed the equations of motion for a $U(1)$ global string and showed that this space-time geometry produced by it is necessarily singular. Gibbons, Oriz and Ruiz [10] considered such a string and showed that this type of string cannot admit any asymptotically well-behaved solution as the deficit angle produced by the string diverges logarithmically for a large radial distance. The only known solution for the full nonlinear Einstein equations in the case of a global string has been provided by Cohen and Kaplan [8]. Letelier [17] has studied model of a string cloud, in which the strings that form the cloud are massive strings instead of geometrical strings. Each massive string is formed by a geometric string with particles attached along its extension. Hence, the strings that form the cloud are the generalization of Takahayaris realistic model of strings which we call p-strings [17, 30]. This is the simplest model wherein we have particles and strings together. In principle we can eliminate the strings and end up with a cloud of particles. This is a desirable property of a model of a string cloud to be used in cosmology, since the strings are not observed at the present time of the evolution of the universe. The cosmology models studied by Letelier [17] of the Bianchi type 1 and of the "Kantowski-Sachs" type which has been chosen because they are supposed to be a reasonable representation of the universe in early epochs and they are simple enough that many of their features can be studied exactly. This has been already done by several authors, (Vilenkin [31], Gott [11], Garfinkle [9]), although the general relativistic treatment of strings was pioneered by Letelier [17] and Stachel [27]. In geometrical string (mass less) models, infinite number of degrees of freedom is possessed by each string for which the end points move at the speed of light. This problem is resolved by considering the realistic (massive) string model of Takabayashi [30]. The energy-momentum tensor for the massive strings has been first formulated by Letelier [20], who considered the massive string being formed by geometric string with particles attached along its extension. Its application to general relativity first appeared in Letelier [18], While Stachel [27] considered mass less strings. So the total energy momentum tensor for a cloud of massive strings can be written as

$$(1.1) \quad T_i^j = \rho u_i u^j - \tau x_i x^j$$

where ρ is the rest energy density for a cloud of strings with particles attached to them (p-strings). This we have

$$(1.2) \quad \rho = \rho_p + \tau$$

ρ_p being the particle energy density and \square being the strings tension, u^i is the four velocity for the cloud of particles and x^i is the four vector representing the strings direction which essentially is the direction of anisotropy. Thus

$$(1.3) \quad u_i u^j = -1 = x_i x^j \text{ and } u_i x^i$$

Banerjee et al. [4] have found some cosmological solution in Bianchi I space time following the technique used by Letelier and Stachel with/without magnetic field. Melvin [21] in his solution for dust and de-electromagnetic field argued that the presence of magnetic field is not as unrealistic as it appears to be, because for a large part of the history of evolution matter was highly ionized, and matter and field were smoothly coupled. Later during cooling as a result of expansion the ions combined to form neutral matter. Some other worker's in this line are Baysal et al. [6] Bali and Pareek [3], Yadav et. al. [33] and Pradhan et. al. [23, 24], Yilmaz [34], Chawla et al. [7] and Wald, R.M.[32].

In this paper we have studied a string cosmological solution in axially symmetric Bianchi – I space time with an without magnetic field using different assumption. To get a deterministic model we have assumed the condition $\lambda = 1 + m\upsilon$ (where 1 and m are constant). Under the different cases $m = 0, 1 = m = 0$ and $1 = 0$ (for magnetic field) & $K = 0$ (in the absence of magnetic field). We have also calculated and discussed various physical parameters of the model.

2. The Field Equations :

Here consider an axially symmetric Bianchi-1 metric given by-

$$(2.1) \quad ds^2 = -dt^2 + e^{2\lambda}.dx^2 + e^{2\upsilon}(dy^2 + dz^2)$$

where λ and υ are function of t only.

Now, the energy – momentum tensor for the string dust with a magnetic field along the direction of the string, i.e the x direction is given by

$$(2.2) \quad T_\mu^\delta + E_\mu^\delta = \rho u_\mu u^\delta - \tau x_\mu x^\delta + \frac{1}{4\pi} \left[F_\mu^i F_i^\delta - \frac{1}{4} F_{ij} F^{ij} \zeta_\mu^\delta \right]$$

where T_μ^δ the stress-energy tensor for a string dust system is, E_μ^δ is that for magnetic field & F_{ij} is the electromagnetic field tensor. The other terms have already been explained in the previous system. In the co-moving co-ordinates system

$$u_\mu = \zeta_4^\delta \text{ and}$$

$$(2.3) \quad T_4^4 = -\rho, T_1^1 = -\tau, T_2^2 = T_3^3 = 0 = T_\mu^\delta \text{ (for } \mu \neq \delta)$$

Again since the magnetic field is being assumed in the X-direction, F_{23} is the only non-zero component of the electromagnetic field tensor.

Maxwell Equation :

$$F_{[\mu\delta,\alpha]} = 0 \text{ and } [F^{\mu\delta} (-g)^{1/2}]_\mu = 0$$

Now lead to the result,

$$(2.4) \quad -F_{23} = K$$

where K is constant. Therefore the components of stress energy tensor for the electromagnetic field are

$$(2.5) \quad E_4^4 = E_1^1 = -E_2^2 = -E_3^3 = \frac{-K^2}{8\pi} \exp(-4\upsilon)$$

Now choosing unit such that $8\square G = 1$, the surviving components of Einstein field equations given by

$$(2.6) \quad R_\mu^\delta - \frac{1}{2} \zeta_\mu^\delta .R = -(T_\mu^\delta + E_\mu^\delta)$$

are

$$(2.7) \quad 2\dot{\lambda}\dot{\upsilon} + \dot{\upsilon}^2 = \rho + \frac{k^2}{8\pi} .\exp(-4\upsilon)$$

$$(2.8) \quad 2\ddot{\upsilon} + 3\dot{\upsilon}^2 = T + \frac{k^2}{8\pi} ,\exp(-4\upsilon)$$

$$(2.9) \quad \ddot{\lambda} + \dot{\lambda}^2 + \dot{\upsilon}^2 + \dot{\lambda}\dot{\upsilon} = -\frac{K^2}{8\pi} .\exp(-4\upsilon)$$

where the dot denotes the differentiation with respect to time t.

The proper volume R^3 ; expansion scalar (\square) and shear scalar \square^2 are respectively given by

$$(2.10) \quad R^3 = \exp(\lambda + 2\upsilon)$$

$$(2.11) \theta = U_{;\mu}^{\mu} = \dot{\lambda} + 2\dot{\upsilon} = \frac{3\dot{R}}{R}$$

$$(2.12) \sigma^2 = \sigma_{\mu\delta} \sigma^{\mu\delta} = \dot{\lambda}^2 + 2\dot{\upsilon}^2 - \frac{1}{3}\theta^2$$

where,

$$(2.13) \sigma_{\mu\delta} = \frac{1}{2} [u_{\mu\delta} + u_{\delta\mu} + u_{\mu} u^i u_{\delta i} + u_{\delta} u^i u_{\mu i}] - \frac{1}{3}\theta(g_{\mu\delta} + u_{\mu} u_{\delta})$$

3. Solution for the field equations :

We have three equations (2.7) to (2.9) in four unknown, λ, υ, ρ and \square . Therefore the system is indeterminate. To make the system determinate, we required one more relation. For this we choose.

$$(3.1) \lambda = 1 + m\upsilon$$

(where 1 and m are constant)

Case - I

Here use $1 = 0$ in equation (3.1), i.e. $\lambda = 1 + m\upsilon$, then we get,

$$(3.2) \lambda = m\upsilon$$

By the use of (3.2) and (2.9) we get

$$(3.3) (m+1)\ddot{\upsilon} + (m^2 + m + 1)\dot{\upsilon} = \frac{-K^2}{8\pi} \exp(-4\upsilon)$$

This equation can be written as an integral equation

$$(3.4) \int d \left[\dot{\upsilon}^2 \exp \left(2 \left(\frac{m^2 + m + 1}{m + 1} \right) \upsilon \right) \right] = \frac{-K^2}{4\pi(m + 1)}$$

$$\int \exp \left(2 \left(\frac{m^2 - m - 1}{m + 1} \right) \upsilon \right) d\upsilon + K_2$$

Where K_2 is constant of integration. So we have

$$(3.5) \dot{\upsilon}^2 = K_2 \exp \left[-2 \left(\frac{m^2 - m - 1}{m + 1} \right) \upsilon \right] - \frac{K^2}{8\pi(m^2 - m - 1)} \cdot \exp(4\upsilon)$$

which can again be written as an integral form as -

$$(3.6) \int \frac{e^{2\upsilon} d\upsilon}{\left[K_2 \exp \left\{ -2 \left(\frac{m^2 - m - 1}{m + 1} \right) \upsilon \right\} - \frac{K^2}{8\pi(m^2 - m - 1)} \right]^{1/2}} = \pm(t - t_0)$$

Where t_0 is another constant of integration. To solve (3.6) we choose m such that

$$(3.7) \frac{m^2 + 2m + 12}{m + 3} = 5$$

It is a quadratic equation in m which can be solved to give :

$$(3.8) m = \frac{3 \pm \sqrt{9 + 12}}{2}$$

$$m = \frac{3 \pm \sqrt{21}}{2}$$

Using (3.8) in (3.6) following by integration, we get

$$(3.9) \quad e^{2v} = \left[\frac{8\pi K_2}{K^2} (5 \pm \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 \pm \sqrt{21})} \right]^{1/2}$$

From (3.9) it is clear that arbitrary constant K_2 must be positive in this case. So we replace K_2 by \square^2 in the following. The other parameters can be found as before :

$$(3.10) \quad \rho = \frac{\frac{9 + 2\sqrt{21}}{(5 + \sqrt{21})^2} \cdot \frac{K^2}{16\pi} (t-t_0)^2 - (5 + \sqrt{21})\delta^2}{\left[\frac{8\pi\delta^2}{K^2} (5 + \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})} \right]^2}$$

From energy condition, $\square > 0$, which demands positive sign before $\sqrt{21}$ in equation (3.9) with this choice other parameters are found explicitly as follows :

$$(3.11) \quad \tau = \frac{9 + \sqrt{21}\delta^2 - \frac{4 + \sqrt{21}}{(5 + \sqrt{21})^2} \cdot \frac{K^2}{16\pi^2} (t-t_0)^2}{\left[\frac{8\pi\delta^2}{K^2} (5 + \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})} \right]^2}$$

$$(3.12) \quad \rho_p = \frac{4\delta^2 + \frac{K^2(t-t_0)^2}{16\pi^2(5 + \sqrt{21})}}{\left[\frac{8\pi\delta^2}{K^2} (5 + \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})} \right]^2}$$

$$(3.13) \quad R^3 = \left[\frac{8\pi\delta^2(5 + \sqrt{21})}{K^2} - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})} \right] \times \frac{(7 + \sqrt{21})}{8}$$

$$(3.14) \quad \theta = - \left(\frac{7 + \sqrt{21}}{5 + \sqrt{21}} \right) \left(\frac{K^2}{8\pi} \right) \left[\frac{t-t_0}{\frac{8\pi\delta^2}{K^2} (5 + \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})}} \right]$$

$$(3.15) \quad \sigma^2 = - \left(\frac{7 + \sqrt{21}}{(5 + \sqrt{21})^2} \right) \left(\frac{K^2}{8\pi} \right) \left[\frac{t-t_0}{\frac{8\pi\delta^2}{K^2} (5 + \sqrt{21}) - \frac{K^2(t-t_0)^2}{2\pi(5 + \sqrt{21})}} \right]$$

It is clear from (3.8) that

$$(3.16) \quad T^2 < \frac{16\pi^2\delta^2}{K^4} (5 + \sqrt{21})^2$$

where $T = t - t_0$, when $t < t_0$ we have $T > 0$ and clearly from equation (3.14).

We have $\theta > 0$ and $\sigma^2 < 0$

This indicates that our model is expanding because $\theta > 0$

Case II : When magnetic field is absent (i.e. $K = 0$)

Then the field equation (2.7), (2.8) and (2.9) take the following form.

$$2\dot{\lambda}\dot{v} + \dot{v}^2 = \rho$$

$$2\ddot{v} + 3\dot{v}^2 = \tau$$

$$\ddot{\lambda} + \dot{\lambda}^2 + \dot{v}^2 + \dot{\lambda}\dot{v} = 0$$

Now assuming (3.2) equation (2.9) reduces to

$$(m + 1)\ddot{v} + (m^2 + m + 1)\dot{v} = 0$$

Or,

$$(3.17) \quad \ddot{v} + \frac{m^2 + m + 1}{m + 1} \dot{v} = 0$$

This equation can be written as an integral equation

$$(3.18) \quad \int d \left[\dot{v} \exp \left\{ 2 \left(\frac{m^2 + m + 1}{m + 1} \right) v \right\} \times v \right] = \gamma$$

where γ is constant of integration. So we have

$$(3.19) \quad \dot{v}^2 = \gamma \left[\exp \left\{ -2 \left(\frac{m^2 + m + 1}{m + 1} \right) v \right\} \right]$$

which can again be written as integral form as ?

$$(3.20) \quad \int \frac{e^{2v} .dv}{\left[\gamma \exp \left\{ -2 \left(\frac{m^2 - m - 1}{m + 1} \right) v \right\} \right]^{1/2}} = \pm (t - t_0)$$

where t_0 is the another constant of integration to solve (3.20) we get,

$$(3.21) \quad e^{2v} = [\gamma(t - t_0)]^{\frac{2(m+1)}{m^2+m+1}}$$

(where m is any constant) Therefore,

$$(3.22) \quad e^{2\lambda} = [\gamma(t - t_0)]^{\frac{2m(m+1)}{m^2+m+1}}$$

The physical parameters are found to be

$$(3.23) \quad R^3 = [\gamma(t - t_0)]^{\frac{(2+m)(m+1)}{m^2+m+1}}$$

$$(3.24) \quad \rho = \frac{2m + 1}{(t - t_0)^2}$$

$$(3.25) \quad \tau = \frac{1}{(t - t_0)^2}$$

$$(3.26) \quad \rho_p = \frac{2m}{(t - t_0)^2}$$

$$(3.27) \quad \theta = \frac{m + 2}{(t - t_0)}$$

$$(3.28) \quad \sigma^2 = \frac{2(m - 1)^2}{3(t - t_0)^2}$$

Special Case :

We choose m such that $m^2 + m - 2 = 0$ (quadratic equation) gives

$$(3.29) \quad m = 1 \text{ or } -2$$

We put the value of m using (3.29) in equation (3.21 – 3.28) and get :

$$(3.30) e^{2\nu} = [\gamma(t-t_0)]^{4/3} \text{ or } [\gamma(t-t_0)]^{-2/3}$$

$$(3.31) e^{2\lambda} = [\gamma(t-t_0)]^{4/3} \text{ or } [\gamma(t-t_0)]^{4/3}$$

The physical parameters are found to be

$$(3.32) R^3 = [\gamma(t-t_0)]^2 \text{ or } [\gamma(t-t_0)]^0 = 1$$

$$(3.33) \rho = \frac{3}{(t-t_0)^2} \text{ or } \frac{-3}{(t-t_0)^2}$$

$$(3.34) \tau = \frac{1}{(t-t_0)^2}$$

$$(3.35) \rho_p = \frac{2}{(t-t_0)^2} \text{ or } \frac{-4}{(t-t_0)^2}$$

$$(3.36) \theta = \frac{3}{(t-t_0)} \text{ or } 0$$

$$(3.37) \sigma^2 = \frac{2}{3} \times 0 = 0 \text{ or } \frac{2}{3} \times \frac{9}{(t-t_0)^2}$$

Now if consider $T = t - t_0$ and $t > t_0$, then for the particular value of m like $m > -2$, $m = -2$ and $m < -2$ we get different results for expansion.

If we consider $m > -2$ in equation (3.27) then we get

$$(3.38) \theta = +ve \Rightarrow \theta > 0, \text{ which indicate our model is expanding.}$$

If $m = -2$ in equation (3.27) then

$$(3.39) \theta = 0, \text{ which indicates that neither expansion nor contraction occurs.}$$

If $m < -2$ in equation (3.27) we get

$$(3.40) \theta = -ve \Rightarrow \theta < 0, \text{ which indicate our model is contracted.}$$

Again if consider $T = t - t_0$ and $t < t_0$, then for the particular value of m like, $m > -2$, $m = -2$ and $m < -2$ we get different results for expansion.

If we consider $m > -2$ in equation (3.27) then we get

$$(3.41) \theta = -ve \Rightarrow \theta < 0, \text{ which indicate our model is contracted.}$$

If $m = -2$ in equation (3.27) then we get

$$(3.42) \theta = 0, \text{ which indicates that neither expansion nor contraction occurs.}$$

If $m < -2$ in equation (3.27) we get

$$(3.43) \theta = +ve \Rightarrow \theta > 0, \text{ which indicate our model is expanding.}$$

4. Discussion:

Here we show that at initial epoch $t = t_0$, $R^3 \rightarrow 0$, While $\rho, \tau, \rho_p, \theta, \sigma^2$ all diverge and $\exp(2\lambda), \exp(2\nu) \rightarrow 0$ the point of singularity. This is the starting point of the string model. Again at a later stage, when $t \rightarrow \infty, R \rightarrow \infty$ but all other physical parameters becomes insignificant. Further we see that for a pure geometric string ($\square_p = 0$) model, we have to take $m = 0$. For this case the universe starts with string and ends up at a stage, where the massive strings them self disappear without any remnant. In the absence of magnetic field we have to take $m > -2$ (for $t > t_0$) then our model is expanding. But if $t < t_0$ then the expansion occurs for $m < -2$. So the range of expansion of model (in absence of magnetic field) depends upon particular value of m and t respectively.

The present work extends the work of Letelier and various other researchers. In addition we have consider a source with and without magnetic field respectively we have obtained four sets of solution with a special choice of metric coefficients, viz, $\square = 1 + m U$. for a particular value of m we observe that our model starts from a string dominated era but at a later instant string vanish and the universe become particle dominated. In this model gravitational field is coupled to magnetic field such that in the absence of the magnetic field (i.e. $K = 0$) the system reduces to pure geometric string. For different value of m we calculate upper range of expansion of the string universe (for magnetic and without magnetic field), proper volume and shear scalar.

References

1. Amirhashchi, H., Zainuddin, H., Pradhan, A. (2012): Rom. J. Phys., 57, 748.
2. Bali, R. and Anjali (2004) : Pramana, J. Phys., 63, 484.
3. Bali, R and Pareek, U.K. (2009): Pramana J. Phys., 72, 787.
4. Banerjee, A., Sanyal, A.K. and Chakraborty, S. (1990): Pramana J. Phys., 34, 1
5. Banerjee, A., Banerjee, N. and Sen, A.A. (1996): Phys. Rev., D53, 5508.
6. Baysal, H. et. al. (2001) : Turk, J. Phys., 25, 283.
7. Chawla, C., Mishra, R.K., Pradhan, A. (2012) : Eur. Phys. J. Plus., 127, 137.
8. Cohen, A.D., and Kalpan, D.B. (1998): Phys. Lett., B215, 65.
9. Garfinkle, D. (1985) : Phys. Rev., D32, 1323.
10. Gibbons, G.W., Oritz, M. and Ruiz, F. (1989): Phys. Rev., D39, 1546.
11. Gott, G.R. (1985) : Astrophys. J., 288, 422.
12. Gregory, R. (1988) : Phys. Lett, B215, 663.
13. Hiscock, W.A. (1985) : Phys. Rev., D 31, 3288.
14. Harari, D. and Sikivie, P. (1988) : Phys. Rev., D 37, 3448.
15. Kibble, T.W.S. (1976) : J. Phys., A9, 1387.
16. Krori, K.D. et. al. (1990) : G.R.G., 22, 123.
17. Letelier, P.S. (1978) : J. Math. Phys., 19, 1908.
18. Letelier, P.S. (1979) : Phys. Rev., D20, 1294.
19. Letelier, P.S. (1981) : NuoveCimento, 63B, 519.
20. Letelier, P.S. (1983) : Phys. Rev., D28, 2414.
21. Melvin, M.A. (1975) : Ann. New York Acad. Sci., 262, 253.
22. Nielson H.B. and P. Olesen (1973) : Nucl. Phys., B61, 45.
23. Pradhan, A., Singh, A.K., Chauhan, D.S. (2013) : Int. J. theor. Phys. 52, 266.
24. Pradhan, A., et. al. (2005) : Zech. J. Phys., 55, 503.
25. Ray Chaudhuri, A.K. (1979): Theoretical Cosmology (Oxford : Clarendon Press).
26. Ray Chaudhuri, A.K. (1990) : Phys. Rev., D41, 3041.
27. Stachel, J. (1980) : Phys. Rev., D21, 2171.
28. Singh, J.P. and Tiwari, R.K. (2008): PRAMANA-Journal of Phys., 70, No. 4, pp. 565-574.
29. Singh, A.K. (2014): International Journal of Engineering Research and Technology, Vol. 3, Issue 05, 342-348.
30. Takabayashi, T. (1978) : Q. mech. Determinam, causality and particles (ed.) M. Flato (Holand : ReidelDorbrecht) P. 179.
31. Vilenkin, a. (1985) : Phys. Rev., 121, 263.
32. Wald, R.M. (1984) : General Relativity and Gravitation (Univ. of Chicago Press).
33. Yadav, M.K. Rai, A. and Pradhan, A. (2007) : Int. J. Theo. Phys., 46, 2677.
34. Yilmaz, I. (2006) : G.R.G., 38, 1397.
35. Zeldovich, Ya. B., et. al. (1974) : Zh. Eksp. Teor. Fiz., 67, 3.
36. Zeldovich, Ya. B., (1980) : Mon. Not. R. Astron. Soc., 192, 663.