

# Some Static Spherically Symmetric Models for the Universe Filled with Anisotropic Ferro Fluid

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## ABSTRACT

The main objective of this paper is to forward some spherically symmetric static models for the universe which is filled with anisotropic ferrofluid under the special choices of dynamical variables.

## 1. Introduction

Various relativists have shown their growing interest in finding solution of Einstein-Maxwell field equations. It has immense importance in general relativity. This solution enhances a universal understanding of the space-time, which is not concerned to a specific selection of parameters and basic conditions. The latest self-sufficing survey utilizing varied examples of isotropic solution of Einstein-field equations pertaining to spherical symmetry is given by Kramer et al. [9], Yodzis et al. [14] has obtained a solution which proves that naked singularities can occur in the spherical gravitational collapse of anisotropic matter. As such for Bowers and Liang [4], the study of static anisotropic spheres is valuable for relativistic astrophysics. Cosenza et al. [6] has evolved heuristic procedure resulting in obtaining interior solutions for Einstein's equations for anisotropic matter from known solutions of isotropic matter. Stewart [13] has initiated successful research pertinent to large collection of anisotropic interior solutions has studied static, spherically symmetric, conformally flat interior solutions in resulting a freedom to specify the functional form of the mass distribution. Cosenza et al. [7] and Bayin [2] have presented a comparison between properties of isotropic and anisotropic relativistic spheres. Ponce de Leon [11, 12] has explored many solutions using new ansatz. Berman [3] has accrued an anisotropic cosmological model with a Bianchi-1 type metric. A class of interior solutions for uniform density source under a specific form of radial pressure is presented by Maharaj and Maartens [10]. Coley and Tupper [5] explored that the anisotropic fluid space times admitting covariantly constant vector must have to satisfy a number of strong restrictions.

The main objective of this paper is to forward some spherically symmetric static models for the universe which is filled with anisotropic ferrofluid under the special choices of dynamical variables. Section 2 deals with Einstein-Maxwell field equations corresponding to anisotropic ferrofluid whereas section 3, 4 and 5 deals with some precise solutions of these operative equations.

## 2. The Field Equations

We take static spherically symmetric time demand given by

$$(2.1) \quad ds^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^\nu dt^2$$

(where  $\lambda$  and  $\nu$  are functions of  $r$  alone)

The attention is confined to the comoving coordinate system for which flow vector  $v^i$  and magnetic field vector  $H^i$  have expressions

$$(2.2) \quad v^i = \delta^i_4 e^{-\nu/2}$$

$$(2.3) \quad H^i = \delta^i_4 e^{-\lambda/2}$$

Thus under comoving system we have

$$(2.4) \quad h^2 = f^4 / r^4$$

$$(2.4) \quad \mu = \phi^2 / f^2$$

where  $\phi$  is constant of integration and  $f$  is function of  $r$ . Therefore the equation provides the value

$$(2.5) \quad m = (1/2)\mu h^2 = (1/2)\left[\left(\phi^2 f^2 / r^4\right)\right]$$

The assumption of static spherical symmetry under comoving co-ordinate system restricts the components of the stress-energy tensor. Accordingly we have

$$T_1^1 = -(P_R - m)$$

$$T_2^2 = -(P_T + m) = T_3^3$$

and  $T_4^4 = (\rho + m)$

Thus the Einstein field equations for the anisotropic ferrofluid system described through the energy momentum tensor generates the following equations

$$(2.6) \quad K(P_R - m) = e^{-\lambda} \left[ (v'/r) + (1/r^2) \right] - 1/r^2$$

$$(2.7) \quad K(P_T + M) = e^{-\lambda} \left[ (v''/2) - (\lambda'v'/4) + (v'^2/4) + (v' - \lambda'/2r) \right],$$

$$(2.8) \quad K(\rho + m) = e^{-\lambda} \left[ (\lambda'/r) - (1/r^2) \right] + 1/r^2$$

Here the overhead prime denotes derivative with respect to r

The sum of equation (2.6) and (2.8) gives

$$(2.9) \quad K(\rho + P_R) = e^{-\lambda} \left[ (v'/r) + (\lambda'/r) \right] = e^{-\lambda} \left[ (v' + \lambda'/r) \right]$$

On subtracting the equation (2.7) from the equation (2.6) it is obtained that

$$(2.10) \quad K(P_R - P_T) = 2Km - e^{-\lambda} \left[ v''/2 + (v'^2/4) - (\lambda'v'/4) - \lambda'/2r - v'/2r - 1/r^2 \right] - 1/r^2$$

On differentiating the equation (2.6) with respect to r and using the equation (2.5) one can get

$$(2.11) \quad KP'_R = e^{-\lambda} \left[ (v''/r) - (v'/r^2) - 2/r^3 - (\lambda'v'/r) - \lambda'/r^2 \right] + 2/r^3 + [(2/r) - (f'/f)](-2Km).$$

Further by utilizing the equation (2.10) it produces

$$(2.12) \quad KP'_R = -(1/2)e^{-\lambda}v'(\lambda' + v')/r - K(P_R - P_T)(2/r) + 2Km(f'/f)$$

Thus the equation (2.9) and (2.11) imply

$$(2.13) \quad P'_R + (\rho + P_R)(v'/2) + (P_R - P_T)(2/r) - (\phi^2ff'/r^4) = 0$$

This equation describes the factors affecting the rate of variation in radial pressure.

### 3. Solution of the Field Equations

Here we start with a physical plausibility that the matter distribution consists of an ionized imperfect gas satisfying the equation of state  $P_R \propto \rho$ . This situation can be well described through the setting

$$(3.1) \quad K_\rho = ie^{-\lambda}/r^2$$

$$(3.2) \quad KP_R = je^{-\lambda}/r^2$$

where i and j are non-negative constants. The value of v is obtained by integration of equation (2.9) as

$$(3.3) \quad v = K \int (\rho + P_R) e^\lambda r dr - \lambda + \log k_1$$

where  $k_1$  is a constant of integration.

The equation (3.1) and (3.2) simplify equation (3.3) to

$$(3.4) \quad v = \log(k_1 r^k e^{-\lambda}),$$

where  $K = i + j$

By making use of the values of m and  $P_R$  from equation (2.5) and (3.2) in equation (2.10), we obtain

$$(3.5) \quad r^2 e^{-\lambda} (\lambda'' - \lambda'^2) + (3/2)kre^{-\lambda}\lambda' - (1/2)(k^2 - 4i - 4)e^{-\lambda} = 2 - 2KP_T r^2 - 2K(\phi^2 f^2 / r^2)$$

Now to solve the field equations we suppose that tangential pressure be expressed in the form

$$(3.6) \quad KP_T = k_2 e^{-\lambda} / r^2$$

With  $k_2$  as constant.

This exhibits the physical plausibility that  $P_T \rightarrow 0$ , as  $r \rightarrow \infty$ .

By choosing  $y = e^{-\lambda}$  the equation (3.5) reduces to

$$(3.7) \quad r^2 y'' + (3/2)kry' + (1/2)(k^2 - 4i - 4k_2 - 4)y = 2k(\phi^2 f^2 / r^2) - 2$$

The introduction of new variable  $z$  as  $r = e^z$  leads the equation (3.7) to

$$(3.8) \quad d^2y/dz^2 + (1/2)(3k - 2)dy/dz + (1/2)(k^2 - 4i - 4k_2 - 4)y = 2K(\phi^2 f^2)e^{-2z} - 2$$

This equation admits the general solution in the form

$$(3.9) \quad y = Ae^{P_1 z} + Be^{P_2 z} + k_3 + k_4 e^{-2z}$$

where  $A$  and  $B$  are constants of integration and  $P_1, P_2, k_3, k_4$  are constants given by

$$(3.10) \quad P_1 = -(1/4) \left[ 3k - 2 - \sqrt{k^2 + 20i - 12j + 36 + 32k_2} \right],$$

$$(3.11) \quad P_2 = -(1/4) \left[ 3k - 2 + \sqrt{k^2 + 20i - 12j + 36 + 32k_2} \right],$$

$$(3.12) \quad k_3 = -4 / (k^2 - 4i - 4k_2 - 4)$$

$$(3.13) \quad k_4 = 4K(\phi^2 f^2) / (k^2 - 10i - 6j - 4k_2 + 8)$$

The equation (3.9) is expressed in terms of  $r$  as

$$(3.14) \quad y = e^{-\lambda} = Ar^{P_1} + Br^{P_2} + k_3 + k_4 / r^2$$

The value of  $e^\nu$  is then obtained from the equation (3.4) as

$$(3.15) \quad e^\nu = k_1 \left[ Ar^{P_1 - C} + Br^{P_2 + C} + k_3 r^C + k_4 r^{(C-2)} \right]$$

Hence the space-time metric in this case reads as

$$(3.16) \quad ds^2 = - \left[ Ar^{P_1} + Br^{P_2} + k_3 + k_4 / r^2 \right]^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + k_1 \left[ Ar^{P_1 + C} + Br^{P_2 + C} + k_3 r^C + k_4 r^{(C-2)} \right] dr^2$$

This describes the static spherically symmetric model consistent with the ferrofluid system.

Implications of the solution (3.16) :

(1) The physical requirements

$$(3.17) \quad \rho \geq 0, P_R \geq 0, P_T \geq 0$$

imply the conditions on  $i, j$  and  $k_2$  as

$$(3.18) \quad i \geq 0, j \geq 0, k_2 \geq 0$$

(2) The values of kinematical parameters associated with time like unit flow vector  $v^i$  corresponding to (3.16) are given by

$$(3.19) \text{ Expansion : } v^i = 0$$

$$(3.20) \text{ Shear : } \sigma^2 = 0$$

$$(3.21) \text{ Rotation : } \omega^2 = 0$$

$$(3.22) \text{ Acceleration } \equiv v^{*2} = -\left(v'^2/4\right)e^{-\lambda}$$

Where  $v$  is given by the equation (3.15) and  $e^{-\lambda}$  by the equation (3.14)

(3) The choice  $j = k_2$  implies that the radial and tangential pressures are equal. The restriction  $j = k_2 = 0$  leads the solution (3.16) to a dust filled universe. The selection  $i = 3j = 3k_2$  procures the radiating model. When one identifies  $i = j = k_2$ , then it reduces to a super dust model.

(4) The metric (3.16) with  $j = -i = k_2 = 0$  and  $k_1 = 1$  assumes the solution in the form

$$(3.23) \text{ } ds^2 = -\left[1 + Ar^2 + B/r + K\phi^2 f^2 / 2r^2\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + Ar^2 + B/r + K\phi^2 f^2 / 2r^2\right] dt^2$$

This describes the static exterior field of the anisotropic ferrofluid distribution.

(5) For  $A = 0$  the solution (3.23) reduces to Reissner-Nordstrom metric describing the gravitational field in the exterior region of a infinitely conducting static sphere which has the metric form

$$(3.24) \text{ } ds^2 = -\left[1 + (B/r) + K\phi^2 f^2 / 2r^2\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + (B/r) + K\phi^2 f^2 / 2r^2\right] dt^2$$

(6) A metric (3.23) with  $A = 0 = \phi = f$  generates the well known Schwarchild's exterior solution.

$$(3.25) \text{ } ds^2 = -\left[1 + (B/r)\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + (B/r)\right] dt^2$$

(7) A metric (3.23) reduces to desitter's model for a static homogeneous universe for the substitution  $B = 0 = \phi = f$  given by

$$(3.26) \text{ } ds^2 = -\left[1 + Ar^2\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + Ar^2\right] dt^2$$

(8) The selection  $A = B = 0$  reduces to solution (3.23) to the line element characterising gravitational field of an electron (Eddington 8).

$$(3.27) \text{ } ds^2 = -\left[1 + K\phi^2 f^2 / 2r^2\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + K\phi^2 f^2 / 2r^2\right] dt^2$$

(9) The treatment  $B = 0$  in the metric (3.23) generates the line element as obtained by Aherkar and Asgekar [1] representing a static spherically symmetric space time model for the universe filled with the magnetofluid.

$$(3.28) \text{ } ds^2 = -\left[1 + Ar^2 + K\phi^2 f^2 / 2r^2\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left[1 + Ar^2 + K\phi^2 f^2 / 2r^2\right] dt^2$$

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