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Fuzzy Lattice with New Algebraic Structure

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ABSTRACT

This paper contains some definitions and results in intuitionistic fuzzy lattice ordered M-groups, which are required in the sequel. Some properties of homomorphism and anti-homomorphism of intuitionistic L-fuzzy M-subgroups are also established.

1. INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh [10].Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, Rosenfield [1] introduced the concept of fuzzy subgroup. Motivated by this,many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], Biswas introduced the concept of anti-fuzzy subgroups of groups. Palaniappan. N and Muthuraj, [7] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups .G.S.V. Satya Saibaba [4] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l-groups. Pandiammal P, Natarajan R and Palaniappan N, [9] defined the homomorphism, antihomomorphism of an anti L-fuzzy M-subgroup, Institutionistic L- fuzzy m-groups. In this paper we define a new algebraic structure of intuitionistic fuzzy lattice ordered M-group and studied some related properties.

2. PRELIMINARIES:

2.1 Definition: Let G be a group, M be any set if i) m x ϵ G. ii) m (x y) = (m x) y = x m y for all x, yϵ G, m ϵ M. Then G is called a M group.

2.2 Definition : Let μ: X→ [0, 1] be a fuzzy set & G be a M group G. A fuzzy set on G,
G ϵ þ(X)) is called a fuzzy m group if i) μ (m(x y)) ≥ min {μ (m x), μ (my)}
ii) $\mu(mx^{-1}) \geq \mu(m x)$ for all x, yϵ G, m ϵ M

2.3 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be L-fuzzy M-subgroup (LFMSG) of G if it satisfies the following axioms:
(i) $\mu_A(mxy) \geq \min\{\mu_A(mx), \mu_A(y)\}$
(ii) $\mu_A(mx^{-1}) \geq \mu_A(mx)$, for all x and y in G.

2.4 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be anti L-fuzzy M-subgroup (LFMSG) of G if it satisfies the following axioms:
(i) $\mu_A(mxy) \leq \min\{\mu_A(mx), \mu_A(y)\}$
(ii)$\mu_A(mx^{-1}) \leq \mu_A(mx)$, for all x and y in G.

2.5 Definition: A lattice ordered group is a system (G,.,≤) if i) (G, ·) is a group ii) (G,≤) is a lattice . iii) x ≤ y implies a x b ≤ a y b (compatibility) For a, b, x, yϵ G

2.6 Definition : Let μ: X → [0, 1] be a fuzzy set & G is a lattice ordered set, G ϵ þ(X).
A function μ on G is said to be a fuzzy lattice ordered group if
i) μ (x y) ≥ min {μ (x), μ(y)}
ii) $\mu(x^{-1}) \geq \mu(x)$ for all x, yϵ G

2.7 Definition: μ: X to [0, 1], G ϵ þ(X), M C X.
A function μ on G is said to be a fuzzy lattice ordered m-group if

i) (G, ·) is a M-group.
ii) (G,·, ≤) is a lattice ordered group.
iii) μ (m(x y)) ≥ min {μ (m x), μ(my)}
iv) $\mu((m x)^{-1}) \geq \mu(m x)$
v) μ (m x v my) ≥ min {μ (m x), μ(m y)}
vi) μ (m x ʌ my) ≥ min {μ (m x), μ(my)} For all x, y ϵ G

2.8 Definition: Let (G, ·) be a M-group. An intuitionistic L fuzzy subset A of G is said to be an intuitionistic L-fuzzy M subgroup (ILFMSG) of G if the following conditions are satisfied:

(i) $\mu_A (mxy) \geq \min \{ \mu_A (mx) , \mu_A (my) \}$
(ii) $\mu_A (mx^{-1}) \geq \mu_A (mx)$,
(iii) $\nu_A (mxy) \leq \max \{\nu_A (mx) \mu_A (my) \}$
(iv) $\nu_A (mx^{-1}) \leq \nu_A (mx)$, for all x and y in G.

2.9 Definition: μ: X to [0, 1], G ϵ ƥ(X), M C X. A function μ on G is said to be an Intuitionistic fuzzy lattice ordered m-group (IFLOMG) if

i) (G, ·) is a M-group.
ii) $(G,\cdot, \leq)$ is a lattice ordered group.
iii) $\mu (m(x y)) \geq \min \{\mu (m x), \mu(my)\}$
iv) $\mu ((m x)^{-1}) \geq \mu (m x)$
v) $\mu (m x \vee my) \geq \min \{\mu (m x), \mu(m y)\}$
vi) $\mu (m x \wedge my) \geq \min \{\mu (m x), \mu(my)\}$
vii) $\nu_A (mxy) \leq \max \{\nu_A (mx), \mu_A (my) \}$
viii) $\nu_A ((m x)^{-1}) \leq \nu_A (m x)$
ix) $\nu_A (mx \vee my) \leq \max \{\nu_A (mx), \mu_A (my) \}$
x) $\nu_A (mx \wedge my) \leq \max \{\nu_A (mx), \mu_A (my) \}$

2.10 Definition: Let (G, ·) and (G', ·) be any two M-groups. Let f : G → G' be any function and A be an IFLOMG in G, V be an IFLOMG in f(G) = G', defined by $\mu_v(y) = \sup\{ \mu_A(x)) / x\epsilon f^{-1}(y) \}$ and $\nu_A(y) = \inf\{ \nu_A(x)) / x\epsilon f^{-1}(y) \}$, for all x in G and y in G'. Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

2.11 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B denoted by A x B , is defined as

A x B = {< (x, y), $\mu_{A\times B}(x, y)$, $\nu_{A\times B}(x, y)$ >/ for all xϵ G and y ϵ H },where

$\mu_{A\times B}(x, y) = \mu_A(x) \wedge \mu_B(y) \}$ and $\nu_{A\times B}(x, y) = \nu_A(x) \vee \nu_B(y)$

2.12 Definition: Let A and B be any two IFLOMG. Then A and B are said to be conjugate IFLOMG if for some g in G, $\mu_A (x) = \mu_B (m g^{-1} x mg)$ and $\nu_A (x) = \nu_B (mg^{-1} x mg)$, for every x in G.

2.13 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the strongest intuitionistic L-fuzzy relation on S, that is an intuitionistic L-fuzzy relation on A is V given by

$\mu_v(x, y) = \min \{\mu_A(x) , \mu_A(y) \}$ and $\nu_v (x, y) = \max \{ \nu_A(x) ,\nu_A(y) \}$, for all x and y in S.

3 – PROPERTIES OF INTUITIONISTIC L-FUZZY M-SUBGROUPS:

3.1 Theorem: If A is an IFLOMG of a M-group (G, ·), then $\mu_A (mx^{-1}) = \mu_A(mx)$ and $\nu_A(mx^{-1})=\nu_A(mx)$, $\mu_A(mx) \leq \mu_A(me)$ and $\nu_A(mx) \geq \nu_A(me)$, for x in G, where e is the identity element in G.

Proof: For x in G and e is the identity element in G.

Now, $\mu_A(mx) = \mu_A((mx^{-1})^{-1}) \geq \mu_A(m x^{-1}) \geq \mu_A(m x)$.
Therefore, $\mu_A(mx^{-1}) = \mu_A (m x)$. And ,
$\nu_A (m x) = \nu_A((mx^{-1})^{-1}) \leq \nu_A (mx^{-1})\leq \nu_A(m x)$.
Therefore, $\nu_A(mx^{-1}) = \nu_A(mx)$, for all x in G.
Now, $\mu_A(me) = \mu_A(m xx^{-1}) \geq \min \{\mu_A(mx) , \mu_A (mx^{-1})\} = \min \{\mu_A(mx) , \mu_A (mx)\}= \mu_A(mx)$.
Therefore, $\mu_A(me) \geq \mu_A(m x)$. And,
$\nu_A(me) = \nu_A(m xx^{-1})\leq \max \{\nu_A(mx), \nu_A(mx^{-1}) \}= \max \{\nu_A(mx), \nu_A(mx) \} = \nu_A(mx)$.
Therefore, $\nu_A(me) = \nu_A(mx)$, for all x in G.

3.2 Theorem: If A is an IFLOMG of a M-group (G, ·), then
(i) $\mu_A(mxy^{-1}) = \mu_A(me)$ gives $\mu_A(mx) = \mu_A(my)$,
(ii) $\nu_A(m xy^{-1}) = \nu_A (me)$ gives $\nu_A (mx) = \nu_A (my)$, for x & y in G, where e is the identity element in G.
Proof: Let x & y in G and e is the identity element in G.
Now, $\mu_A (mx) = \mu_A (m xy^{-1}y) \geq \min \{\mu_A (mxy^{-1}) ,\mu_A (my) \}$
$= \min \{\mu_A (me), \mu_A (my) \}$
$= \mu_A (my) = \mu_A (myx^{-1}x)$
$\geq \min \{\mu_A (myx^{-1}) ,\mu_A (mx) \}$
$= \min \{\mu_A (me), \mu_A (mx) \}$
$= \mu_A (mx)$.
Therefore, $\mu_A (mx) = \mu_A (my)$, for all x and y in G.
And, $\nu_A (mx) = \nu_A (mxy^{-1}y)\leq \max \{ \nu_A (m xy^{-1}), \nu_A (my)\}$
$= \max \{\nu_A (me), \nu_A (my)\}$
$= \nu_A (my)$
$= \nu_A (myx^{-1}x)$
$\leq \max \{\nu_A (m yx^{-1}) , \nu_A (mx) \}$
$= \max \{\nu_A (me), \nu_A (mx)\}$
$= \nu_A (mx)$.
Therefore, $\nu_A A(mx) = \nu_A A(my)$, for all x and y in G.

3.3 Theorem: A is an IFLOMG of a M-group (G, ·) if and only if
$\mu_A(mxy^{-1}) \geq \min \{\mu_A(mx) , \mu_A(my) \}$, $\nu_A(mxy^{-1}) \leq \max \{\nu_A(mx),\nu_A (my)\}$,
$\mu_A(mx \vee my^{-1}) \geq \min \{ \mu_A (mx), \mu_A (my)\}$, $\mu_A(mx \wedge my^{-1}) \geq \min \{ \mu_A (mx), \mu_A (my) \}$
$\nu_A(mx \vee my^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my) \}$, $\nu_A(mx \wedge my^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my) \}$
for all x , y in G.
Proof: Let A be an IFLOMG of a M-group (G, ·).
Then, $\mu_A(mxy^{-1})\geq \min \{\mu_A (mx), \mu_A (my^{-1})\}= \min \{ \mu_A (mx), \mu_A (my) \}$
Therefore, $\mu_A (mxy^{-1}) \geq \min \{ \mu_A (mx), A(my) \}$, for all x & y in G. And,
$\nu_A (mxy^{-1}) \leq \max \{\nu_A (mx), \nu_A (my^{-1})\}= \max \{\nu_A (mx), \nu_A (my)\}$.
Therefore, $\nu_A (mxy^{-1}) \leq \max \{\nu_A (mx), \nu_A (my)\}$, for all x & y in G.
$\mu_A(mx \vee my^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\}= \min \{ \mu_A (mx), \mu_A (my) \}$
Therefore $\mu_A(mx \vee my^{-1}) \geq \min \{ \mu_A (mx), \mu_A (my) \}$
$\mu_A(mx \wedge my^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\}= \min \{ \mu_A (mx), \mu_A (my) \}$
Therefore $\mu_A(mx \wedge my^{-1}) \geq \min \{ \mu_A (mx), \mu_A (my) \}$
$\nu_A(mx \vee my^{-1}) \leq \max \{\nu_A (mx), \nu_A (my^{-1})\}= \max \{ \nu_A (mx), \nu_A (my) \}$
Therefore $\nu_A(mx \vee my^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my) \}$
$\nu_A(mx \wedge my^{-1}) \leq \max \{\nu_A (mx), \nu_A (my^{-1})\}= \max \{ \nu_A (mx), \nu_A (my) \}$
Therefore $\nu_A(mx \wedge my^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my) \}$

Conversely, if $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$ and $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), A(my)\}$,
replace y by x, then
$\mu_A(me) \geq \min\{\mu_A(mx), \mu_A(mx)\} = \mu_A(mx)$ and
$\nu_A(me) \leq \max\{\nu_A(mx), A(mx)\} = \nu_A(mx)$
i) $\mu_A(mx^{-1}) = \mu_A(mex^{-1}) \geq \min\{\mu_A(me), \mu_A(mx)\} = \mu_A(mx)$.
Therefore, $\mu_A(mx^{-1}) \geq \mu_A(mx)$.
ii) $\mu_A(mxy) = \mu_A(m\,x(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mxy) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
iii) $\nu_A(mx^{-1}) = \nu_A(mex^{-1}) \leq \max\{\nu_A(me), \nu_A(mx)\} = \nu_A(x)$.
Therefore, $\nu_A(mx^{-1}) \leq \nu_A(mx)$.
iv) $\nu_A(mxy) = \nu_A(mx(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\nu_A(mxy) \leq \max\{\nu_A(mx), \nu_A(my)\}$ for all x and y in G.
v) $\mu_A(mx \vee my) = \mu_A(mx \vee m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
vi) $\mu_A(mx \wedge my) = \mu_A(mx \wedge m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
vii) $\nu_A(mx \vee my) = \nu_A(mx \vee m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \leq \max\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
viii) $\nu_A(mx \wedge my) = \nu_A(mx \wedge m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \leq \max\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
Hence A is an IFLOMG of G.

3.4 Theorem: Let A be an intuitionistic L-fuzzy subset of a group (G, ·). If $\mu_A(m\,e) = 1$ and $\nu_A(me) = 0$ and $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$ and $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$,
$\mu_A(mx \vee my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$, $\mu_A(mx \wedge my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$
$\nu_A(mx \vee my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$, $\nu_A(mx \wedge my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$
for all x , y in G then A is an IFLOMG of a M-group G.

Proof: Let x & y in G and e is the identity element in G.
i) $\mu_A(mx^{-1}) = \mu_A(mex^{-1}) \geq \min\{\mu_A(me), \mu_A(mx)\} = \min\{1, \mu_A(mx)\} = \mu_A(mx)$
Therefore, $\mu_A(mx^{-1}) \geq \mu_A(mx)$, for all x in G.
ii) $\nu_A(mx^{-1}) = \nu_A(mex^{-1}) \leq \max\{\nu_A(me), \nu_A(mx)\} = \max\{0, \nu_A(mx)\} = \nu_A(mx)$.
Therefore, $\nu_A(mx^{-1}) \leq \nu_A(mx)$, for all x in G.
iii) $\mu_A(mxy) = \mu_A(m\,x(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mxy) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
iv) $\nu_A(mxy) = \nu_A(mx(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\nu_A(mxy) \leq \max\{\nu_A(mx), \nu_A(my)\}$ for all x and y in G.
v) $\mu_A(mx \vee my) = \mu_A(mx \vee m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
vi) $\mu_A(mx \wedge my) = \mu_A(mx \wedge m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \geq \min\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
vii) $\nu_A(mx \vee my) = \nu_A(mx \vee m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \leq \max\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
viii) $\nu_A(mx \wedge my) = \nu_A(mx \wedge m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \leq \max\{\mu_A(mx), \mu_A(my)\}$, for all x and y in G.
Hence A is an IFLOMG of G.Hence A is an intuitionistic L-fuzzy M-subgroup of a M-group G.

3.5 Theorem: If A is an IFLOMG of a M-group (G, ·), then H = {mx / x∊ G : $\mu_A(mx) = 1$,
$\nu_A(mx) = 0$} is either empty or is a M-subgroup of a M-group G.

Proof: If no element satisfies this condition, then H is empty.
If m x and m y in H, then
$\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\} = 1$.
Therefore, $\mu_A(mxy^{-1}) = 1$, for all x and y in G.
And, $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu(mx), \nu(my)\} = 0$.
Therefore, $\nu_A(mxy^{-1}) = 0$, for all x and y in G. We get mxy^{-1} in H.
Therefore, H is a M-subgroup of a M-group G.
Hence H is either empty or is a M-subgroup of M-group G.

4. CONCLUSION

In this paper, we define a new algebraic structure of Intuitionistic fuzzy lattice ordered M-groups of M-groups.

REFERENCES

[1] Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications 35, (1971) 512-517.
[2] Biswas R, Fuzzy subgroups and anti-fuzzy subgroups,Fuzzy Sets and Systems 35(1990), 121-124.
[3] Goguen J.A, L-fuzzy sets, J. Math. Anal.Appl.18(1967),145-179.

[4] G.S.V. Satya Saibaba. Fuzzy lattice ordered groups, South east Asian Bulletin of Mathematics 32,749-766 (2008).
[5] Mohamed Asaad, Groups and Fuzzy Subgroups, Fuzzy Sets and Systems 39(1991)323-328.
[6] Prabir Bhattacharya, Fuzzy subgroups: Some characterizations, J. Math. Anal. Appl. 128 (1981) 241-252.
[7] Palaniappan, R.Muthuraj, The homomorphism, Antihomomorphism of a fuzzy and an anti-fuzzy group, Varahmihir Journal of mathematical Sciences, 4 (2)(2004) 387-399.
[8] Palaniappan , S. Naganathan, & K. Arjunan, A Study on Intuitionistic L-Fuzzy Subgroups, Applied mathematical Sciences, 3 (53) (2009) 2619-2624.
[9] Pandiammal. P, Natarajan. R, and Palaniappan. N , Anti Lfuzzy M-subgroups, Antarctica J. Math., Vol. 7, number 6, 683-691, (2010).
[10] L.A. Zadeh, Fuzzy sets, Information and control, 8, (1965) 338-353.