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Study of Numerical Solution of System of Nonlinear Equations by Newton's Method

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ABSTRACT

We think about an arrangement of nonlinear conditions . Another iterative strategy for taking care of this issue mathematically is recommended. The logical conversations of the technique are given to uncover its 6th request of assembly. A conversation on the proficiency list of the commitment with correlation with the other iterative techniques is additionally given. At last, mathematical tests delineate the hypothetical perspectives utilizing the programming bundle Mathematicas. System of nonlinear equations arises in many parts of the engineering, physics and biology applications. In most cases, the system arisen almost impossible to be solved analytically, hence numerical analysis become the fashion. For many years, Newton's method seems to be the best method in solving the problems but some issues came out afterwards. For example, the Jacobian matrix, J that need to be supplied in each iteration required a lot of calculation works. and consumed a lot of time.

1. Introduction

Nonlinear problems are of interest to engineers, biologists, physicists, mathematicians, and many other scientists because most systems are inherently nonlinear in nature. To specify, nonlinear system of equations arises in many parts of the engineering applications such as pressure driven analysis of water distribution network in earth and atmospheric science, heat and thermal conduction in mechanical engineering, power distribution system in electrical science, and so on (Balaji et al., 2017). The solution of nonlinear system of equations is often the crucial step in the solution of practical problems arising in physics and engineering. These equations can be expressed as the simultaneous zeroing of a set of functions, where the number of functions to be zeroed is equal to the number of independent variables. If the model of the system arise from a well constructed of engineering or physical system, the solution will be meaningful to some state of the system (Broyden, 1965).

2. Direct methods v/s Iterative methods for nonlinear equations

Direct methods give accurate value of the roots in a finite number of steps. Moreover, generally, direct methods give all the roots at the same time. For example, to solve a quadratic equation

$$ax^2 + bx + c = 0$$

the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

gives precise estimations of the roots and yields both the roots at the same time. Consequently this is an immediate technique for settling the quadratic condition. Then again, iterative strategies (otherwise called mathematical techniques) depend on progressive estimate which means that we start with one or then again more suitable introductory approximations of the root and by a specific plan, get an arrangement of approximations (or repeats) which unites to the specific root. Obviously, iterative techniques give rough estimation of the root. Additionally, such strategies give just each root in turn

3. Fixed Point Theorem

Definition 1.2..

A point x ∈ R is said to be a fixed point of a mapping T : R → R if T x = x

Remark 1.2.8. A mapping may or may not have any fixed point. A mapping may even have more than one fixed point. For example, a translation map

$$Tx = x + a, \quad x \in \mathbb{R},$$

for any a ∈ R, a = 0, does not have any fixed point. A rotation map

$$Tx = e^{ix}, \quad x \in [0, 2\pi]$$

has only one fixed point which is the center of the corresponding circle. The mapping

$$Tx = x^2, \quad x \in \mathbb{R}$$

has two fixed points, namely, 1 and -1. It has been an important question "when does a mapping have one and only one fixed point ?" The answer to this question is given in terms of the following definition:

Definition 1.2.9. A mapping T : R → R is said to be a contraction map if there exists a real number α ∈ (0, 1) such that

$$|Tx - Ty| \leq \alpha|x - y| \quad \forall x, y \in \mathbb{R}.$$

Now the existence and uniqueness of a fixed point is known by the following well known theorem:

Historical Background

Nonlinear conditions emerge in practically all zones of sciences, specifically, in physical and numerical sciences. Basically, it is once in a while conceivable to address a nonlinear condition scientifically. So iterative strategies are by and large utilized in such circumstances. The most well-known among such techniques for settling a nonlinear condition f(x) = 0 are the Newton strategy

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

and the secant method

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n).$$

It is known that the order of convergence of Newton method is 2 while that of secant method is 1.618. These methods have numerous applications, e.g., in complex dynamics.

Over the years, tremendous variants of these methods have appeared showing one or the other advantages over these methods in some sense. It is not possible to give a exhaustive record of the research that has taken place so far in this direction. We shall try to trace the investigation that we came across while doing our own work. Note that in order to apply Newton's method (1.3.1), the function needs to be differentiable.

4. Concerning the Efficiency Index

Presently we survey the productivity record of the new iterative technique rather than the current strategies for frameworks of nonlinear conditions. In the iterative technique , three direct frameworks dependent on LU disintegrations are expected to acquire the 6th request of intermingling. The fact of the matter is that for applying to enormous scope issues one may tackle the direct frameworks by iterative solvers and the quantity of straight frameworks ought to be in congruity with the combination rate. For instance, the technique for Sharma et al. requires three diverse direct frameworks for huge inadequate nonlinear frameworks, that is, the equivalent with the new strategy (, however its assembly rate is just 4, which unmistakably shows that technique (is better)

As was emphatically called attention to constantly official, considering just the quantity of assessments for scalar capacities can't be the affecting component for assessing the proficiency of nonlinear solvers. The quantity of scalar items, network items, decay LU of the main subordinate, and the goal of the three-sided direct frameworks are critical in evaluating the genuine proficiency of such plans. Some broad conversations on this issue can be found in the new writing.

5. Conclusion

To accomplish this objective, we in what follows think about an alternate way. Allow us to tally the quantity of lattice remainders, items, summations, and deductions alongside the expense of tackling two three-sided frameworks, that, depends on lemon (the genuine expense of settling frameworks of direct conditions). For this situation, we comment that the failures for getting the LU factorization are , and to tackle two three-sided framework, the lemon would be . Note that if the right-hand side is a lattice, the cost (flops) of the two three-sided frameworks is , or generally as considered in this paper. Table likewise uncovers the examinations of failures and the lemon like proficiency file. Note that to the best of the creators' information, such a list has not been given in some other work. Consequences of this are accounted for in Table and Figure too. It is seen that the new plan again contends all the new or notable emphasess when contrasting the computational effectiveness files.

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