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Some Bianch Type-III String Cosmological Models with Bulk Viscosity

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ABSTRACT
The present paper provides solution for Bianchi type III string cosmological model with bulk viscosity by taking suitable relations between shear scalar, bulk viscosity and expansion scalar. Various physical and geometrical features have been also discussed.

1. Introduction

In fact, the general relativistic treatment of strings was initiated by Letelier [11] and Stachel [20]. This model has been used as a source for Bianchi types-I and Kantowski-Sachs cosmologies by Letelier [11]. After wards, Krori et al. [9, 10] have discussed about cloud string. Recently Bali and Dava [4] have presented Bianchi type-III string cosmological model with bulk viscosity, where the constant coefficient of bulk viscosity is considered. However, it is known that the coefficient of bulk viscosity is not constant but decreases as the universe expands [1, 3, 7]. Arbab [2[, Bali and Tinker [6], Pradhan et. al. [12-14], Ray and Mukhopadhay [20], Singh and Singh [21], Singh and Pradhan [22], Singh and Kumar [23-24], Yadav and Pradhan [39] are some of the authors who have studied various aspects of interacting fields in the framework of Bianchi type-III string cosmological model with bulk viscosity in general relativity.

In this paper, we study the Bianch type-III string cosmological model with bulk viscosity. To obtain a determinate solution, we assume that the coefficient of the bulk, viscosity is a power function of the scalar of expansion $\xi = k\theta^{a+bm}$ and the shear scalar is proportional to scalar of expansion $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. The physical and geometric features of the model are also discussed.

2.The Field Equations And Their Solutions

The Bianchi type-III space-time metric we considered here is [5]

$$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2 \tag{2.1}$$

Where □, □ and □ are only the functions of time t.

The energy-momentum tensor for a cloud of string with bulk viscosity is [5]

$$T_{ij} = \rho u_i u_j - \lambda \chi_i \chi_j - \xi\theta(u_i u_j + g_{ij}) \tag{2.2}$$

where $\rho = \rho_p + \lambda$, is the rest energy density of the cloud of strings with particles attached to them □$_p$ is the rest energy density of particle, □ is the tension density of the cloud of strings, $\theta = u^i_{;i}$, is the scalar of expansions, and □ is the coefficient of bulk viscosity. Accordint to Letelier [11] the energy density for the coupled system □ and □$_p$ is restricted to be positive, while the tension density □ may be positive or negative. The vector u^i describes the cloud four-velocity and □^{i} represents a direction of anisotropy, i.e. the direction of string. They satisfy the standard relation [14].

$$u^i u_j = -\chi^i \chi_j = -1, \; u^i \chi_i = 0 \tag{2.3}$$

The expressions for scalar of expansion and shear scalar are (kinematical parameters)

$$\theta = u^i_{;i} = \frac{\dot{\alpha}}{\alpha} + \frac{\dot{\beta}}{\beta} + \frac{\dot{\beta}}{\beta} + \frac{\dot{\gamma}}{\gamma} \tag{2.4}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{\alpha}^2}{\alpha^2} + \frac{\dot{\beta}^2}{\beta^2} + \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} - \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}\right) \tag{2.5}$$

Einstein's equation we consider here is

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$$R_{ij} - \frac{1}{2} R g_{ij} = J_{ij} \tag{2.6}$$

Where we have choose the units such that c = 1 and 8□G = 1. In the co-moving coordinates $u^i = \delta^i_0$ and $u^i = -\delta^0_i$, and with the help of equations (2.1), (2.3), the Einstein equation (2.6) can be written as [5]

$$\frac{\ddot{\beta}}{\beta} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} = \xi\theta \tag{2.7}$$

$$\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} = \xi\theta \tag{2.8}$$

$$\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{1}{2} = \lambda + \xi\theta \tag{2.9}$$

$$\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\gamma} - \frac{1}{\alpha^2} = \rho \tag{2.10}$$

$$\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\beta}}{\beta} = 0 \tag{2.11}$$

Where the dot denotes the differentiation with respect to time t. From Eq.(2.11), we have

$$\alpha = H\beta \tag{2.12}$$

Where H is the constant of integration. In order to obtain a more general solution, we assume

$$\xi = k\theta^{\alpha+gm} \tag{2.13}$$

where a, b, k and m are the positive constants.

We note that the five independent equations (2.8) – (2.11) and (2.13) connect six unknown variable $(\alpha,\beta,\gamma,\lambda,\rho,\xi)$. thus one more relation connecting these variables is needed to solve these equations. In order to obtain explicit solutions, one additional relation is needed and we adopt an assumption that the shear scalar is proportional to the scalar of expansion $\sigma \propto \theta$, which leads to

$$\beta = a + b\gamma^{\mu+1} \tag{2.14}$$

where □ is a constant.

Now we consider a = 0 and b = 1, then equation (2.13) and (2.14) reduces to

$$\xi = k\theta^m \tag{2.15}$$

$$\beta = \gamma^{\mu+1} \tag{2.16}$$

Substituting equation (2.16) into Equation (2.4) and using equation (2.15) we have

$$\theta = (2\mu+3)\frac{\dot{\gamma}}{\gamma} \tag{2.17}$$

$$\xi\theta = K\frac{\dot{\gamma}^{m+1}}{\gamma^{m+1}} \tag{2.18}$$

$$K = k(2\mu+3)^{m+1} \tag{2.19}$$

With the help of equations (2.16) and (2.18), equation (2.7) reduces to

$$\frac{\dot{\gamma}}{\gamma} + \frac{(\mu+1)^2}{\mu+2}\frac{\dot{\gamma}^2}{\gamma^2} = \frac{K}{\mu+2}\frac{\dot{\gamma}^{m+1}}{\gamma^{m+1}} \tag{2.20}$$

To solve equation (2.20), we denote $\dot{\gamma} = \theta$, then $\ddot{\gamma} = \theta \frac{d\theta}{d\gamma}$, and the equation (2.20) can be reduced to the first order differential equation in the following form :

$$\frac{d\theta}{d\gamma} + \ell \frac{\theta}{\gamma} = \frac{K}{\mu + 2} \frac{\theta^m}{\gamma^m} \tag{2.21}$$

where

$$\ell = \frac{(\mu+1)^2}{\mu+2} \tag{2.22}$$

Equation (2.21) can be written as (for $m \neq 1$)

$$\frac{d}{d\gamma}\left(\theta^{1-m}\gamma^{(1-m)\ell}\right) = \frac{(1-m)K}{\mu+2}\gamma^{(1-m)\ell - m} \tag{2.23}$$

Thus the solution of equation (2.21) can easily be obtained :

$$\theta = \left[\frac{K\gamma^{1-m}}{(\mu+2)(\ell+1)} + D\gamma^{(m-1)\ell}\right]^{\frac{1}{1-m}} \tag{2.24}$$

Where D is the constant of integration. With the help of equation (2.24), the line-element (5.2.1) reduces to

$$ds^2 = -\left[\frac{K\gamma^{1-m}}{(\mu^2+3\mu+3)} + D\gamma^{(m-1)\ell}\right]^{\frac{-2}{1-m}} d\gamma^2 \tag{2.25}$$

$$+H^2\gamma^{2\mu+2}dx^2 + \gamma^{2\mu+2}e^{2x}dy^2 + \gamma^2 dz^2$$

Under suitable transformation of coordinates, equation (2.25) reduces to

$$ds^2 = -\left[\frac{KJ^{1-m}}{(\mu^2+3\mu+3} + DT^{(m-1)\ell}\right]^{\frac{-2}{1-m}dT^2} \tag{2.26}$$

$$+H^2T^{2\mu+2}dx^2 + T^{2\mu+2}e^{2x}dy^2 + T^2dz^2$$

For the model of equation (2.26), the other physical and geometrical parameters can easily be obtained. The expressions for the energy density □, the string tension density, the particle density □$_p$, the coefficient of bulk viscosity □, the scalar of expression □ and the shear scalar □^{2} are, respectively, given by

$$\rho = (\mu+1)(\mu+3).\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell+1)}\right]^{\frac{2}{1-m}} - \frac{T^{-(2\mu+2)}}{H^2} \tag{2.27}$$

$$\lambda = \frac{\mu}{1-m}.\left[\frac{KT^{1-m}}{(\mu^2+3\mu+3)} + DT^{(m-1)\ell}\right]^{\frac{1+m}{1-m}}. \tag{2.28}$$

$$\left[\frac{(1-m)KT^{-(1+m)}}{(\mu^2+3\mu+3)} + D(m-1)\ell J^{(m-1)\ell-2}\right]$$

$$+\mu(2\mu+2).\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell+1)}\right]^{\frac{2}{1-m}} - \frac{T^{-(2\mu+2)}}{H^2}$$

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$$\rho_p = \frac{-\mu}{1-m}.\left[\frac{KT^{1-m}}{(\mu^2+3\mu+3)} + DT^{(m-1)\ell}\right]^{\frac{1+m}{1-m}} \left[\frac{(1-m)KT^{-(1+m)}}{(\mu^2+3\mu+3)} + D(m-1)LT^{(m-1)\ell-2}\right]$$

(2.29)

$$+(\mu+1)(3-\mu).\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell+1)}\right]^{\frac{2}{1-m}}$$

$$\theta = (2\mu+3).\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell+1)}\right]^{\frac{1}{1-m}}$$

(2.30)

$$\sigma^2 = \frac{\mu^2}{3}.\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell-1)}\right]^{\frac{2}{1-m}}$$

(2.31)

From equation (2.27) it is observed that the standard condition $\rho \geq 0$ is fulfilled when

$$(\mu+1)(\mu+3)\left[\frac{K}{(\mu^2+3\mu+3)} + DT^{(m-1)(\ell-1)}\right]^{\frac{2}{1-m}} \geq \frac{T^{-(2\mu+2)}}{H^2}$$

(2.32)

3. Discussion

It is seen that in the case m < 1, the scalar of expansion □ tends to infinitely large and the energy density $\rho \to \infty$ when $T \to 0$, but □ tends to finite and □ tends to finite when $T \to \infty$ due to the presence of bulk viscosity (in the absence of bulk viscosity $K = 0,\ \theta \to 0$ and $\rho \to 0$ when $T \to \infty$). Hence the model represents the shearing and non-rotating expanding universe with the big-bang start. However, in the case m > 1, it is observed that $\theta \to 0$ when $T \to \infty$, but □ tends to finite when $T \to 0$ due to the presence of bulk viscosity (in the absence of bulk viscosity K = 0, $\theta \to \infty$ when $T \to 0$). Here $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$. Therefore the model describes a shearing non rotating expanding universe without the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of universe [37, 38]. Furthermore, since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of T. The shear scalar □ is zero when □ = 1, hence □ = 1 is the isotropy condition.

4. Summary

In this paper, we study the Bianchi type-III string cosmological model with bulk viscosity. To obtain a determinate solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion $\xi = K\theta^{\alpha+bm}$ and the shear scalar is proportional to scalar of expansion $\sigma \propto \theta$, which leads to the relation between metric potential $\beta = a + b\gamma^{\mu+1}$. The physical and geometric features of the model are also discussed. It is found that the power index m has significant influence on the string model. There is a big-bang start in the model when m $\leq$ 1 but there is not any big-bang in the start when m > 1 [37, 38]. In particular when m = 0 the model reduces to the string model of constant coefficient of bulk viscosity, which was previously given by Bali and Dave [5].

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