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Some Static Charged Fluid Spheres with Spin

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ABSTRACT

The present paper provides solutions of Einstein-Cartan – Maxwell equations for a static spherically symmetric charged fluid sphere by a different method under certain assumptions on the metric potential and total charges Q. We have assumed that the spins of the Individual particles composing the fluid are all aligned in radial direction. We have taken $e^A = (\alpha + \beta r^\mu), Q = Hr^s$ and have obtained three sets of solutions for general value of □.

1. Introduction

Attempts have been made by Trautmen [12, 13] and others (Kuchowicz [17-19], Hehl [1-2], Kopczynsid [3], Prasanna [5-6] to link up the spin with geometry by considering the affinities to be non-symmetric. The basic geometry is thus non-Riemannian and the field equations are obtained from a variational principle where the metric tensor components and affinities are varied independently. The antisymmetric part of the affinity is coupled with the intrinsic spin density of material particles and in particular vanishes in the absence of spin. Prasanna [5]and Singh and Yadav [9] have considered the problem of static fluid spheres in the framework of Einstein-Cartan theory (or E-C theory). A doping Hehl's approach [1-2] to E-C theory, Prasanna [5] has obtained the solutions analogous to solutions of Tolman [11] in general relativity whereas Singh and Yadav [9] have solved them by a different technique. Further attempts have been made to investigate the problem of charged fluid spheres in E-C theory (Nduka 1975, Singh and Yadav [8]). Prasanna [6], Kopczynsid [3] and Raychaudhuri [7] have considered the generalization of Maxwell's equations in space having torsion but this idea leads to a breakdown in the gauge invariance and charge conservation principle. However, Raychoudhuri [7] and Nduka [4] have taken the equations in a form so as to preserve the charge conservation principle. With this formulation Raychoudhuri has investigated the possibility of bounce in the presence of a magnetic field for Bianchi type I universes with p = 0 and p = □. Further Nduka [4] has discussed the charged static fluid spheres in E-C theory and has found that the pressure is discontinuous at the boundary of the fluid sphere.

Some other workers in this line are Yadav et al. [15-16], Singh and Kumar [10] and Sah and Chandra [14].

In this before we have solved the Einstein-Cartan – Maxwell equations for a static spherically symmetric charged fluid sphere by a different method under certain assumptions on the metric potential and total charges Q. We have assumed that the spins of the Individual particles composing the fluid are all aligned in radial direction. We have taken $e^A = (\alpha + \beta r^\mu), Q = Hr^s$ and have obtained three sets of solutions for general value of □. In particular if we put □ = 0 in these solutions, then by suitable adjustement of constants we get the results already reported by Nduka [4]. We have also calculated different physical parameters for the solution.

2. The field equations and their solution

Thus the Einstein – Cartan – Maxwell equations are

$$\text{(2.1)} \quad R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij},$$

$$\text{(2.2)} \quad Q^k_{ij} - \delta^\ell_{j\ell} Q^k_{i\ell} - \delta^k_j Q^\ell_{i\ell} = -8\pi\, S^K_{ij}$$

$$\text{(2.3)} \quad \left[(-g)^{1/2} F^{ij}\right]_{,j} = (-g)^{1/2} J^i,$$

$$\text{(2.4)} \quad F_{[i\ j;k]} = 0$$

Where R_{ij} is the Ricci tensor of the asymmetric connection and also the energy momentum tensor t_{ij} is not symmetric, $F_{\mu\upsilon}$ is the electromagnetic field tensor Q^i_{jk} is to torsion tensor, S^i_{jk} is spin tensor, T_{ij} is energy mome and J□ is current four – vector (we have set c = 1 and the gravitational constant x = 8□).

For a static spherically symmetric system we taken the metric as

$$\text{(2.5)} \quad ds^2 = e^{B(r)} dt^2 - e^{A(r)} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2).$$

For the system under study the energy momentum tensor T^i_j has two parts t^i_j and E^i_j for matter and electromagnetic field respectively (Prassanna [5], Anderson [21]).

(2.6) $T^i_j = t^i_j + E^i_j$

where

(2.7) $t^i_j = \left[(\rho + p)u^i u_j - g^i_j p\right], u^i u_j = 1$

The only non-vanishing components of t^i_j are $t^4_4 = \rho, t^1_1 = t^2_2 = t^3_3 = -p$. Due to spherical symmetry the only non-vanishing components of F^{ij} are $F^{14} = -F^{41}$. By this choice of F^{ij} equation (2.4) is clearly satisfied. It then follows that the non-zero components of F^i_j are

$$E^4_4 = E^1_1 = -E^2_2 = -E^3_3 = -\frac{1}{8\pi} g_{44} g_{11} (F^{41})^2$$

Now we get from equation (2.3)

(2.8) $F^{41} = \dfrac{Q(r)e^{-\frac{(\lambda+\upsilon)}{2}}}{r^2}$

where Q(r) represents total charge contained within a sphere of radius r

$$Q(r) = 4\pi \int_0^r J^4 r^2 e^{\left(\frac{\lambda+\upsilon}{2}\right)} dr$$

From above it is clear that outside the fluid sphere Q(r) is a constant Q_0 which is total charge. It follows from equation (2.8) that the asymptotic form of the electric field is $\dfrac{Q_0}{r^2}$

It then follows from equations (2.1) and (2.6) that the field equations are (Prasanna [5], Hehl [1], [2])

(2.9) $8\pi\rho + 8\pi E^4_4 = 16\pi^2 K^2 + \dfrac{1}{r^2} - e^{-A\left(\frac{1}{r^2} - \frac{A'}{r}\right)}$

(2.10) $8\pi\rho - 8\pi E^1_1 = 16\pi^2 K^2 - \dfrac{1}{r^2} + e^{-\lambda\left(\frac{1}{r^2} + \frac{B'}{r}\right)}$

(2.11) $8\pi\rho - 8\pi E^2_2 = 16\pi^2 K^2 + e^{-\lambda\left(\frac{B''}{2} + \frac{B'^2}{4} - \frac{A'B'}{4} + \frac{B'-A'}{2r}\right)}$

where

(2.12) $K = H_1 e^{-B/2}$

and K is called the spin density of the distribution. Here H_1 is constant of integration and a prime denotes differentiation with respect to r.

It is clear from these equation that it is $\bar{p} = p - 2\pi K^2$ and not the p which is continuous across the boundary r = r_0 of the fluid sphere. The continuity of $\bar{p}$ across the boundary ensures that of $B' \exp(B)$. Further, with $\bar{p}$ and $\bar{\rho} = p - 2\pi K^2$ replacing p and □ respectively we are assured that the metric coefficients are continuous across the boundary. Hence we shall apply the usual boundary conditions to the solutions of equations (2.9), (2.10) and (2.11).

The exterior metric is taken as usual Reissner-Nordstrom line element given by

(2.13) $ds^2 = \left(1 - \dfrac{2M}{r} + \dfrac{Q_0^2}{r^2}\right) dt^2$

$$-\left(1-\frac{2M}{r}+\frac{Q_0^2}{r^2}\right)^{-1} dr^2$$
$$-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

where $Q_0 = Q(r_0)$ and M is the total mass of the fluid sphere. The total mass, as measured by an external observer, inside the fluid sphere of radius r_0 is given by

(2.14) $$M = 4\pi\int_0^{r_0}\bar{\rho}(r)r^2dr$$
$$= 4\pi\int_0^{r_0}\rho(r)r^2dr - 8\pi^2\int_0^{r_0}K^2(r)r^2dr.$$

Thus the total mass of the fluid sphere is modified by the correction

$$8\pi^2\int_0^{r_0}K^2(r)r^2dr$$

Now eliminating p from (2.10) and (2.11) we get

(2.15) $$B'' - \frac{B'^2}{2} - \frac{1}{r}\left(B'+A'\right) - \frac{A'B'}{2}$$
$$+\frac{1}{2r^2}\left(2-32\pi E_1^1 r^2 e^{-\lambda}\right) - \frac{2}{r^2} = 0$$

We choose B such that

(2.16) $$y = e^{B/2}$$

Use of equations (2.8) and (2.16) in (2.15) gives the second order differential equation

(2.17) $$y'' - \left(\frac{1}{r}+\frac{A'}{2}\right)y' + \left(\frac{e^A}{r^2} - \frac{A'}{2r} - \frac{1}{r^2} - \frac{2Q^2e^A}{r^4}\right)y = 0$$

Which is generalization of Wyman's equation [22]

Equation (2.17) contains three unknows y, A and Q(r). For complete determinacy of this set, we should be accompanied with two more conditions. For this purpose we assume.

(2.18) $$A = \log(\alpha+\beta r^\mu)$$

(2.19) $$Q(r) = H\, r^s$$

where $\alpha, \beta H, s$ and μ are constants.

Substitution of (2.18) and (2.19) in (2.17) yields

(2.20) $$y'' - \left[\frac{1}{r} + \frac{\mu\beta r^{\mu-1}}{2(\alpha+\beta r^\mu)}\right]y' + \left[\frac{\alpha+\beta r^\mu}{r^2} - \frac{\mu\beta r^{\mu-2}}{2\left(\alpha+\beta r^\mu\right)} - \frac{1}{r^2}\right.$$
$$\left. -2H^2r^{2s-4}(\alpha+\beta r^\mu)\right]y = 0$$

3. Solution of the field equations

Equation (2.20) is a second order differential equation in y for the general value of μ, β, H, α and s. Giving different values to these constants it is possible to obtain a number of solutions from this equation, some of which are already known. The whole classes of solutions can be divided into the following cases :

Case (i) : Solutions corresponding to uncharged cases i.e. when H = 0

Case (ii) : Solution corresponding to charged cases i.e. when $H \neq 0$

Here we shall confine ourselves to the case (ii) in which charge is present

Case I : He we take $s = 1,\ \alpha = 1,\ \beta = 0$

Then (2.20) yields

(3.1) $r^2y'' - ry' - 2H^2y = 0$

This is Euler's homogeneous equation. To solve it there are three possible cases (Ritger and Rosa [20], viz. depending upon the value of determinant $\Delta = 2\psi^2 + 1$according as

(i) $\Delta > 0$, (ii) $\Delta = 0$ (iii) $\Delta < 0$

or (i) $2H^2 + 1 > 0$ (ii) $2H^2 + 1 = 0$, (iii) $2H^2 + 1 < 0$

But $H^2 < 0$ is not physically significant and so in what follows we shall take $H^2 > 0$. Thus the solution of equation (3.1) is

(3.2) $y = C' r^{1+\eta} + D' r^{1-\eta}$

Where C' and D' are constants and

(3.3) $\eta = \sqrt{1 + 2H^2}$

From (3.2) we get metric coefficient B and other metric coefficient A can be obtained from (2.18). The electromagnetic energy tensor is given by

(3.4) $E_1^1 = E_4^4 = -E_2^2 = -E_3^3 = \frac{1}{8\pi}\frac{H^2}{r^2}$

Through equation (2.9) and (2.10) pressure and density can be obtained in a straight forward manner

(3.5) $8\pi r^2 \rho(r) = 16\pi^2 K^2 - H^2$

(3.6) $8\pi r^2 p(r) = 2\left[1 + \eta(C'r^{2n} - D')\right] / (C'r^{2\eta} + D') + H^2 + 16\pi^2 K^2$

The spin density K is given by

(3.7) $K = H_1\left(C'r^{1-\eta} + D'r^{1-\eta}\right)^{-1}$

Using the boundary conditions discussed in section (2). the constant $C^1_; D'$ are given by

(3.8) $C' = \frac{1}{2\eta}\left(1 - 2\zeta + \eta\zeta - \frac{Q_0^2}{r_0^2}\right) r_0^{-(1+\eta)}$

(3.9) $D' = -\frac{1}{2\eta}\left(1 - 2\zeta - \eta\zeta - \frac{Q_0^2}{r_0^2}\right) r_0^{-(1-\eta)}$

where

$$\zeta = 1 - \frac{2m}{r_0} + \frac{Q_0^2}{r_0^2}$$

Also H_1 is determined from

$$8\pi\rho(r_0) = \frac{16\pi^2 H_1^2}{r_0^2}\left(C'r_0^{1+\eta} + D'r_0^{1-\eta}\right)^{-2} - \frac{H^2}{r_0^2}$$

From which we get

(3.10) $H_1^2 = \frac{(C'r_0^{1+\eta} + D'r_0^{1+\eta})^2}{16\pi^2}\left(8\pi\rho(r_0)r_0^2 + H^2\right)$

Also

(3.11) $H = \frac{Q_0}{r_0}$

Case II : Here we take s = 1, □ = 0, $H^2 = \frac{1}{2}$

Then equation (2.20) yields

$$y''-\left(\frac{1}{r}+\frac{\mu}{2r}\right)y'-\left(\frac{\mu}{2r^2}+\frac{1}{r^2}\right)y=0 \tag{3.12}$$

Which can be written s

$$r^2y''-\left(\frac{\mu}{2}+1\right)(ry'+y)=0 \tag{3.13}$$

Which can be put to the form of Euters equation

$$r^2y''+pry'+qy=0$$

$$\left[\text{Here } p=q=-\left(\frac{\mu}{2}+1\right)\right]$$

As in case I, (3.13) can be solved and the metric function B(r) is obtained and equation (2.18) provides other metric function A(r). As in case I, to solve (3.13), there are three possible cases :

Sub case (i) $\Delta=(p-1)^2-4q=\mu^2+16\mu+32>0$

The solution in this case is

$$y=a_1\,r^{1+\frac{1}{4}(\mu+N)}+b_1r^{1+\frac{1}{4}(\mu-N)},e^A=\beta_1r^{\mu} \tag{3.14}$$

Where as a_1 and $\square_1$ are constants to be fixed by boundary conditions and

$$N=\sqrt{\mu^2+16\mu+32} \tag{3.15}$$

Now the metric functions A and B are known. The electromagnetic energy tensor is given by

$$8\pi E_1^1=8\pi E_4^4=-8\pi\ E_2^2=-8\pi\ E_3^3=\frac{1}{2r^2} \tag{3.16}$$

Hence from equations (2.12) and (2.13) we get □ and p as

$$8\pi\rho=16\pi^2K^2+\frac{1}{\beta_1r^2}\left[(\mu-1)r^{-\mu}+\frac{\beta_1}{2}\right] \tag{3.17}$$

$$8\pi p=16\pi^2K^2-\frac{2}{\beta_1r^{\mu+2}}\left[\frac{a_1\left\{1+\frac{1}{4}(\mu+N)\right\}r^{\frac{(\mu+N)}{4}}}{a_1r^{1+\frac{1}{4}(\mu+N)+b_1}}\right. \tag{3.18}$$

$$\left.+b_1\left\{1+\frac{1}{4}(\mu-N)\right\}r^{\frac{(\mu-N)}{4}}+\frac{1}{2}-\frac{\beta_1r^{\mu}}{4}\right]$$

The spin density K is given by

$$K=H_1e^{-\beta/2}=H_1\left[a_1r^{1+\frac{1}{4}(\mu+N)}+b_1r^{1+\frac{1}{4}(\mu-N}\right]^{-1} \tag{3.19}$$

using the boundary conditions, the constants a_1, b_1 and $\square_1$ are found to be

$$\beta_1=r_0^{-\mu}\zeta^{-1} \tag{3.20}$$

$$a_1=\left[1-\zeta-2\left(1+\frac{(M-N)}{4}\right)\zeta-\frac{Q_0^2}{r_0^2}\right]\frac{r_0^{-[1+\frac{(\mu+N)}{4}]}}{N\sqrt{\zeta}} \tag{3.21}$$

$$b_1=-\left[1-\zeta-2\left\{1+\frac{(M+N}{4}\right\}\zeta-\frac{Q_0^2}{r_0^2}\right]\frac{r_0^{-[1+\frac{(\mu+N)}{4}]}}{N\sqrt{\zeta}} \tag{3.22}$$

with $\zeta = 1 - \frac{2M}{r_0} + \frac{Q_0^2}{r_0^2}$

Also, H_1 is determined from

$$8\pi\rho(r_0) = 16\pi^2 H_1^2 \left[a_1 r_0^{1+\frac{1}{4}(\mu+N)} + b_1 r_0^{1+\frac{1}{4}(\mu-N)} \right]^{-2} + \frac{1}{p_1 r_0^2}\left[(\mu-1) r_0^{-\mu} + \frac{\beta_1}{2} \right] \tag{3.23}$$

Sub case (ii) : when $\Delta = 0 \text{ i.e. } \mu^2 + 16\mu + 32 = 0$

which $\Rightarrow (\mu + 8 + 4\sqrt{2})(\mu + 8 - 4\sqrt{2}) = 0$

Therefore $\mu = -4(2 \pm \sqrt{2})$

Solution can be obtained as in previous case

In addition, the conditions p > 0 and □ > 0 will impose further restrictions on our solutions. We therefore restrict our solutions to only those values of constants for which the pressure and density are positive.

By putting □ = 0 and by suitable adjustment of constants, some of these results coincide with those of Nduka [4]. Thus our solutions may be considered as generalization of those already reported by Nduka [4].

References

1. Hehl, F.W. (1973) : Gen. Rel. Grav., 4, 333.
2. Hehl, F.W. (1974) : Gen. Rel. Grav., 5, 491.
3. Kopczynski, W. (1975) : Scripts Fac. Sci. Net. Ujep Brunensis Physica, 3-4, 5, 255.
4. Nduka, A. (1977) : Gen. Rel. Grav., 8, 371.
5. Prasanna, A.R. (1975 a) : Phys. Rev., D. 11, 2076.
6. Prasanna, A.R. (1975 b) : Phys. Rev., D. 11, 2083.
7. Raychaudhuri, A.K. (1975) : Phys. Rev., D 12, 952.
8. Singh, T. and Yadav, R.B.S. (1978) : Acta Phys., Pol. B9, 831.
9. Singh, T. and Yadav, R.B.S. (1978) : Acta Phys. Pol., B9, 837.
10. Singh, S. and Kumar, M. (2019), Res. Rev., J, 4, p. 1684-88.
11. Tolman, R.C. (1939) : Phys., Rev., 55, 364.
12. Trautman, A. (1973a) : Nature (Phsy., Sci.,) 242, 7.
13. Trautman, A. (1973 b): Symposium Mathematics, 12, 139.
14. Sah, A. and Chandra, P. (2016), Int. J. Astronomy Astrophys. 6, 494.
15. Yadav, R.B.S. and Kushwaha P.N.P. (2020), S.I.P.N., 40, p. 2250-2266.
16. Yadav, R.B.S. and Prasad, U. (1993) Astrophys. Space Sci., 203, p. 37-42.
17. Kuchowicz, B. (1975 a): Acta Phys., Pol., B6, 173.
18. Kuchowicz, B. (1975 b) : Acta Phys., Pol., B6, 555.
19. Kuchowicz, B. (1975 c) : Astro. Phys., Space Sci., D13, 33.
20. Ritger, P.D. and Rose, N.J. (1968) : Differantial Equations with applications, Mc Graw-Hill, New York, p. 144.
21. Anderson, J.L. (1967) : Principles of Relativity Physics, Academic Press, New York, Chap. 11.
22. Wyman, M. (1949) : Phys. Rev., 75, 1930.