RESEARCH REVIEW JOURNALS
Volume-03
Issue-08
August-2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

On Viscous Fluid Bianchi Type-IX Cosmological Models

[1]Rahul Raj and [2]Dr. R.N. Roy

[1]Research Scholar, Dept. of Mathematics, J.P. University, Chapra
[2]Dept. of Mathematics, J.P. University, Chapra

ARTICLE DETAILS

Article History
Published Online: 07 August 2018

Keywords
Viscous, metric, bulk viscosity, shear, expansion.

ABSTRACT

Considering Bianchi type-IX viscous fluid cosmological model we have found solution by assuming suitable relations between metric potentials α, β and $\eta \propto \theta$ and ξ is constant. Various physical and geometrical properties have been also discussed.

1. Introduction

Bianchi Type –IX Cosmological models are interesting because these models allow not only expansion but also rotation and shear and in general are anisotropic. Many relativists have been taken interest in studying Bianchi type-IX universe because familiar solutions live Robertson walker universe with positive curvature the de-sitter universe, the Taub – NUT solutions e.t.c. are Bianch type-IX space-time. In these models, neutrino viscosity does not guarantee isotropy at the present epoch. Viscosity is important in cosmology for a number of reason. Krori et al. [5] and Wang [9, 10] studied the exact solutions of string cosmology for Bianchi type-II, VI_0, VIII and IX space – times. Pradhan et al. [8] have investigated the generation of Bianchi type – V cosmological models with varying ^ term.

Bali et. al. [3, 4] have studied Bianchi type – IX viscous tluid cosmological models in general relativity. Robertson walker cosmological models with bulk viscosity and equation of state $P = (\gamma - 1)\rho, 0 \leq \gamma \leq 2$ is investiaged by Mohanty and Pradhan [7]. Some other workers in this field are Bali [5] and Banerjee et. al. [1].

Here in this paper we have considered Bianchi Type – IX viscous fluid cosmological model in General Relativity. To get a deterministic model we have assumed suitable relations between metric potential □, □ and $\eta\alpha\theta$ where □ is the co-efficient of shear viscosity and □ is the scalar of expansion in the models. Further we have taken co-efficient of bulk viscosity (ζ) to be constant various physical geometrical properties of the model have been found.

2. The field equations

Here we take Bianchi Type – IX line element in the form given by

(2.1) $$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 dy^2 + (\beta^2 \sin^2 y + a^2 \cos^2 y)$$
$$dz^2 - 2\alpha^2 \cos y \; dx \; dz$$

Where □ and □ are functions of t alone

The energy momentum tensor T_μ^v for viscous fluid distribution is given by landau and Lifshitz [6]

(2.2) $$T_\mu^v = (\rho + p)u_\mu u^v + pg_\mu^v - \eta\left(u_\mu^v + u_{;\mu}^v + u^v u^\ell u_{\mu,\ell} + u_\mu u^\ell u_\ell^v\right)$$
$$-\left(\zeta - \frac{2}{3}\eta\right)u_{;\ell}^\ell\left(g_\mu^v + u_\mu u^v\right)$$

Here p is the isotropic pressure, □ the density, η and ζ are the co-efficient of viscosity, u□ the flow vector satistying

(2.3) $$g_{\mu v}u^\mu u^v = -1$$

Here we use co-moving co-ordinates, so that

(2.4) $$u^1 = 0 = u^2 = u^3 \text{ and } u^4 = 1$$

The Einstein's field equation

(2.5) $R_\mu^v - \frac{1}{2}Rg_\mu^v + \wedge g_\mu^v = -8\pi T_\mu^v$ (C = 1, G = 1 in gravitation unit) For the line element (2.1) Leads to

(2.6) $\left[\frac{2\beta_{44}}{\beta}+\frac{\beta^2{}_{44}}{\beta^2}+\frac{1}{\beta_2}-\frac{3\alpha^2}{4\beta^4}+\wedge\right]=-8\pi\left[p-2\eta\frac{\alpha_4}{\alpha}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right]$

(2.7) $\left[\frac{\alpha_{44}}{\alpha}+\frac{\alpha_4\beta_4}{\alpha\beta}+\frac{\beta_{44}}{\beta}+\frac{\alpha^2}{4\beta^2}+\wedge\right]=-8\pi\left[p-2\eta\frac{\beta_4}{\beta}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right]$

(2.8) $\left[\frac{2\alpha_4\beta_4}{\alpha\beta}+\frac{\beta_4^2}{\beta^2}-\frac{\alpha^2}{4\beta^4}+\frac{1}{\beta^2}+\wedge\right]=8\pi\rho$

Here 4 over □ and □ denotes ordinary differentiation with respect to t and expansion □ is given by

(2.9) $\theta=u^{\mu}_{;\mu}$

3. Solution of the field equations

Equations (2.6) – (2.8) are the three equations in Six unknowns $\alpha,\beta,\rho,p\zeta$ and η. To make the system determinate we need three more relations or conditions. For this we firstly assume co-efficients of shearviscouty □ divectly proportional to expansion □ and co-efficient of bulk viscosity ζ to be constant i.e.

(3.1) $\eta\alpha\theta$ and

(3.2) $\zeta=\text{constant}$

We further assume suitable relations between metric co-efficients □ and □ to solve the field equations.
Here we take

(3.3) $\alpha=\lambda\sqrt{\beta}$

Now equation (2.7)and (2.8) provide

(3.4) $\left[\frac{\beta_{44}}{\beta}+\frac{\beta_4^2}{\beta^2}-\frac{\alpha_4\beta_4}{\alpha\beta}-\frac{\alpha^2}{\beta^4}+\frac{1}{\beta^2}\right]=16\pi\eta\left(\frac{\alpha_4}{\alpha}-\frac{\beta_4}{\beta}\right)$

Conditions (3.1) and (3.2) leads to

(3.5) $\eta=H\left(\frac{\alpha_4}{\alpha}+\frac{2\beta_4}{\beta}\right)$

Where H is Constant
Equation (3.1), (3.4) and (3.5) yield

(2.6) $\beta\beta_{44}+D\beta_4^2=2\beta^{-1}-2$

Here we have taken □ = 1 to avoid mathematical complexity

and

(2.7) $D=\frac{3}{2}+40\pi H$

Equation (3.6) gives

(3.8) $\frac{d}{d\beta}(\psi^2)+\frac{2D}{\beta}(\psi^2)=\frac{4}{\beta^2}-\frac{4}{\beta}=\frac{4}{\beta^2}(1-\beta)$

(3.9) $\beta_4=\psi(\beta)$

Equation (3.8) fields.

(3.10) $(\beta\beta_4)^2=\frac{4\beta}{2D-1}+\frac{L}{\beta^2(D-1)}-\frac{2\beta^2}{D}$

Now metric (2.1) goes to the form

(3.11) $ds^2=-\left(\frac{dt}{d\beta}\right)^2+\beta dx^2+\beta^2dy^2+(\beta^2\sin^2y+\beta\cos^2y)dz^2$

$-2\sqrt{\beta}\cos y\,dx\,dz$

Use of (3.10) reduces (3.11) to the form

(3.12) $$ds^2 = \frac{dT^2}{4T^{-1}L \qquad 2} + Tdx^2 + T^2dy$$

$$2D - 1^{+}T^{2D} - 2D$$

$$+(T^2\sin^2 y + T^2\cos^2 Y)dz^2 - 2T\cos y\, dx\, dz$$

Where we have used

$$\beta = T, x = X, y = Y, z = Z$$

4. Physical and geometrical aspect

Pressure and density for the model (3.12) are found to be

(4.1) $$8\pi p = \frac{320\pi H - 6D + 27}{12(2D-1)T^3} + \frac{L(80\pi H + 18D - 3)}{12T^{2(D+4)}} + \frac{3 - 80\pi H}{6DT^2}$$

$$+20\pi\zeta\left(\frac{4}{(2D-1)t^3} + \frac{L}{T^{2(D+1)}} - \frac{2}{DT^2}\right)^{1/2} - \wedge$$

And

(4.2) $$8\pi\rho = \frac{33 - 2D}{(2D-1).4T^3} + \frac{2L}{T^{2(D+1)}} + \frac{D-4}{DT^2} + \wedge$$

The energy condition given by Ellis [11] are

(i) (□ + p) > 0 and (ii) (□ + 3p) > 0

The condition (i) Leads to

(4.3) $$\left[\frac{63 - 6D + 160\pi H}{6(2D-1)T^3} + \frac{6D - 80\pi H - 21}{6DT^2} + \frac{L(80\pi H + 18D + 21)}{12T^{2(D+1)}}\right.$$

$$\left. +20\pi\zeta\left(\frac{4}{(2D-1)T^3} + \frac{1}{T^{2(D+1)}} - \frac{2}{DT^2}\right)^{1/2}\right] > 0$$

And the condition (ii) leads to

(4.4) $$\left[\frac{80\pi H + 15 - 2D}{(2D-1)T^3} + \frac{L}{4}\frac{(80\pi H + 18D + 5)}{T^{2(D+1)}} + \frac{2D - 80\pi H - 5}{2DT^2}\right.$$

$$\left. +60\pi\zeta\left(\frac{4}{(2D-1)T^3} + \frac{L}{T^{2(D+1)}}\frac{2}{DT^2}\right)^{1/2}\right] > 2\wedge$$

Which puts restriction on $\wedge$. Thus energy condition (i) □ + p > 0 (ii) □ + 3p > 0 are satisfied.

The expansion (□) and shear (□) in the model 5.3.12 are given by

(4.5) $$\theta = \frac{5}{2}\left[\frac{4}{(2D-1)T^3} + \frac{L}{T^{2(D+1)}}\left(1 - \frac{2T^{2D}}{LD}\right)\right]^{1/2}$$

(4.6) $$\theta = \frac{1}{\sqrt{6}}\left[\frac{4}{(2D-1)+3} + \frac{1}{T^{2(D-1)}}\left(1 - \frac{2T^{2D}}{LD}\right)\right]^{1/2}$$

To the absence of viscosity i.e. when H $\longrightarrow$ 0 the metric vedues to

(4.7) $$ds^2 = \frac{-3dT^2}{6T^{-1} + 3LT^{3-4}} + Tdx^2 + T^2dy^2$$

$$+(T^2\sin^2 y + T\cos^2 y)dz^2 - 2T\cos y dx\, dz$$

From (2.6) we get

(4.8) $D = 40\pi H + \frac{3}{2}$

From the above equation we find that in the absence of viscosity $D = \frac{3}{2}$ as H = 0. The expression for pressure, density, expansion (□) and (□) are given by

(4.9) $8\pi p = \frac{3}{4T^3} + \frac{2L}{T^3} - \frac{1}{3T^2} - \wedge$

(4.10) $8\pi\rho = \frac{15}{4T^3} + \frac{2L}{T^3} - \frac{5}{3T^2} + \wedge$

(4.11) $\theta = \frac{5}{2}\left[\frac{2}{T^3} + \frac{L}{T^5} + \frac{4}{3T^2}\right]^{1/2}$

(4.12) $\sigma = \frac{1}{\sqrt{6}}\left[\frac{2}{T^3} + \frac{L}{T^3} + \frac{4}{3T^2}\right]^{1/2}$

The energy condition (i) □ + p > 0 and (ii) □ + 3p > 0 leads to

(4.13) $\frac{1}{T^3} + \frac{2L}{T^5} + \frac{1}{T^2} > 0$

(4.14) $\frac{16L}{T^5} - \frac{5}{T^3} - \frac{16}{3T^2} > 4\wedge$

5. Discussion

Which gives Condition on $\wedge$. In the absence of viscosity. The expansion in the model starts with a big bang at T = 0 and the expansion in the model decreases with time when T $\rightarrow$ 0 then $\rho \rightarrow \infty, p \rightarrow \infty$ and when $T \rightarrow \infty$ then $\rho - \wedge$ since lim $T \rightarrow \infty \left(\frac{\sigma}{\theta}\right) \neq 0$. Hence the model does not approach isotropy for large values of T in the absence of viscosity in general.

Reference

1. Banerjee, A. Sanyal AK and chkraborty S, (1990) pramana-J. Phys. 34.
2. Bali R. Electron. J. (2008) Theor. Phys. 5, 105.
3. Bali R, and Anjali 2006. Astro phys. Space. Sci 302, 201.
4. Bali R, and yadav M.K. (2005) pramana J. Phys. 64, 187.
5. Krori KD, Chodhury T. Mahanta CR and Mazumder a, Gen. Relativ.
6. Landan, L.D. and lifshitz, E.M. (1963) Fluid mechanics 6, 503.
7 Mohanty G and Pradhan, B.D. (1991) Astrophy, space-sci, 181, 1 (1991)
8. Pradhan A, Yadav AK and Yadav L, (2005) Cz ech. J. Phys, 55, 503.
9. Wang xx, 2003 Chin phys. Lett. 20, 615.
10. Wang xx, 2003 Chin. Phys. Lett., 20, 1205.