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A Study of Densities Probability of Mellin Transforms in Statistics

Mohit Kumar

MDU Rohtak, Haryana

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ABSTRACT

The Mellin basic transform is outstanding for finding the probability densities for sums and contrasts of random variables. We utilize the Mellin essential transforms to infer various properties in statistics and probability densities of single persistent random variable. We likewise talk about the connection between the Laplace and Mellin essential transforms and utilization of these indispensable transforms in inferring densities for algebraic mix of random variables. The utilization of the Mellin transform to assess the convolution fundamental for the pdf of an item is less outstanding, yet similarly direct, from a certain point of view. Practically speaking, however the utilization of Mellin or Laplace transforms for sums of random variables is generally utilized and clarified in each propelled statistics message, the Mellin transform remain.

1. Introduction

The study of this paper, we characterize a random variable (RV) as an incentive in some space , state $\Re$, speaking to the result of a procedure dependent on a probability laws .By the data over the probability dispersion ,we coordinate the probability thickness function (p d f)in the instance of the Gaussian , the p d f is

$$f(x)= \frac{1}{\sigma\sqrt{2\pi}} E^{-\frac{1}{2}(\frac{X-\pi}{\sigma})^2}, -\infty < x < \infty$$

when µ is mean and σ^2 is the variance.

We characterize the p d f for X+Y and XY ,where X and Y are the R Vs, by utilizing the short foundation on probability hypothesis and see the convolution by utilizing Laplace – Mellin basic transforms.

In this paper we characterize Mellin vital transform , persistent random variable for X and its p d fs , constant conveyance function, desires and minutes about root and mean for autonomous CRVs X , mode , middle , quartiles , deciles , percentiles , slant ness (by utilizing mean and mode and furthermore by utilizing quartiles) , kurtosis (by utilizing minutes) : Transform for the whole of the random variables, Convolution Algebra on ( ) 1 L $\Re$,: The Mellin indispensable transform, and connection with Laplace basic transform connection in the middle of expected qualities and snapshots of CRVs X ,one dimensional nonstop random variable and its p d fs , minor thickness functions, hypotheses of expansion and augmentation of CRVs X and Y, relations in the middle of expected estimations of CRVs X and Y and Mellin vital transform.

2. Mellin Transforms

The Mellin transforms have been utilized all things considered in numerous fields independently. On consolidating transforms for example Mellin transforms is gotten. Mellin transforms are utilized for fathoming differential and essential conditions. The algebraic properties of Mellin transforms are (quickly) turned out in a progression of activities in [8]. For the more algebraically slanted, one can build up a dynamic hypothesis of convolution and Mellin investigation on gatherings. See [7], for a full treatment and all spread the Mellin transform, the last two in the probability setting [3]. contains a broad table of Mellin transforms. A progressively dynamic view is given by [10], which incorporates a treatment of essential transforms of (Schwartz) dispersions [5]. contains an extremely complete treatment of properties of the Mellin transform, with proofs. In any case, here in this paper we discover the relationship of Infinite Mellin transforms and how Mellin transform can be gotten from transform for some function which is useful for comprehending differential conditions with complex applications.

Significant and intriguing as those refered to references may be, we realize that the Mellin transform has a characteristic complex structure and is (more) normally characterized on R (instead of R+).Here we consider [Q] and utilize the beneath criteria

(a) The integral transform is unitary.

(b) The integral kernel is Mellin.

(a) And (b) are understood by well-known Mellin -transform properties:

$$\hat{f}(x) = F(f)(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ixy} f(y)dy$$

$$f(x) = F(\hat{f})(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ixy} \hat{f}(y)dy$$

so that we have $FF = id,$ where *id* denotes the identity operator, and the kernel is $k(xy) = e^{ixy}/\sqrt{2\pi}$ Thus, the Mellin transform itself is unitary, with a kernel $k(xy),$ satisfying both (a) and (b) above.

A characterizing condition on *k(xy)* in terms of the Mellin transform has been obtained in

$$\tilde{k}(s)\tilde{k}(1-s) = 1$$

where $\tilde{k}(s) = \int_{R_+} x^{s-1}\, k(x)dx, Re(s) > 0$

Interrelation of Mellin transform

The Mellin transform of *f* with argument $s \in C$ is defined as

$$F(s) = M[f]s = \int_0^{\infty} f(u)u^{s-1}ds$$

where $a \leq Re(s) \leq b$ (if 1 converges for $s = c \in R,$ then it can be easily shown that it converges for $s = c + it, t \in R)$ For $(s) = M[f]s,$ the inverse Mellin transform is

$$F(x) = M^{-1}\{M[f]\}x = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)\, x^{-s} ds$$

For the existence of inverse the condition is that $F(s)x^{-s}$ is analytic in a strip $(a, b) \times (-i\infty, i\infty)$ such that $c \in (a, b)$ In many engineering texts, statistics, the Mellin transform is defined as

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i\xi x}dx$$

Using the transform *T* and substituting $\xi = -(\frac{\eta - c}{2\pi i})$ for all real $c \geq 0$

$$\hat{T}\hat{f}(\xi) = \int_{-\infty}^{\infty} f(e^x)e^{2\pi i(\frac{\eta-c}{2\pi i})\, x}dx$$

On substituting $y = e^x$ we get

$$\hat{T}\hat{f}\left(-\frac{\eta - c}{2\pi i}\right) = \int_{-\infty}^{\infty} f(e^x)e^{(\eta-c)\log y}\frac{1}{y}dy$$

$$= \int_{-\infty}^{\infty} f(y)y^{-c}y^{\eta}\, \frac{1}{y}dy$$

$$= \int_{-\infty}^{\infty} f^*(y)y^{\eta-1}\, dy \;\; for\; f^*(y) = f(y)y^{-c}$$

(An aside on the substitution $\xi = -(\frac{\eta - c}{2\pi i})$. The factor of 2π is a consequence of the way we define the Mellin transform. In statistics, and in many engineering texts, the Mellin transform is defined as $\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{i\xi x}dx$ (basically estimating recurrence in radians per time unit rather than cycles per time unit), which streamlines the induction of the Mellin transform from the Mellin transform. For a synopsis of the various ways the Mellin transform and its converse are spoken to. A similar procedure can be utilized to infer Mellin opposite transform equation as

$$f(y) = \int_{-\infty}^{\infty} \hat{f}(\xi)e^{2\pi i log\,(y)\xi}d\xi$$

On substituting $\xi = -(\frac{\eta - c}{2\pi i})$

$$f(y) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \hat{f}\left(-\frac{\eta - c}{2\pi i}\right)e^{-(\eta-c)\log y}d\eta$$

$$= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \hat{f}\left(-\frac{\eta}{2\pi i}\right)y^{c}y^{-\eta}d\eta$$

$$f(y)y^{-c} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \hat{f}\left(-\frac{\eta}{2\pi i}\right)y^{-\eta}d\eta$$

$$= f^*y$$

In some cases the transformation T provides an easier way to invert Mellin transforms, by using Fourier inversion techniques.

3. Preliminary Results

The Mellin integral transform of the function f(x) with its kernel x^{r-1} and r>0 is the parameter , is denoted by M[f(x),r] and defined as

$$M\,[f(x), r] = \int_0^\infty x^{r-1} f(x)dx , 0<x<\infty , r>0,$$

At whatever point this essential is exist. This is a significant basic transform, whose utilization in statistics is identified with recuperating of probability appropriations, item or remainder of autonomous nonnegative nonstop random variables.

One of the principle issues emerging in the applications is that of upsetting Mellin necessary transform, I. e. the assurance of the first function f(x) from the transform

M [f(x), r] .This problem, formally solved by the inverse formula

$$f(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c\_i\infty} x^{-r} M[f(x)mr]dr$$

cannot be solved ,in most of cases, analytically. Therefore the only possible of inverting the transformed functions is by numerical means.

Continuous Random Variable

On the off chance that X is a random variable which can take all qualities (I .e. endless number of qualities)in an interim , then X is known as a persistent random variable.

Its probability dissemination is known as constant probability dispersion . On the off chance that function f(x) is said to be a function of persistent random variable X ,in the event that it fulfills the accompanying conditions

$$(1)\ f(x) \geq 0 \quad \text{and} \quad (2) \int_{-\infty}^{\infty} f(x)dx = 1$$

Probability Density Function of Continuous Random Variable

A continuous function y=f(x) such that

(1) f (x) is integrable

$$(2) \int_a^b f(x)dx = 1 \text{ if X lies in [a ,b] and}$$

$$(3) \int_\alpha^\beta f(x)dx = P(\alpha \leq x \leq \beta), \text{ where } a < \alpha < \beta < b$$

probability function of a continuous random variable X.

Thus for a continuous random variable

$$\int_\alpha^\beta f(x)dx = P(\alpha \leq x \leq \beta)$$

Clearly $\int_\alpha^\beta f(x)dx$ represents the area under the curve f(x) ,the x-axis

and the ordinates x=α and x= β

Continuous Distribution Function

Probability distribution of X or the probability density function of X helps os to find the probability that X will be within a given interval [a ,b] i.e

$$P(a \leq x \leq b) - \int_a^b f(x)dx,$$

other conditions being satisfied.

If X is a continuous random variable, having the probability density function f(x) then the function

$$P(x) = P(X \leq x) = \int_{-\infty}^{\infty} f(t)dt$$

$$P(x) = P(X \leq x) = \int_{-\infty}^{\infty} f(x)dx \qquad , -\infty \leq x \leq \infty$$

is called distribution function or cumulative distribution function of the continuous random variable X.

Expectation and Moments about origin by using MIT DMIT

The expectation of continuous random variable X is denoted by E[X] and defined as

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

For the Mellin Integral Transform the Probability Density Function is

$$f(x)=\frac{1}{\Gamma(r)}x^{r-1}e^{-x}, \; r>0$$

where x^{r-1} is the Mellin kernel. Then

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

For the Mellin Transform

$$\int_{0}^{\infty} f(x)dx = \int_{0}^{\infty} \frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^{r-1}e^{-x}dx$$

$$= \frac{\Gamma(r)}{\Gamma(r)} = 1, \text{ where } \int_{-\infty}^{\infty} f(x)dx = 1$$

If $f(x) = \frac{1}{\Gamma(r)}e^{-x}$ is the function of continuous random variable X, then

$$M[f(x),s] = \int_{0}^{\infty} x^{r-1}f(x) = \int_{0}^{\infty}\frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^{r-1}e^{-x}dx$$

$$= \frac{\Gamma(r)}{\Gamma(r)} = 1 = \mu'_{x0}$$

M [f(x), s] =1, is the moment about origin , denoted by μ'_0, $\mu'_0=1$

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x^{r-1}\frac{1}{\Gamma(r)}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{-\infty}^{\infty} x^{r}e^{-x}dx$$

$$= \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^{r+1-1}e^{-x} = \frac{1}{\Gamma(r)}\Gamma(r+1) = \frac{(r+1)\Gamma(r)}{\Gamma(r)} = (r+1) = \mu'_1$$

$$E[X] = \mu'_1 = (r+1)$$

$$E[X^2] = \int_{0}^{\infty} x^2 f(x)dx = \int_{0}^{\infty} x^2\frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^2 x^{r-1}e^{-x}dx$$

$$= \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^{r+1}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^{r+2-1}e^{-x}dx$$

$$= \frac{\Gamma(r+2)}{\Gamma(r)} = \frac{(r+1)(r+2)\Gamma(r)}{\Gamma(r)} = (r+1)(r+2) = \mu'_2$$

$$E[X^2] = \mu'_2 = (r+1)(r+2)$$

$$E[X^3] = \int_{0}^{\infty} x^3 f(x)dx = \int_{0}^{\infty} x^3\frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx = \frac{1}{\Gamma(r)}\int_{0}^{\infty} x^3 x^{r-1}e^{-x}dx$$

$$=\frac{1}{\Gamma(r)}\int_0^\infty x^3 x^{r+3-1} e^{-x}dx$$

$$=\frac{\Gamma(r+3)}{\Gamma(r)}=\frac{(r+1)(r+2)(r+3)\Gamma(r)}{\Gamma(r)}=(r+1)(r+2)(r+3)=\mu_3^{'}$$

$$E[X^3]=\mu_3^{'}=(r+1)(r+2)(r+3)$$

$$E[X^4]=\int_0^\infty x^4 f(x)dx=\int_0^\infty x^4\frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx\ \frac{1}{\Gamma(r)}\int_0^\infty x^4 x^{r-1}e^{-x}dx$$

$$=\frac{1}{\Gamma(r)}\int_0^\infty x^{r+4-1}e^{-x}dx$$

$$=\frac{\Gamma(r+4)}{\Gamma(r)}=\frac{(r+1)(r+2)(r+3)(r+4)\Gamma(r)}{\Gamma(r)}$$

$$=(r+1)(r+2)(r+3)(r+4)=\mu_4^{'}$$

$$E[X^4]=\mu_4^{'}=(r+1)(r+2)(r+3)(r+4)$$

The Mellin Integral Transform and Moments (Moments about origin)

If X is a cintineous random variable, then the expectations are as follows

$$1:\ E[x^{r-1}]=\int_0^\infty x^{r-1}f(x)dx=M[f(x),r]=\mu_{x0}^{'}=1$$

$$2:\ E[x^{r}]=\int_0^\infty x^{r}f(x)dx=\int_0^\infty x^{r+1-1}f(x)dx=M[f(x),r+1]=\mu_{x1}^{'}=(r+1)$$

$$3:\ E[x^{r+1}]=\int_0^\infty x^{r+1}f(x)dx=\int_0^\infty x^{r+2-1}f(x)dx=M[f(x),r+2]=\mu_{x2}^{'}=(r+1)(r+2)$$

$$4:\ E[x^{r+2}]=\int_0^\infty x^{r+2}f(x)dx=\int_0^\infty x^{r+3-1}f(x)dx=M[f(x),r+3]=\mu_{x3}^{'}=(s+1)(s+2)(s+3)$$

$$5:\ E[x^{r+3}]=\int_0^\infty x^{r+3}f(x)dx=\int_0^\infty x^{r+4-1}f(x)dx=M[f(x),r+4]$$

$$=\mu_{x4}^{'}=(s+1)(s+2)(s+3)(s+4)$$

If $f(x) = \frac{1}{\Gamma(s)}$ is the function of continuous random variable Y , then

6: $E[Y^{r-1}] = \int_0^\infty y^{s-1} f(y)dy$ =M[f(y),r]= $\mu'_{y0} = 1$

7: $E[Y^r] = \int_0^\infty y^s f(y)dy = \int_0^\infty x^{s+1-1} f(x)dx$ =M[f(y),s+1]= μ'_{y1} =(s+1)

8: $E[Y^{r+1}] = \int_0^\infty y^{s+1} f(y)dy = \int_0^\infty y^{s+2-1} f(y)dy$ =M[f(y),s+2] = μ'_{y2} =(s+1)(s+2)

9: $E[Y^{r+2}] = \int_0^\infty y^{s+2} f(y)dy = \int_0^\infty y^{r+3-1} f(y)dy$ =M[f(y),s+3]= μ'_{y3} =(s+1)(s+2)(s+3)

10: $E[Y^{r+3}] = \int_0^\infty y^{s+3} f(y)dy = \int_0^\infty y^{s+4-1} f(y)dy$ =M[f(y),s+4]

$= \mu'_{y4}$ =(s+1)(s+2)(s+3)(s+4)

Moments Abour Mean, Variance, Skewness and Kurtosis

The varience of the random variable X is denoted by V(X) and defined as

1: Varience of X=V(X)

$=E[(X-m)^2] = E[X^2]-(E[X])^2 = \mu_2$

$= \mu_{x2} = \mu'_{x2} - (\mu'_{x1})^2$ $=(r+1)(r+2)-(r+1)^2$ =(r+1)(r+2-r-1)

V(X) =(r+1)= μ_{x2}

The other moments about mean are obtained using relations in between moments about a origin and moments about mean.

2: $\mu_{x3} = \mu'_{x3} - 3\mu'_{x2}\mu'_{x1} + 2(\mu'_{x1})^3$

$=(r+1)(r+2)(r+3)-3(r+1)(r+2)(r+1)+2(r+1)^3$

$=(r+1)[(r+2)(r+3)-3(r+1)(r+2)+2(r+1)^2]$

$=(r+1)(r^2 + 5r + 6 - 3r^2 - 9r - 6 + 2r^2 + 4r + 2)$

μ_{x3} =2(r+1)

3: $\mu_{x4} = \mu'_{x4} - 4\mu'_{x3}\mu'_{x1} + 6\mu'_{x2}(\mu'_{x1})^2 - 3(\mu'_{x1})^4$

$=(r+1)(r+2)(r+3)(r+4)-4(r+1)(r+2)(r+3)(r+1)+6(r+1)(r+2)(r+1)^2-3(r+1)^4$

$=(r+1)[(r+2)(r^2 + 7r + 12 - 4(r+2)(r^2 + 4r + 3) + 6(r+2)(r^2 + 2r + 1) - 3(r^3 + 3r^2 + 3r + 1)]$

$=(r+1)(r^3 + 7r^2 + 12r + 2r^2 + 14r + 24 - 4r^3 - 16r^2 - 12r - 8r^2 - 32r - 24$

$+ 6r^3 + 12r^2 + 6r + 12r^2 + 24r + 12 - 3r^3 - 9r^2 - 9r - 3)$

=9(r+1)

μ_{x4} =9(r+1)

Measure of Skewness $(\beta_1 \gamma_1)$

Karl Pearson's characterized the four coefficients dependent on minutes about mean These are utilized to gauge the skewness and kurtosis By utilizing the minutes about mean, we characterize the Karl Pearson's skewness and as follows skewness $=\gamma_1=\sqrt{\beta_1}=\sqrt{\frac{\mu_3^2}{\mu_2^3}}=\sqrt{\frac{8(r+1)^2}{(r+1)^3}}=\sqrt{\frac{8}{r+1}}$

If β_1 =0 , the distribution is symmetric

If β_1 <0 , the distribution is negative skew

If β_1 >0 , the distribution is positive skew

For r= 1 to n , β_1 >0 ,the distribution is positive skew for Mellin integral transform.

Measure of Kurtosis (β_2,γ_2)

Kurtosis= $\gamma_2=\beta_2-3=\frac{\mu_3}{\mu_2^2}$ -3= $\frac{9(r+1)}{(r+1)^2}$ -3= $\frac{9}{r+1}-3$

(1) If γ_2 =0, the distribution is normal or mesokurtosis

for β_2=3 when r=2

If γ_2 >0, the distribution is more peaked or leptokurtosis

for β_2>3 when r=1

If γ_2 <0, the distribution is more flat or platykurtosis

for β_2<3 when $r\geq 3$

Mode

If f(x) be a function of continuous random variable X, then $\frac{dy}{dx}=f'(x)=0$

We get values of X i.e. X_1, X_2 ,…., X_n , and if $[\frac{d^2y}{dx^2}]_{X=X_{11}}$ <0 , then $X=\overline{X}$ is the mode

If $f(x)=\frac{1}{\Gamma(r)}e^{-x}x^{r-1}$ be the contineius function of random variable X, then

$$f'(x)=\frac{1}{\Gamma(r)}[x^{r-1}(-e^{-x})+(r-1)x^{r-2}e^{-x}]$$

$$=\frac{1}{\Gamma(r)}[x^{r-1}(-e^{-x})+(r-1)x^{r-2}e^{-x}]$$

$$=\frac{x^{r-2}e^{-x}}{\Gamma(r)}[-x+(r-1)]$$

$$=\frac{x^{r-2}e^{-x}}{\Gamma(r)}[-x+r-1]$$

f'(x)=0 , then
-x+r-1=0
x=r-1, is the point

$$f''(x)=\frac{-e^{-x}}{\Gamma(r)}[-x^{r-1}+(r-1)x^{r-2}]+\frac{e^{-x}}{\Gamma(r)}[-(r-1)x^{r-2}+(r-1)(r-2)x^{r-3}]$$

$$=\frac{e^{-x}}{\Gamma(r)}[x^{r-1}-(r-1)x^{r-2}-(r-1)x^{r-2}+(r-1)(r-2)x^{r-3}]$$

$$=\frac{e^{-x}}{\Gamma(r)}x^{r-3}[x^2-2(r-1)x+(r-1)(r-2)]$$

$$[f''(x)]_{x=r-1}=\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-3}[(r-1)^2-2(r-1)(r-1)+(r-1)(r-2)]$$

$$=\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-3}[(r-1)^2-2(r-1)^2+(r-1)(r-2)]$$

$$=\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-3}[-(r-1)^2+(r-1)(r-2)]$$

$$=\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-3}[(r-1)(r-2-r+1)]$$

$$=\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-3}[-(r-1)]$$

$$=-\frac{e^{-(r-1)}}{\Gamma(r)}(r-1)^{r-2}$$

$$[f''(x)]_{x=r-1}$$

Then f(x) is maximum at x=r-1
The value of the mode is r-1
Mo=x=r-1

Mean Deviation from Mean

If X be a continuous randon variabl then its p .d. function if

$$f(x)=\frac{1}{\Gamma(r)}e^{-x}x^{r-1}$$

$$MD=\int_{-\infty}^{\infty}|X-\overline{X}|f(x)dx=\int_{0}^{\infty}|X-\overline{X}|\frac{1}{\Gamma(r)}e^{-x}x^{r-1}dx$$

$$=\frac{1}{\Gamma(r)}\int_{-\infty}^{\infty}|x-\bar{x}|e^{-x}x^{r-1}=\frac{1}{\Gamma(r)}[\int_0^{\infty}xe^{-x}x^{r-1}dx-\int_0^{\infty}\bar{x}e^{-x}x^{r-1}dx]$$

$$=\frac{1}{\Gamma(r)}[\int_0^{\infty}e^{-x}x^{r+1-1}dx-\bar{x}\int_0^{\infty}e^{-x}x^{r-1}dx]=\frac{1}{\Gamma(r)}[\Gamma(r+1)-\bar{x}\Gamma(r)]$$

$$=\frac{1}{\Gamma(r)}[(r+1)\Gamma(r)-\bar{x}\Gamma(r)]=\frac{\Gamma(r)}{\Gamma(r)}[r+1-\bar{x}]$$

$$MD=r+1-\bar{x}$$ where r is positive

Probability

If $F(x)=\frac{d}{dx}f(x)$, and $f(x)\geq 0$ then

$P(a\leq x\leq b)=F(b)-F(a)$ or

$$P(a\leq x\leq b)=\int_a^b f(x)dx=\int_a^b x^{r-1}f(x)dx$$

For Mellin Integral Transform

$$P(0\leq x\leq \infty)=\int_0^\infty x^{r-1}f(x)dx, \text{ where}$$

f(x)= $\frac{e^{-x}}{\Gamma(r)}$ is the continuous .function then

$$P(0\leq x\leq \infty)=\int_0^\infty \frac{1}{\Gamma(r)}x^{r-1}e^{-x}dx=1$$

The Sum of Random Variables by using MIT

Suppose that the RV X has p d f $f_X(x)$ and Y has p d f $f_Y(y)$ and X and Y are independent .Consider the transformation $\psi:\Re^2\rightarrow\Re^2$ given by

$$\psi(x,y)=(x.x+y)=(x,z)$$

If we can determine the joint density function $f_{XZ}(x,z)$ then the marginal density

$$\text{function } f_z(z)=\int_\Re f_{XZ}(x,z)dx=\int_\Re f_{XZ}[\psi^{-1}(x,z)]dx$$

$$=\int_\Re f_{XZ}(x,z-x)dx=\int_\Re f_X(x)f_Y(z-z)dx\ ,$$

where X and Y are independent

$$=f_X * f_Y(z)$$

The nest-to-last line above is intuitive it says that we find the density for Z=X+Y by integrating the joint density of X ,Y over all points where X+Y=Z .i.e. Y=Z-X.

By using the Mellin transform [7]

$$\int_\Re \frac{1}{\Gamma(r)}x^{r-1}e^{-x}e^{-2\pi i\xi x}dx=\frac{1}{\Gamma(r)}\int_\Re x^{r-1}e^{-(1+2\pi i\xi)x}dx$$

substitute $(1+2\pi i\xi)x=q$,x= $\frac{q}{1+2\pi i\xi}$, dx= $\frac{dq}{1+2\pi i\xi}$,then

$$=\frac{1}{\Gamma(r)}\int_\Re \left(\frac{q}{1+2\pi i\xi}\right)^{r-1}e^{-q}\frac{dq}{1+2\pi i\xi}$$

$$=\frac{1}{\Gamma(r)}\frac{1}{(1+2\pi i\xi)^r}\int_\Re q^{r-1}e^{-q}dq$$

$$=\frac{1}{\Gamma(r)}\frac{\Gamma(r)}{(1+2\pi i\xi)^r}$$

$$=\frac{1}{(1+2\pi i\xi)^r}$$

By using Laplace Transform

$$\int_\Re \frac{1}{\Gamma(r)}x^{r-1}e^{-x}e^{-\xi x}dx=\frac{1}{\Gamma(r)}\int_\Re x^{r-1}e^{-(1+\xi)x}dx$$

substitute $(1+\xi)x = q$, x=$\frac{q}{1+\xi}$, dx=$\frac{dq}{1+\xi}$,then

$$=\frac{1}{\Gamma(r)}\int_{\Re}(\frac{q}{1+\xi})^{r-1}e^{-q}\frac{dq}{1+\xi}=\frac{1}{\Gamma(r)}\frac{1}{(1+\xi)^{r}}\int_{\Re}q^{r-1}e^{-q}dq$$

$$=\frac{1}{\Gamma(r)}\frac{\Gamma(r)}{(1+\xi)^{r}}=\frac{1}{(1+\xi)^{r}}$$

The Mellin Integral Transform and relation with Laplace Transform

If $f\in M_{c}(\Re)$ for all $c\in[a,b]$, we say that $f\in M_{[a,b]}(\Re)$,then we define Mellin integral transform of *f* with argument

$$F(s)=M\{f(u),s\}=\int_{0}^{\infty}u^{s-1}f(u)du \text{ ,where } a\leq \text{Re}(s)\leq b$$

The inverse Mellin transform is

$$F(x)=M^{-1}[f](s)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}x^{-s}F(s)ds$$

The condition that the inverse exists is that $F(s)x^{-s}$ is analytic in a strip $(a,b)X(-\infty,\infty)$ such that $c\in[a,b]$ The mellin integral transform is derived from Laplace integral transforms follows

$$L[f(t),s]=\int_{-\infty}^{\infty}e^{-st}f(t)dt,$$

substitute x=e^{-t}, t=-log(x), dt= - $\frac{dx}{x}$,if t=-∞ then x=∞ and if t=∞ then x=0

$$L[f(t),s]=\int_{-\infty}^{\infty}(e^{-t})^{s}f(t)dt=\int_{\infty}^{0}x^{s}f(-\log x)(\frac{-dx}{x})$$

$$=\int_{0}^{\infty}x^{s-1}f(x)dx=M[f(x),s],$$

this is the Mellin integral transform of f(x) of the Mellin kernel x^{s-1} ,s>0 is the parameter. The inverse Mellin integral transform is

$$f(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}x^{-s}F(s)ds,$$

whenever this integral is exists. The some technique is used to obtain the Mellin inversion theorem from the Laplace inverse

$$f(y)=T^{-1}[\overline{f}(.)](s)=\int_{-\infty}^{\infty}\overline{f}(s)e^{s\log y}ds$$

substitute $s=-(\eta-c)$, $ds=-d\eta$, limits are $c-i\infty$ to $c+i\infty$,then

$$f(y)=T^{-1}[\overline{f}(.)](y)=\int_{c-i\infty}^{c+i\infty}\overline{f}(-(\eta-c))e^{-(\eta-c)\log y}d\eta$$

$$=\int_{c-i\infty}^{c+i\infty}\overline{f}(-(\eta-c))y^{-\eta}y^{c}d\eta$$

$$=y^{c}\int_{c-i\infty}^{c+i\infty}\overline{f}(-\eta+c))y^{-\eta}d\eta$$

$$f(y)\,y^{-c}=\int_{c-i\infty}^{c+i\infty}\overline{f}(\eta)y^{-\eta}d\eta=f^{*}(\eta)$$

4. Conclusion

Mathematical science is not the just a single application territory for the Mellin transform however numerous different applications likewise cause the Fourier to transform and its variations all inclusive somewhere else in practically all parts of science and designing, i.e, it has flexible applications in numerous fields. The Mellin transform is utilized in software engineering for investigation of utilizations to systematic number hypothesis and Mellin himself created it regarding his inquires about in the hypothesis of functions, number hypothesis, and fractional differential conditions. The outcome can likewise be personality framework that will additionally improve the portraying state of Mellin transform. The utilization of the Laplce essential transform for a portion of the random variables is for the most part utilized and clarified in each propelled statistics content, presently short hypothesis of Mellin basic transform for statistics and probability is given in this paper .It appears for any analysts, mathematicians and specialists will likewise look into creating Mellin transform with statistics and probability. We have exhibited some foundation on statistics and probability hypothesis and propelled to process probability thickness functions for whole and augmentation of nonstop random variables. The utilization of the Laplace transform to assess the convolution essential for the p d f of aggregate is moderately basic .The utilization of the Mellin necessary transform to assess the convolution basic for the p d f of an item is known in the hypothesis of vital transforms.

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