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Mathematical Model in terms of System of Partial Differential Equations

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ABSTRACT

Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over arching or meta-principles phrased as questions about the intentions and purposes of mathematical modeling. These metaprinciples are almost philosophical in nature. Mathematical modeling in phrases of differential equations arises once the scenarios modelled involves several constant variables altering with regard to various other constant variables and we've some sensible hypothesizes regarding the rates of change of reliant variables with respect to impartial variables. So, in this paper we will study about the mathematical modelling in terms of system of partial differential equations.

1. INTRODUCTION

Mathematical modeling is actually a representation in mathematical terms of the actions of genuine equipment as well as items. Right here we attempt to learn how to create or maybe produce versions or representations mathematical, precisely how to confirm them, how you can utilize them and when and how the use of theirs is restricted. But before talking about into these crucial issues, it's really worth discussing the reason we do mathematical modeling. Actually the modeling of equipment plus phenomena is actually crucial to both sciences and engineering, engineers & scientists have extremely practical reasons for doing mathematical modeling. Additional engineers, scientists as well as mathematicians wish to feel the large pleasure of formulating as well as solving mathematical issues. In the latest times mathematical modeling has been effectively utilized by just about all scientists as well as engineers.

It value as a discipline to be studied as well as cultivated has been recognized only throughout the previous 3 or maybe 4 years. A lot of good succeeds on mathematical modeling have also appeared. These're both self-discipline - centred and perhaps strategy - situation-centred or centred. Mathematical modeling in phrases of differential equations arises once the scenarios modelled involves several constant variables altering with regard to various other constant variables and we've some sensible hypothesises regarding the rates of change of reliant variables with respect to impartial variables. When we've one reliant adjustable x (say population size) based on a single impartial varying (says time't), we get a mathematical model in phrases of regular differential equation of initial order, if the hypothesis is all about the speed of change dx/dt. The unit is going to be in phrases of a regular differential equation of 2nd order in case the hypothesis consists of the speed of change of dx/dt. When generally there are numbers of reliant adjustable the hypothesis might provide a mathematical model in phrases of devices of higher or first order regular differential equations. When there's one dependent constant varying (say velocity of substance u) as well as a selection of independent constant variables (say room coordinates x, y, z as well as time't), we get a mathematical model in phrases of a partial differential situation. When generally there is actually a selection of dependent constant variables as well as a variety of independent constant variables, we are able to get a mathematical model in phrases of method of partial differential equations.

2. LINEAR GROWTH AND DECAY MODELS

A. Exponential Growth Model

The basic principle of exponential growth for human populations was initially propounded by Thomas R. Malthus (1766 1834) an English clergyman as well as political economist in the very first edition of the popular book of his entitled' An Essay on the Principle of Population' posted in 1798.

Formulation of the Model

During formulating the population growth model, Malthus made the following three assumptions:

i. The population is sufficiently large.

ii. Population is homogeneous, that is, it is evenly spread over the living space.

iii. There are no limitations to growth i.e., no limitations of food, space and so on. Population changes only by the occurrence of births and deaths.

Let x(t) be the size of the population at time t which is taken to be positive integer with x(0) = x0.

Let us think that all changes in the population effect from deaths as well as births, consequently, there's no immigration or perhaps emigration. Let B(t) and D(t) denote respectively, the statistics of deaths and births at time t. Than per capita birth rate b and per capita death rate m are provided by

$$b = \frac{1}{x(t)}\frac{dB}{dt} \quad \textbf{(1)}$$

$$b = \frac{1}{x(t)}\frac{dD(t)}{dt} \quad \textbf{(2)}$$

per capita growth rate of the population at any time t is given by

$$\frac{1}{x(t)}\frac{dB(t)}{dt} - \frac{1}{x(t)}\frac{dD(t)}{dt} = b - m \quad \textbf{(3)}$$

which gives

$$\frac{1}{xt}\frac{d}{dt}(B - D) = b - m \quad \textbf{(4)}$$

Or

$$\frac{1}{xt}\frac{d}{dt} b - m = a \quad \textbf{(5)}$$

Or

$$\frac{1}{xt}\frac{dx}{dt} = ax, \text{ where } x(0)=x_0 \quad \textbf{(6)}$$

The distinction between the per capita birth as well as death rates α ≡ b' m, plays an especially crucial role and it is referred to as the intrinsic rate of net development rate or development.

B. **Effects of immigration and Emigration on Population**

First of all we define Immigration and Emigration.

Immigration: The process in which some individuals are added from outside to the population is known as Immigration.

Emigration: The task in which a lot of people went out of the population, is actually widely known as Emigration. If immigration into the population from outside is actually at a speed proportional to the population size, the influence is actually equivalent to raising the birth rate. Likewise, in case emigration from the population is actually at a speed proportional to the population size, the influence is actually equivalent to increasing in the death rate.

When immigration as well as emigration occur at frequent rates I and e respectively, then the speed of change in population is actually provided by

$$\frac{dx}{dt} = ax + i - e \quad \textbf{(7)}$$

$\boldsymbol{ax + \beta}$ where $i - e = \beta(constant)\ say$

$$\textbf{Or } \frac{dx}{ax+\beta} = dt \quad \textbf{(8)}$$

On integrating, we get

$\frac{1}{\alpha}\boldsymbol{log(\alpha x+\beta)=t+D}$, where$D$ being a constant.

This equation gives the population at time t in the presence of immigration and emigration.

3. NON-LINEAR GROWTH AND DECAY MODELS

As public increases, thanks to limits and overcrowding of resources, the birth rate b decreases as well as the death rate d improves with the population size x. The easiest assumption is to take

$b=b_1-b_2x$, $d=d_1+d_2x$,$b_1b_2,d_1,d_2>0$

so that (2) becomes

$$\frac{dx}{dt}=\big((b_1-d_1)-(b_2-d_2)x\big)=x(a-bx),a>0,b>0$$

3.1 Non-Linear Growth and Decay models

A. Logistic Growth Model

Whenever a population is actually growing in a small space the density steadily arises until ultimately the existence of some other organism lessen the fertility and longevity that decrease the speed of expansion of the population until eventually the public ceases to develop. The development curve outlined by such a public follows S-shaped or sigmoid pattern. The sigmoid curve arises when the density of the population increases with regard to the environmental opposition. If the environmental factors are linearly proportional to the density then this kind of kind of development is known as logistic and the progress equation is known as the logistic situation.

$$\frac{dx}{dt}=x\alpha\left[1-\frac{x}{k}\right]$$

Where α > 0 and K > 0

since, $$\frac{d\lambda}{dx}=\frac{-\alpha}{K}<0,\ \forall x>0$$

Formulation: We shall have a single public i.e., a single species to create a model. Let x(t) be the number of individual species at a time t, which should be constructive integer, x(t) is usually taken to be the population density or even to be Bio mass instead of the number of people. The speed of expansion of the public is directly proportional to the population at that time.

Therefore $\frac{dx}{dt}=\lambda x$, where λ is a growth rate.

Assuming $\lambda$to be positive and equals to $\alpha\left[1-\frac{x}{K}\right]$

So, the equation becomes,

(9)

B. Law of Mass Action: Chemical Reactions

Two chemical substances combine in the ratio a : b to form a third substance Z. If $z(t)$ is the amount of the third substance at time t, then a proportion $az(t)/(a+b)$ of it consists of the first substance and a proportion $bz(t)/(a+b)$ of it consists of the 2nd material The speed of development of the third material is proportional to the item of the quantity of the 2 component substances which haven't yet coupled together. In case A and B are the original quantities of the 2 substances, then we geT

$$\frac{dz}{dt}=k\left(A-\frac{az}{a+b}\right)\left(B-\frac{bz}{a+b}\right)$$

(10)

This is the non-linear differential equation for a second order reaction. Similarly for an nth order reaction, we get the non-linear equation

$$\frac{dz}{dt} = k(A_1 - a_1 z)(A_2 - a_2 z)....(A_n - a_n z), \qquad \textbf{(11)}$$

4. MATHEMATICAL MODELLING IN DYNAMICS THROUGH ORDINARY DIFFERENTIAL EQUATIONS OF FIRST ORDER

Let a particle travel a distance x in time i in a straight line, then its velocity v is given by dx/dt and its acceleration is given by

$$dv/dt = (dv/dx)(dx/dt) = vdv/dx = d^2x/dt^2$$

(A) Simple Harmonic Motion

Here a particle moves in a straight line in such a manner that its acceleration is always proportional to its distance from the origin and is always directed towards the origin, so that

$$v\frac{dv}{dx} = -\mu x \qquad \textbf{(12)}$$

Integrating

$$v^2 = \mu(a^2 - x^2), \qquad \textbf{(13)}$$

where the particle is initially at rest at x = a. Equation (13) gives

$$\frac{dx}{dt} = -\sqrt{\mu}\sqrt{a^2 - x^2} \qquad \textbf{(14)}$$

(B) Motion under Gravity in a Resisting Medium

A particle falls below gravity in a medium in which the resistance is actually proportional to the velocity. The situation of movements is

$$m\frac{dv}{dx} = mg - mkv \qquad \textbf{(15)}$$

Or

$$\frac{dv}{V - v} = k\,dt; \quad V = \frac{g}{k} \qquad \textbf{(16)}$$

5. CONCLUSION

When we discover the model of ours is actually insufficient or even that it fails in a way, we then enter an iterative loop in which we cycle back again to an earlier stage of the model building and re examine the assumptions of ours, the recognized parameter values of ours, the process selected, the equations used, the ways of calculation, etc. This particular iterative procedure is actually important since it's the sole method in which models could be improved, corrected, and validated.

REFERENCES

1. Berry, J.S., Burghes, D.N. and Huntley, I.D. (2008): Mathematical modelling : Methodology, Models and Micros, Ellis Harwood and John Wiley.
2. Burghes, D.N. (2009) : Modelling with differential equations, Ellis Harwood and John Wiley
3. Giordano, F.R. and Weir, M.D. (2010), A first course in Mathematical Modelling, Brooks Cole California
4. Kapur J.N. (2005) Mathematical models in Biology and Medicines, East-West Publishers Pvt. Ltd., New Delhi.
5. Kapur J.N. (2007) Mathematical Modelling, New Age Inst. Publishers, New Delhi.
6. Pundir, S.K. (2010): Biomathematics, PragatiPrakashan Meerut.
7. Ward, J.P and J.R. King (2008): Mathematical Modelling Monolayer Cultures. Math. Biosci., 181, 177-207.
8. Dacorogna, Bernard. Introduction to the calculus of variations. London: Imperial College, cop. 2004. ISBN 1860945082