RESEARCH REVIEW JOURNALS
Volume-03
Issue-08
August-2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

An Exact Static Spherically Symmetric Solution of Einstein's Field Equations for the Zeldouich Fluid Distribution

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ARTICLE DETAILS

Article History
Published Online: 07 August 2018

Keywords
Pressure, Static, Spherically, Energy density

ABSTRACT

This paper provides an exact, static spherically symmetric solution of Einstein's field equation for the perfect fluid with p = *p*

1. Introduction

Einstein's field equations have been solved using a simple equation of state in various cases, e.g. for $\rho = p$ (letelier [7], Letelier and Tabensky [8]), $\rho + 3p =$ constant (Whittaker [13]), $\rho + 3p$ (Klain [6], Singh and Abdussatar [9], Fenstein and Senovilla, [4]) for $p = \rho +$ constant (Buchdahal and Land [13]) and for $\rho = (1+a)\sqrt{p} - ap$ (Buchdahal [2]). But if one takes e.g. polytropic fluid sphere $p = a\rho^{1+\frac{1}{n}}$ (Klein [5]. Tooper [12], Buchdahl [1], or a mixture of ideal gas and radiation (Suhonen [11]), one soon has to use numerical method Singh and Yadav [10] have also studied the static fluid sphere with the equation of state p = ρ (i.e. Zeldovich fluid). Further study in this line has been done by Yadav and Saini [14] which is more general than one due to Singh and Yadav [10].

In this paper we have obtained an exact, static spherically symmetric solutions of Einstein's field equations for the perfect fluid with p = ρ (i.e. stiff matter). To overcome the difficulty of infinite density at the centre, it is assumed that the distribution has a core of radius r_0 and constant density p_0 which is surrounded by the fluid with pressure equal to energy density.

2. The field equations and Their Solutions

We use the static spherically symmetric metric given by

$$ds^2 = e^{\beta}dt^2 - e^{\alpha}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.1}$$

Where α and β are function of r only. The field equations.

$$R^i_j - \frac{1}{2}R\delta^i_j = -8\pi T^i_j \tag{2.2}$$

For the metric (2.1) for the Zeldovich fluid which can be regarded as a perfect fluid having the energy momentum tensor

$$T^i_j = (\rho + p)u^1 u_j - \sigma^i_j p \tag{2.3}$$

Characterized by the equation of state p = ρ in comoving co-ordinates.

(i.e. $u_1 = u_2 = u_3 = 0$ and $u_4 = e^{-\frac{\beta}{2}}$) are

$$8\pi\rho = e^{-\alpha}\left(\frac{B'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} \tag{2.4}$$

$$8\pi\rho = e^{-\alpha}\left(\frac{B''}{r} - \frac{\alpha'\beta'}{4} + \frac{B^2}{4} + \frac{\beta^{-\alpha}}{2r}\right) \tag{2.5}$$

$$8\pi\rho = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{2.6}$$

From equations (2.4), (2.6) and using p = ρ we have

$$e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{2.7}$$

Equation (2.7) shows that if β is known α can be found, so we choose

$$e^{\beta} = Ar,\ \ A = \text{constant} \tag{2.8}$$

Using (2.8) equation (2.7) takes the form

$$\frac{e^{-\alpha}\alpha'}{r} - \frac{3e^{-\alpha}}{r^2} + \frac{2}{r^2} = 0 \tag{2.9}$$

Putting $z = e^{-\alpha}$ the equation (2.9) is reduced to

$$\frac{dz}{dr} + \frac{3z}{r} = \frac{2}{r^2} \tag{2.10}$$

Which is a linear differential equation whose solution is

$$z = \frac{2r^3}{3} + c \tag{2.11}$$

Therefore we get

$$e^{-\alpha} = \frac{2}{3}r^3 + c \tag{2.12}$$

where C is an integration constant

Hence the metric (2.1) after suitable adjustment of constants takes the form

$$ds^2 = rdt^2 - r^{-3}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.13}$$

The non zero component of R_{hijk} for the metric (2.13) are

$$R_{1212} = \frac{R_{1313}}{\sin^2\theta} = \frac{-R_{2323}}{2r\sin^2\theta} = -\frac{1}{2}, \tag{2.14}$$

$$R_{1414} = \frac{-3R_{2424}}{r} = \frac{-3R_{3434}}{r\sin^2\theta} = -\frac{9}{2r^5},$$

Choosing the orthonormal tetrad $\lambda_{(j)}^{-i}$ as

$$\lambda_{(1)}^{-i} = (r^{3/2},0,0,0), \lambda_{(2)}^{-i} = (0,\frac{1}{r},0,0)$$

$$\lambda_{(3)}^{-i} = (0,0,\frac{1}{r\sin\theta'}0),\ \lambda_{(4)}^{-i} = (0,0,0,\frac{1}{\sqrt{r}}) \tag{2.15}$$

The physical component $R_{(abcd)}$ of the curvature tensor are

$$R_{1212} = R_{(1313)} = \frac{r^4R_{(1414)}}{9} = \frac{-r^4R_{2323}}{2} = -\frac{r}{2} \tag{2.16}$$

$$R_{(2424)} = R_{(3434)} = \frac{3}{2r^7}$$

Also for the metric (2.13) the fluid velocity u^i is found to be

$$u^1 = u^2 = u^3 = u_1 = u_2 = u_3 = 0 \text{ and } u^4 = \frac{1}{\sqrt{r}},\ u_4 = \sqrt{r} \tag{2.17}$$

The scalar of expansion $\theta = u^i;i$ is identically zero.

The non-zero components of the tensor of rotation w_{ij} and shear tensor σ_{ij} are

$$(2.18)\quad w_{14} = -w_{41} = -\frac{1}{2\sqrt{r}}$$

$$(2.19)\quad \sigma_{14} = \alpha_{41} = -\frac{1}{2\sqrt{r}}$$

3. Solutions for the Perfect fluid core

Pressure and density for metric (2.13) are

$$(3.1)\quad 8\pi\rho = 8\pi\rho = -4r + \frac{1}{r^2}$$

It follows from (3.1) that the density of the distribution tends to infinity as r tends to zero. In order to get rid of the singularity at r = 0 in the density we visualize that the distribution has a core of radius r_0 and constant density ρ_o. The field inside the core is given by the Schwarzschild internal solution.

$$(3.2)\qquad e^{-\alpha} = 1 - \frac{r^2}{R^2}$$

$$e^{\beta} = \left[A - B\left(1 - \frac{r^2}{R^2}\right)^{1/2}\right]^{1/2}$$

$$8\pi P = \frac{1}{R^2} = \left[\frac{3B\left(1 - \frac{r^2}{R^2}\right)^{1/2} - A}{A - B\left(1 - \frac{r^2}{R^2}\right)^{1/2}}\right]$$

Where A and B are constants and

$$R^2 = \frac{3}{8\pi\rho_0}$$

The constants appearing in the solution can be evaluated by the continuity conditions for the metric (2.13) and (3.2) at the boundary $r = r_0$.

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