

# On LRS Cosmological Models with Zero-Mass Scalar Field

Dr. Purushottam

Deptt. of Mathamatics, Govt. Women's Polytechnic, Patna

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## ABSTRACT

The present paper provides some LRS Bianchi I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution on it. Various physical and geometrical features of the models have been also found.

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## 1. Introduction

Janis et al. [15] obtained the solutions of the Einstein-scalar and Brans-Dicke [3] field equations for static space time and also gave a procedure to generate static solutions of the coupled Einstein-Maxwell-scalar field equations. The solutions of axially symmetric Einstein-Maxwell-scalar field equations have been given by Eris and Gurses [9]. Singh et. Al. [32] have found a method to obtain solutions to the cylindrically symmetric gravitational field coupled to massless scalar and non-null Maxwell fields. They have shown that starting from any solution to the electrovacuum field equations it is possible to generate a whole class of solutions to the coupled Einstein-Maxwell-scalar field equations by a suitable redefinition of one of the space-time metric coefficients. They have further applied the technique to the solution due to Chitre et. al. [7] and have also obtained the dual solution by an extension of Bonnor's theorem [4].

As a matter of fact following the development of inflationary models, the importance of scalar fields (mesons) in cosmology has become well known [16]. The study of interacting fields, one of the fields, being a zero-mass scalar field is basically an attempt to look into the yet unsolved problem of the unification of gravitational and quantum theories [27, 28]. Considerable interest has been focused on a set of field equations representing zero-mass scalar-fields coupled with the gravitational field for the last three decades. Bergamann and Leipnik [1] and Brahmachary [2] have investigated the spherically symmetric fields associated with zero-rest-mass. The static solutions for axially symmetric fields have been investigated by Buchdahl [5]. Jain et. al. [14-15] in an attempt to present an extension of Israel's idea of a singular even horizons [13] have considered the spherically symmetric solutions of the field equations of general relativity containing zero-rest-mass meson fields. Penny [20] and Gautreau [10] have extended the study of the case of axially symmetric fields and have found that the scalar fields obey a flat space Laplace equation and a large class of solution exist. Singh [30, 31], Patel [19] and Reddy [25] have investigated plane symmetric solutions of the field equations corresponding to zero-mass scalar fields. Stephenson [33], Rao et. al. [23], Chatterjee and Roy [6], Reddy and Rao [24], Verma [34], Shanthy and Rao [29], Pradhan et. al. [21] are some of the authors who have studied various aspects of interacting fields in the framework of general relativity. At the present state of evolution, the universe is spherically symmetric with isotropic and homogenous matter distribution. But in its early stages of evolution, it could have not had a smoothed out picture. Close to the big bang singularity, neither the assumption of spherically symmetric nor of isotropy can be strictly valid. So we, consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study in homogeneities. For simplification and description of the large scale behavior of the actual universe, locally rotationally symmetric (LRS) Bianchi I space-time has been widely studied. Mazumdar [17] has obtained solutions of an LRS Bianchi I space-time filled with a perfect fluid. Hajj-Boutros and Steila [11] and Sri Ram [26] have also obtained some solutions for the same field equations by using their solution-generating techniques. Pradhan et. al. [22] have studied LRS Bianchi I space-time with zero-mass scalar field. In fact cosmological models based on scalar fields of various kinds have had enormous success in solving cosmological problems among which are the causality, entropy, initial singularity and cosmological constant problem.

Here in this paper, we have considered some LRS Bianchi I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution in it. We have also discussed various physical and geometrical features of the models.

## 2. The Field Equations

We take metric for the LRS Bianchi I space-time in of the form given by [18]

$$(2.1) \quad ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2)$$

Where A and B are functions of the cosmic time t. The energy momentum tensor of a perfect fluid together with a zero-mass scalar field is given by

$$(2.2) \quad T_{ij}^{(m)} T_{ij}^{(s)}$$

Where

$$(2.3) \quad T_{ij}^{(m)} = (\rho + p)u_i u_j + p g_{ij}$$

is the energy momentum tensor corresponding to perfect fluid distribution with the four vector velocity  $u^i$  satisfying  $u_i u^i = -1$ ,  $p$  the pressure and  $\rho$  the mass – energy density. The energy momentum tensor  $T_{ij}^{(s)}$  corresponds to zero-mass scalar field  $\phi$  and is

$$(2.4) \quad T_{ij}^{(s)} = \phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} g^{ab} \phi_{,a} \phi_{,b}$$

Where  $\phi(t)$  (a function of  $t$  only) is the zero – mass scalar field which satisfies the wave equation

$$(2.5) \quad g^{ij} \phi_{,ij} = 0$$

The scalar field  $\phi$  is not directly coupled to matter. It interacts with matter indirectly through gravity. The Einstein's field equations

$$(2.6) \quad R_{ij} - \frac{1}{2} R g_{ij} = k T_{ij}$$

together with energy momentum tensor defined by equation (2.2) given the following equations

$$(2.7) \quad -K\rho + \phi^2 = \frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2}$$

$$(2.8) \quad -K\rho + \phi^2 = \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A}$$

and

$$(2.9) \quad K_p - \phi^2 = \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}$$

where  $k = 8\pi G$ ,  $G$  the gravitational constant. The overdot indicates a derivative with respect to time  $t$ . The wave equation (2.5) yields

$$(2.10) \quad \left( \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \right) \dot{\phi} + \ddot{\phi} = 0$$

and the energy conservation equation for the matter  $T_{ij,i}^{(m)} = 0$  leads to

$$(2.11) \quad \dot{\rho} + \left( \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \right) (\rho + p) = 0$$

### 3. Solutions of the Field Equations

From equation (2.7) and (2.8) we obtain

$$(3.1) \quad \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} = 0$$

Which has first integral

$$(3.2) \quad B^2 \dot{A} - AB \dot{B} = D$$

Where  $D$  is an integrating constant

Equation (3.2) is a linear differential equation in  $A(t)$  and has an exact solution

$$(3.3) \quad A = C_1 B + DB \int \frac{df}{B^3(t)}$$

Similarly equation (3.2) is also a linear differential equation in  $B(t)$ , which has an exact solution,

$$(3.4) \quad B^2 = C_2 A^2 - 2DA^2 \int \frac{df}{A^3(t)}$$

On integration, equation (2.10) yields

$$(3.5) \quad \phi = C_4 + \int \frac{C_3 dt}{A(t)B^2(t)}$$

Where  $C_1, C_2, C_3$  and  $C_4$  are integration constants.

Thus, for any arbitrary  $B(t)$ , equation (3.3) gives  $A(t)$  and then  $\square$  is known from equation (3.5). Similarly for an arbitrary  $A(t)$  one can calculate  $B(t)$  and  $\square$  from equation (3.4) and (3.5). Then from equations (2.7) and (2.9),  $p$  and  $\square$  can be obtained and hence the solution of the field equations is completely known

To illustrate our problem, we choose  $B = t^{\frac{1}{2}(1-3f)}$ . From equation (3.3) and (3.5) we obtain

$$(3.6) \quad A = C_1 t^{k(1-3f)} + \frac{2D}{9f-1} t^{3f}$$

and

$$(3.7) \quad \phi^2 = C_3^2 \left[ C_1 t^{k(1-3f)} + \frac{2D}{(9f-1)} t^{3f(1-3f)} \right]$$

where  $k$  and  $f (\neq 1)$  are real constants. So, in this case, the geometry of our universe is given by metric.

$$(3.8) \quad ds^2 = -dt^2 + \left[ C_1 t^{k(1-3f)} + \frac{2D}{(9f-1)} t^{3f} \right] dx^2 + t^{2k(1-3f)} (dy^2 + dz^2)$$

For the metric (3.8) from the equations (2.7) – (2.9) find the expression for  $p$  and  $\square$ .

$$(3.9) \quad Kp = C_3^2 \left[ C_1 t^{k(1-3f)^2} + \frac{2D}{(9f-1)} t^{3f(1-3f)} \right]^{-2} + \frac{(1-3f)(1+9f)}{4t^2}$$

$$(3.10) \quad K_p = C_3^2 \left[ C_1 t^{k(1-3f)^2} + \frac{2D}{(9f-1)} t^{3f(1-3f)} \right]^{-2} + \frac{2D(1-3f)(1+9f)t^{k(9f-1)} + 3C_1(1-3f)^2(9f-1)}{4t^2 \left[ 2Dt^{\frac{1}{2}(9f-1)} + k_1(9f-1) \right]}$$

When  $k = \frac{1}{2}$  we get solution due to Pradhan et. al. [22] by suitable adjustment of constants. However, when  $k = \frac{1}{2}$  we get

$$(3.11) \quad ds^2 = -dt^2 + \left[ C_1 t^{\frac{1}{2}(1-3f)} + \frac{2D}{(9f-1)} t^{3f} \right] dx^2 + t^{(1-3f)} (dy^2 + dz^2)$$

Also  $p$  and  $\square$  are given by

$$(3.12) \quad Kp = C_3^2 \left[ C_1 t^{\frac{1}{2}(1-3f)^2} + \frac{2D}{(9f-1)} t^{3f(1-3f)} \right]^{-2} + \frac{(1-3f)(1+9f)}{4t^2}$$

$$(3.13) \quad K_p = C_3^2 \left[ C_1 t^{\frac{1}{2}(1-3f)^2} + \frac{2D}{(9f-1)} t^{3f(1-3f)} \right]^{-2} + \frac{2D(1-3f)(1+9f)t^{\frac{1}{2}(9f-1)} + 3C_1(1-3f)(9f-1)}{4t^2 \left[ 2Dt^{\frac{1}{2}(9f-1)} + C_1(9f-1) \right]}$$

When  $f = \frac{1}{3}, p = \rho = \text{constants}$ , whereas in the absence of scalar field we get  $p = \square = 0$  [17].

The energy conditions [8]

- (i)  $(\rho + p) > 0$
- (ii)  $(\rho + 3p) > 0$  and

(iii)  $\rho > 0$

are satisfied when  $C_1 > 0, D > 0$ , and  $\frac{1}{9} < f < \frac{1}{3}$  and the dominant energy conditions [12].

(i)  $(\rho - p) \geq 0$  and

(ii)  $(\rho + p) \geq 0$

when  $D > 0, C_1 > 0$  and  $\frac{1}{9} < f < \frac{2}{9}$

The expansion scalar  $\theta$ , the shear tensor  $\sigma_{\alpha\gamma}$ , the rotation  $\omega_{\alpha\gamma}$  and the acceleration vector  $a_\alpha$  for the velocity field  $U_\alpha$  are defined by

(3.14)  $\theta = u^\alpha_{;\alpha}$

(3.15)  $\sigma_{\alpha\gamma} = \frac{1}{2}(u_{\alpha;\gamma} + u_{\gamma;\alpha}) - \frac{1}{2}(u_\alpha a_\gamma + u_\gamma a_\alpha) - \frac{1}{3}\theta(g_{\alpha\gamma} + u_\alpha u_\gamma)$ ,

(3.16)  $\omega_{\alpha\gamma} = u_{\alpha;\gamma} - \sigma_{\alpha\gamma} - \frac{1}{3}\theta(g_{\alpha\gamma} + u_\alpha u_\gamma) - u_{\alpha;\beta} u^\beta u_\gamma$

and

(3.17)  $a_\alpha = u^\gamma u_{\alpha;\gamma}$

Here the semicolon indicates covariant differentiation. The spatial volume is given by

$V = AB^2$

For the velocity field  $U_\alpha$  these kinematical parameters are found to have the following expressions :

(3.18)  $V = \frac{t \left( C_1(9f - 1) + 2Dt^{\frac{1}{2}(9f-1)} \right)}{(9f - 1)t^{\frac{1}{2}(9f-1)}}$

(3.19)  $\theta = \frac{4Dt^{\frac{1}{2}(9f-1)} + 3C_1(1-f)(9f-1)}{2t \left[ 2Dt^{\frac{1}{2}(9f-1)} + C_1(9f-1) \right]}$

(3.20)  $\sigma = \frac{1}{\sqrt{6}} \left[ \frac{2Dt(9f-1)t^{\frac{1}{2}(9f-1)}}{t \left[ 2Dt^{\frac{1}{2}(9f-1)} + C_1(9f-1) \right]} \right]$

and

(3.21)  $\omega = 0$

(3.22)  $a_\alpha = [0, 0, 0, 0]$ ,

**4. Remarks and Conclusion**

From above equations [3.19 – 3.22] we see that our model is expanding, shearing and non-rotating. The acceleration vector  $a_\alpha$  is zero and consequently the stream links of the perfect fluid are geodesic. As the shear tensor is not zero, the model is clearly anisotropic.

From  $f = \frac{1}{3}$ , the metric (3.11) represents a non-static cosmological model filled with a stiff fluid, the pressure and density of which are given by

$$(4.1) \quad K\rho = K\rho = \frac{C_3^2}{(C_1 + D)^2}$$

The models with  $\square = p$  are important in relativistic cosmology for the description of very early stages of the universe. Choosing  $A = h_1 t^{k(1-3f)} + h_2 t^{3f}$  and  $D = 0$  in equation [3.3], we find

$$(4.2) \quad B^2 = g_1 t^{2k(1-3f)} + g_2 t^{k+3f(1-k)} + g_3 t^{6f}$$

$$\text{Where } g_1 = C_2 h_1^2, g_2 = 2C_2 h_1 h_2, g_3 = C_2 h_2^2$$

Hence, in this case, the geometry of our universe is given by metric.

$$(4.3) \quad ds^2 = -dt^2 + \left( h_1 t^{k(1-3f)} + h_2 t^{3f} \right)^2 dx^2 \\ + \left[ g_1 t^{2k(1-3f)} + g_2 t^{k+3f(1-k)} + g_3 t^{6f} \right] (dy^2 + dz^2)$$

## References

1. Bergmann, O. and Leipnik, R. (1957) : Phys. Rev., 107, 1157.
2. Brahmachary, R.L. (1960) : Prog. Theor. Phys., 23, 749.
3. Brans, C.H. and Dicke, R.H. (1961) : Phys. Rev., 124, 925.
4. Bonnor, W.B. (1954) : Proc. Phys. Soc. (London), A67, 225.
5. Buchdahl, H.A. (1959) : Phys. Rev., 115, 1325.
6. Chatterjee, B. and Roy, A.R. (1982) Acta. Phys. Pol., B13, 385.
7. Chitre, D.M. Guven, R. and Nutku, Y. (1975) : J. Math. Phys. (U.S.A.), 12, 475.
8. Ellis, G.F.R. (1971) : General Relativity and Cosmology, R.K. Sachs (Ed.), Academic Press, New York, P. 117.
9. Eris, E. and Gurses, M. (1977) : J. Math. Phys. (U.S.A.), 12, 475.
10. Gautreau, R. (1969) : Nuovo Cimento, B6, 360.
11. Hajj-Boutros, J. and Sfeila, J. (1987) : Int. J. Theor. Phys. 26, 98.
12. Hawking, S.W. and Ellis, G.F.R. (1973) : The Large Scale Structure of Space Time, Cambridge University Press, Cambridge, P. 94.
13. Israel, W. (1967) : Phys. Rev., 164, 1176
14. Janis, A.I., Newman, E.T. and Winicour, J. (1968) Phys. Rev. Lett., 20, 878.
15. Jains, A.I., Robinson, D.C. and Winicour, J. (1969) : Phys. Rev. Letter, 186, 1729.
16. Linde, A. (1982) : Phys. Lett., 108B, 389.
17. Mazumdar, A. (1994) : Gen. Rel. Grav., 26, 307.
18. M.A.H. Mac Callum, In, General Relativity : An Einstein Centenary Survey (Eds : S.W. Hawking and W. Isreal), Cambridge Univ. Press, Cambridge, 1979.
19. Patel, L.K. (1975) : Tensor, N.S., 29, 237.
20. Penny, R. (1968) : Phys. Rev., 174, 1578.
21. Pradhan, A., Tiwari, K.L. and Beesham, A. (1995) : A Class of Plane Symmetric Models in the presence of Zero-Mass Scalar Fields, Preprint, University of Zululand, S.A., 1995.
22. Pradhan, et. al. (2001) : Ind. J. Pure. Appl. Maths, 32, 789.
23. Rao, J.R. and Rao, A.R. and Tiwari, R.N. (1972) : Ann. Phys., 69, 473.
24. Reddy, D.R.K. and Rao, V.U.M. (1983) : J., Australian. Math. Soc., B24, 461.
25. Reddy, D.R.K. (1988) : Astrophys. Space Sci., 140, 161.
26. Sri Ram (1987) : Int. J. Theor. Phys., 28, 917.
27. Santill, R.M. (1997) : Found. Phys. Letter, 10, 307.
28. Santill; R.M. (1997) : Infinite Energy, 22, 33.
29. Shanthi, K., Rao, V.U.M. (1990) : Astrophys. Space Sci., 172, 77.
30. Singh, T. (1974) : Gen. Rel. Grav., 5, 657.
31. Singh, T. (1978) : Acad. Prog. Math, (India), 12, 27.
32. Singh et. al. (1983). J. Math. Phys., 17, 245.
33. Stephenson, G. (1962) : Proc. Cambridge Phil. Soc., 58, 521.
34. Verma, M.N. (1987) : Astrophys. Space., Sci., 135, 197.