RESEARCH REVIEW JOURNALS
Volume-04
Issue-03
March-2019

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com[UGC Listed Journal]

On Static Charged Fluid Sphere in General Relativity

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ARTICLE DETAILS

Article History
Published Online: 13 March2019

Keywords
Mass, Density, Pressure, Metric, Fluid, Matter

ABSTRACT

The present paper provides solution of Einstein-Maxwell field equations for static charged fluid sphere using specific choice of g_{11} and g_{44}.

1. Introduction

Shi chang [6] has found some conformal flat interior solutions of the Einstein- Maxwell equations for a charged stable static sphere. These solutions satisfy physical conditions inside the sphere. Some exact static solutions of Einstein- Maxwell equations representing a charged fluid sphere were obtained by Singh & Yadav[5]. Xingxiang[7] has obtained an exact solution by specifying matter distribution & charge distribution. Buchdahl[2] has also considered some regular general relativistic charge fluid spheres. Some other cases of interior solutions for charged fulid sphere have been presented by Baliyn[1], Cooperstock & Cruz[3], Nduka[4], Yadav & Purushottam[8].

In this paper considering spherically symmetric metric we have obtained some solutions of Einstein-Maxwell field equation using different assumptions on metric potentials g_{11} and g_{44}. The pressure, matter, density electric field & charge density have been found.

2. The Field Equations

We consider the metric in the form

(2.1) $ds^2 = e^{\beta} dt^2 - e^{\alpha} dr^2 - r^2(d\theta^2 + sin^2\theta d\emptyset)$

Where α *and* β are functions of r only.

The Einstien- Maxwell field equations for the charged perfect fluid distribution are

(2.2) $R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij}$

(2.3) $F^{ij}_{;j} = 4\pi J^i = 4\pi\sigma u^i$

(2.4) $F_{[ij;k]} = 0$

Where T_{ij} is the energy momentum tensor, J^i is the charged current four vector, R_{ij} is the Ricci tensor and R the scalar of curvature tensor. For the system under study the energy momentum tensor T^i_j splits up to into two part viz. $\bar{T}^i_j$ and E^i_j for matter and charges respectively.

(2.5) $T_{ij} = \overline{T^i_j} + E^{\alpha}_{\beta}$

where

(2.6) $\overline{T^i_j} = \left[(\rho + p)u^i u_j - p\delta^i_i\right]$

With

(2.7) $u^i u_i = 1$

The non- vanishing component of $\bar{T}^i_j$ are

(2.8) $\bar{T}^1_1 = \bar{T}^2_{j2} = \bar{T}^3_3 = -p \; and \; \bar{T}^4_4 = \rho$

Here p is internal pressure, ρ and σ are densties of matter and charges respectively, u^i is the velocity vector of the matter.

The static condition is given by

(2.9) $u^1 = u^2 = u^3 = 0$ and $u^4 = (g_{44})^{-1/2}$

i.e $u^4 = e^{-\beta/2}$

The electromagnectic energy momentum tensor E_j^i is given by

(2.10) $E_j^i = -F_{jk}F^{ik} + \frac{1}{4}\delta_j^i F_{lm}F^{lm}$

We assume the field to be purely electrostatic i.e $F_{ik} = 0$ and $F_{4k} = \emptyset, k = \emptyset_k$ and where $\emptyset$ is the electrostatic potential.

Thus the Einstien-Maxwell field equations are reduces into form

(2.11) $e^{-\alpha}\left(\frac{1}{r^2} - \frac{\alpha}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E$

(2.12) $\frac{1}{r^2} - e^{-\alpha}\left(\frac{1}{r^2} + \frac{\beta'}{r}\right) = -8\pi p + E$

(2.13) $e^{-\alpha}\left[\frac{1}{4}\beta'\alpha' - \frac{1}{4}\beta'^2 - \frac{1}{2}\beta'' - \frac{1}{2}\left(\frac{\beta' - \alpha'}{r}\right)\right] = -8\pi p - E$

Where

(2.14) $E = -F^{41}F_{41}$

(2.15) $4\pi\rho = \left(\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\alpha' + \beta'}{2}F^{41}\right)e^{\beta/2}$

3. Solution of the Filled Equations

We have four equations (2.11)-(2.13) and (2.15) in six variables$(\rho, E, p, \alpha, \beta, \sigma)$. Here we take α *and* β are two free variables.

We choose

(3.1) $e^{\alpha} = \frac{Ar^3 + Br^2 + C}{r^3 + C}$

(3.2) $e^{\beta} = \frac{Lr^3 + Mr + C}{4N}$

Where A, B, C, L, M , N are arbitrary constants

Pressure, matter densit, electric field and charge density are found to be

(3.3) $8\pi p = \frac{r^3 + C}{2(Ar^3 + Br^2 + C)}\left[\frac{36L^2r^5 + 54LM^4 + 32CLr^2 + 5M^2r^2 + 6CM}{4r(Lr^3 + M + C)^2}\right]$

(3.4) $8\pi\rho = \frac{r^3 + C}{2(Ar^3 + Br^2 + C)}$

$$\left[\frac{\frac{9}{4}r^3C\,(A-1) + (12L^4 + 11Mr^2 + 10Cr)}{2r(r^3 + C)(Ar^3 + Br^2 + c)(Lr^3 + Mr + C)} + \frac{4L^2r^5 + 6LMr^4 + 3M^2r + 2CMr}{4r(Lr^3 + Mr + C)^2} - \frac{1}{r^2}\right] + \frac{1}{2r^2}$$

(3.5) $E = \frac{r^3 + C}{2(Ar^3 + Br^2 + C)}\left[\frac{\frac{9}{4}r^3C(1-A)\{Lr^3 + (M + 2L)r + (C + M)\}}{(r^3 + C)(Ar^3 + Br^2 + C)(Lr^3 + Mr + C)} - \frac{1}{r^2}\right] + \frac{1}{2r^2}$

(3.6) $4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{1}{2}\left\{\frac{3r^2(CA - Br^2 + C) + 3r^4B + 2CBr}{(r^3 + C)/(Ar^3 + Br^2 + C)} + \frac{3r^2L + M}{Lr^3 + Mr + C}\right\}F^{11}\right] X \left(\frac{Lr^3 + Mr + C}{4N}\right)^{1/2}$

The constants appearing in the solution can be found using boundary conditions.

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