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An Ideal Solution of the Tarry Escott Problem

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The Tarry Escott problem in Diophantine analysis is that of finding two sets of integer (a), (b) such that

$$\sum_{i=1}^{S} a_i^h = \sum_{i=1}^{S} b_i^h \qquad (h = 1,\ 2,\ \&,\ k) \tag{1}$$

We denoted (1) by the notation

$$a_1, a_2, \ldots, a_s \overset{k}{=} b_1, b_2, \ldots, b_s$$

The problem has attracted a good deal of interest from the time of Goldhach and Euler and has severed interesting applications. In this note we describe a method to find all solution of

$$a_1, a_2, a_3, \ldots \overset{2}{=} b_1, b_2, b_3, \ldots \tag{2}$$

Other methods of finding all solutions of this system of equations are given in [1], [2] and [3].

Let $a_1 = A_1 + S, a_2 = A_2 + S, a_3 = A_3 + S,$

$$b_1 = B_1 + S, b_2 = B_2 + S, b_3 = B_3 + S,$$

Where $3S = a_1 + a_2 + a_3$

But if $a_1 + a_2 + a_3$ is not divisible by 3, put

$$S = a_1 + a_2 + a_3, 3a_1 = A_1 + S, 3a_2 = A_2 + S,$$

$$3a_3 = A_3 + S, 3b_1 = B_1 + S, 3b_2 = B_2 + S, 3b_3 = B_3 + S.$$

Thus is either case,

$$A_1 + A_2 + A_3 = 0 = B_1 + B_2 + B_3 \tag{3}$$

Which these substitution, the system (2) implies

$$A_1A_2 + A_1A_3 + A_2A_3 = B_1B_2 + B_1B_3 + B_2B_3$$

Which in turn gives (in virtue of (3))

$$A_1^2 + A_1A_2 + A_2^2 = B_1^2 + B_1B_2 + B_2^2 \tag{4}$$

Hence the problem reduces to solving the equation (4). To find all its solutions, let *N* be any number all of whose prime factors are of the form 6*k* + 1 or 3, besides any factor common to. $A_1, A_2, B_1, B_2.$ It is well known that any prime of the form 6*k* + 1 or 3 besides any factor common to A_1, A_2, B_1, B_2,. It is well know that any prime of the form 6*k* +1 can be expressed in the form $p^2 + pq + q^2$ and no other prime except 3 can be expressed in this form. Moreover, the product of two numbers each of the form $p^2 + pq + q^2$ can be expressed as a number of the same form in two ways in view of the identities.

$$(p^2 + pq + q^2)(p^{1^2} + p^1q^1 + q^2)$$

$$= \quad p_1^2 + p_1q_1 + q_1^2 = p_2^2 + p_2q_2 + q_2^2$$

where,

$$p^1 = pp^1 - qq^1, q^1 = qp^1 + (p+q)q^1$$

$$p_2 = pq^1 - qp^1, q_2 = pq^1 + \left(p+q\right)p^1$$

Also in the expressions $p^2 + pq + q^2$ we may take *p* > 0, *q* > *o*, since

$$p^2 + pq + q^2 = (p+q)^2 + (p+q)(-q) + (-q)^2$$

$$= (p+q)^2 + (p+q)(-p) + (-p)^2$$

$$N = 91 = 7.13 = (1^2 + 1.2 + 2^2)(1^2 + 1.3 + 3^2)$$

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Ex. $$= 6^2 + 6.5 + 5^2 = 1^2 + 1.9 + 9^2$$

$$A_1 = 6, A_2 = 5, A_3 = -(A_1 + A_2) = -11$$

Taking

$$A_1 = 6, A_2 = 5, A_3 = -(A_1 + A_2) = -11$$

$$B_1 = 1, B_2 = 9, B_3 = -\left(B_1 + B_2\right) = -10$$

We set

$$a_1 = 6 + S, a_2 = 5 + S, a_3 = -11 + S$$

$$b_1 = 1 + S, b_2 = 9 + S, b_3 = -10 + S$$

Taking S = 12, we thus obtain 1, 17, 18 $\overset{2}{=}$ 2, 13, 21

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