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A Smoking cessation model with Holling type-II Anti-smoking Media campaign

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ABSTRACT

A new smoking cessation epidemic model has proposed with the effect of media coverage. In this paper, we depict the smoking cessation model with the effect of media awareness program (Anti-Smoking campaign) among smokers as well as non-smokers. When non-smokers will aware about the bad impact of smoking on society and its related disease, they will try to avoid smoking and also try to stay away with smokers and will make less interactions with smokers. By considering these facts, we proposed and analyzed a smoking cessation model with the effect of media in the form of Holling type-II functional response. In the feasible region, the equilibria are computed and their stability behavior have analyzed. Further, some numerical and computer simulations are performed with the help of MATLAB to validate our analytical findings. In last section, discussion and conclusion of our findings are discussed.

1. Introduction

Smoking, in particular, is not a disease but it causes the many of deadly disease like stroke, cerebrovascular disease, hypertension and cancer. For the cessation of smoking, government has raised many steps like increasing the tax on tobacco, by the mass media and a short add in between the movies or TV programs. Smoking has become a fashionable trend for the youngsters. The rate of smoking among youngsters having smoker friend circle is 10 times higher than that of which have non-smoker friend circle. According to WHO report, more than 8 million people dies each year by the use of tobacco, which is approx half of its users and among this death statistics more than 7 million people dies from direct use of tobacco and around 1.2 million people dies from indirect contact of tobacco i.e. second-hand smokers [1]. The government and health care organizations continuously focus in the prevention and controlling of addiction of smoking and alcoholism which also encourage the researchers to focus in this area [3,14]. Hence, for lessening the smoking use, education and counseling are more preventive measure than of treatment [3]. Education in people about the bad impact of smoking on the health as well as on society via an anti-smoking campaign is a rational strategy which help in the smoking cessation. The effect of anti-smoking media campaign can be seen in several papers which help in lowering the prevalence of smoking significantly among the adolescents [7,9,11]. Most of the researchers focus on the smokers and influence them by media to quit smoking, but in real life it is very hard for smokers for quit smoking because of the addiction. Therefore, in this paper by keeping the above facts in the mind we also focus the effect of media on the non-smoker population for keeping distance with smoker individual. The first straightforward mathematical model for the dynamics of giving up smoking was proposed by the Castilo-Garsow et al. [3]. They considered an unchanging community categorized into three groups: potential smokers (P), smokers (S) and quitters (Q). Later on, Sharomiand Gumel[10] inserted the two classes of smokers who temporarily quit smoking and who permanently quit smoking. Furthermore, Zaman [13] analyzed the giving up smoking model by taking into account the occasional smokers compartment. For more detail about the giving up smoking model with different strategy.

In this paper, we see the effect of media on the non-smokers population. The rest of the paper is organized as follows: In section 2 a more realistic smoking cessation model is proposedwith media coverage on non-smokers, we also use the Holling type-II (Saturation Function) to manifest the effect of media motivated by the Wang et al. [12]. Section 3 deals with the positivity and boundedness of the model. In section 4, the existence and stability behavior of the possible equilibria are explored. Finally some numerical simulations and conclusion of the paper are given in section 5 and 6 respectively

2. Model development

We consider a feasible region of total population T. The total population is splitted into three compartments, namely: non-smokers (N), Smokers (S), quitters (Q) with an additional anti-smoking media compartment (M). It is assumed that the total population is constant in a manner that the mortality rate is balanced with the influx rate of the people. The proposed mathematical model is given below:

$$\frac{dN(t)}{dt} = \omega - \beta\left(1 - \frac{kM(t)}{a+kM(t)}\right) N(t)S(t) - \omega N(t),$$

$$\frac{dS(t)}{dt} = \beta\left(1 - \frac{kM(t)}{a+kM(t)}\right) N(t)S(t) - \mu S(t)M(t) + \delta S(t)Q(t) + \alpha Q(t) - \omega S(t),$$

$$\frac{dQ(t)}{dt} = \mu S(t)M(t) - \delta S(t)Q(t) - (\alpha+\omega)Q(t), \qquad (2.1)$$

$$\frac{dM(t)}{dt} = \varepsilon S(t) - \varepsilon_0(M(t) - M_0)$$

With $N(0)>0$, $S(0)>0$, $Q(0)\geq 0$, $M(0)=M_0\geq 0$. Where N(t), S(t), Q(t) and M(t) denotes the densities of Non-smokers population, Smokers population, population who quit smoking and anti-smoking media campaign respectively. The parameters used in the model are described in the Table 1.

Table 1 Parameters of the model.

Parameters	Description	Unit
ω	Influx rate and mortality rate	$time^{-1}$
β	Transmission rate	$time^{-1}$
a	Saturation rate	---
k	Number of people influenced by media	---
μ	Dissemination rate of anti-smoking campaign among smokers	$time^{-1}$
δ	Rate at which quitters smoke again due to peer influence	---
α	Rate at which quitters smoke again without peer influence	$time^{-1}$
ε	Implementation rate of anti-smoking campaign	$time^{-1}$
ε_0	Fading rate of anti-smoking campaign	$time^{-1}$
M_0	Baseline media campaign	$time^{-1}$

3. Positivity and Boundedness

3.1. Positivity of the model

In this subsection, we have shown that the state variable of the model system (2.1) are non- negative for all $t>0$. We have the following result:

Lemma 3.1 Let the initially $N(0)>0$, $S(0)>0$, $Q(0)>0$ and $M(0)>0$. Then the solution $(N(t),S(t),Q(t),M(t))$ of model system (2.1) are non-negative, $\forall\ t>0$.

Proof: Suppose that conclusion doesn't satisfy, then at least one of N(t), S(t), Q(t), M(t) is not positive. Thus, we have one of the following four case.

(1) There exists a first time t_1 such that,

$N(t_1)=0$, $N^{'}(t_1)<0$, $S(t)>0$, $Q(t)>0$, $M(t)>0$, $0\leq t\leq t_1$.

(2) There exists a first time t_2 such that,

$S(t_2)=0$, $S^{'}(t_2)<0$, $N(t)>0$, $Q(t)>0$, $M(t)>0$, $0\leq t\leq t_2$.

(3) There exists a first time t_3 such that,

$Q(t_3)=0$, $Q^{'}(t_3)<0$, $N(t)>0$, $S(t)>0$, $M(t)>0$, $0\leq t\leq t_3$.

(4) There exists a first time t_4 such that,

$M(t_4)=0$, $M^{'}(t_4)<0$, $N(t)>0$, $S(t)>0$, $Q(t)>0$, $0\leq t\leq t_4$.

In case (1), we have,

$N^{'}(t_1)=\omega >0$,

Which is contradiction to $N^{'}(t_1)<0$.

In case (2), we have,

$S^{'}(t_2)=\alpha Q(t)>0$,
Which is contradiction to $S^{'}(t_2)<0$.

In case (3), we have,

$Q^{'}(t_3)=\mu S(t)M(t)>0$,
Which is contradiction to $Q^{'}(t_3)<0$.

In case (4), we have,

$M^{'}(t_4)=\varepsilon S(t)+\varepsilon_0 M_0>0$,

Which is contradiction to $M^{'}(t_4)<0$. Thus, the solution $(N(t),S(t),Q(t),M(t))$ of model system (2.1) are non-negative for all $t>0$.

3.2 Boundedness of the model

It is important to find the boundedness of the model (2.1), means the natural restriction togrowth because of limited resources. Hence, we can establish the following lemma.

Lemma 3.2 All feasible solution N(t), S(t), Q(t), M(t) of equation (2.1) are bounded by the region of attraction:

$$\Gamma = \{(N,S,Q,M) \in \mathbb{R}^4_+ : 0 \leq N+S+Q=T \leq 1,\ 0 \leq M \leq \frac{\varepsilon}{\varepsilon_0}+M_0\}$$

Proof. We have,

$T(t) = N(t) + S(t) + Q(t)$.

Taking the time derivative along the solution of the model system (2.1), we obtain

$\frac{dT(t)}{dt} = \frac{dN(t)}{dt} + \frac{dS(t)}{dt} + \frac{dQ(t)}{dt}$,

This gives $\frac{dT(t)}{dt} = \omega - \omega(N(t)+S(t)+Q(t)) = \omega - \omega T(t)$,

Which implies ,

$$\lim_{t\to\infty} sup T(t) \le 1.$$

Now, from the fourth equation of system (2.1), we have

$\frac{dM(t)}{dt} = \varepsilon S(t) - \varepsilon_0(M(t) - M0)$

it follows that,

$\lim_{t\to\infty} sup M(t) \le \frac{\varepsilon}{\varepsilon_0} + M_0$.

Hence, for the analysis of system (2.1), we get the following set of region of attraction:

$$\Gamma = \{(N, S, Q, M) \in \mathbb{R}^4_+ : 0 \le N+S+Q=T \le 1,\ 0 \le M \le \frac{\varepsilon}{\varepsilon_0} + M_0\},$$

which is positively invariant set for the model (2.1). So we need to consider the dynamics of model inside the feasible region Γ.

4 Possible equilibria and their stability analysis

4.1 Existence of equilibria

In this subsection, we find the possible equilibria for the system (2.1). Since the total population is constant over time i.e. N + S + Q = 1; therefore, by using Q = 1 - N – S, the system (2.1) can be reduced to the following system,

$$\frac{dN(t)}{dt} = \omega - \beta\left(1 - \frac{kM(t)}{a+kM(t)}\right) NS - \omega N,$$

$$\frac{dS(t)}{dt} = \beta\left(1 - \frac{kM}{a+kM}\right) NS - \mu SM + \delta S(1-N-S) + \alpha(1-N-S) - \omega S, \qquad (4.1)$$

$$\frac{dM(t)}{dt} = \varepsilon S - \varepsilon_0(M - M0).$$

This system (4.1) always possess a smoking-free equilibrium $E_0(1,0,M_0)$ and smoking present equilibriumE*(N*,S*,M*). By putting all the derivatives of system (4.1) equal to 0, we have

$N^* = \frac{\omega(\varepsilon_0(a+kM_0)+\varepsilon k S^*)}{\beta a \varepsilon_0 S^* + \omega(\varepsilon_0(a+kM_0)+\varepsilon k S^*)}$, $M^* = M_0 + \frac{\varepsilon}{\varepsilon_0} S^*$.

Using these values in the second equation of system (4.1), i.e. in $\frac{dS}{dt} = 0$, we get the following polynomial $p_1 S^{*2} + p_2 S^* - p_3 = 0$,

where, $p_1 = \delta\varepsilon_0(\beta a\varepsilon_0 + \omega k\varepsilon) + \mu\varepsilon(\beta a\varepsilon_0 + \omega\varepsilon)$,

$p_2 = \varepsilon_0(\beta a\varepsilon_0 + \omega k\varepsilon)(\mu M_0 + \alpha + \mu) + \omega\varepsilon_0(a + kM_0)(\mu\varepsilon + \delta\varepsilon) - \delta\beta a\varepsilon^2_0$,

$p_3 = \omega\varepsilon^2_0(a + kM_0)(\mu M_0 + \alpha + \mu)(\mathcal{R}_0 - 1)$.

Clearly $p_1 > 0$ for all parameter. Sign of p_2 and p_3 may vary for different parameter values. Sign of p_3 depends upon $(\mathcal{R}_0-1)$, where

$\mathcal{R}_0 = \frac{\beta\ a\ (\alpha+\omega)}{\omega(a+k\ M_0)(\alpha+\mu M_0+\omega)}$.

If $\mathcal{R}_0>1$, then there exist a unique positive equilibrium E* and if $\mathcal{R}_0<1$, then both roots may exist positively with the condition $p_2<0$ and $p^2_2 + 4p_1 p_3 > 0$, otherwise none. This implies that multiple equilibria may exist for $\mathcal{R}_0<1$.

4.2 Stability analysis

In this subsection, we will discuss about the stability behavior of the equilibrium point E_0 and E*

4.2.1 Local stability of smoking free equilibrium

The Jacobian matrix corresponding to model system (4.1), at smoking free equilibrium E_0 given by

$$J_{E0} = \begin{pmatrix} -\omega & -\frac{\beta a}{a+kM_0} & 0 \\ -\alpha & \frac{\beta a}{a+kM_0} - (\mu M_0 + \alpha + \omega) & 0 \\ 0 & \varepsilon & -\varepsilon_0 \end{pmatrix}.$$

From the above matrix, we can easily obtain that the one eigenvalue is $-\varepsilon_0$ and the remaining two eigen values can be obtain by the following quadratic equation

$$\lambda^2 + (2\omega + \mu M_0 + \alpha - \frac{\beta a}{a+kM_0})\lambda + \omega(\mu M_0 + \alpha + \omega)(1 - \mathcal{R}_0) = 0.$$

Here, it is clear that all eigenvalues of J_{E0} are negative if $\mathcal{R}_0<1$, whereas $\mathcal{R}_0>1$ produces one positive eigenvalue. Therefore, we can assure that the smoking free equilibrium is locally asymptotically stable if $\mathcal{R}_0<1$, otherwise unstable.

4.2.2 Local stability of smoking present equilibrium

For the local stability analysis of smoking present equilibrium E* , the corresponding jacobian matrix of model system (4.1) is given by

$$J_{E^*} = \begin{pmatrix} -\frac{\beta a S^*}{a+kM^*} - \omega & -\frac{\beta a N^*}{a+kM^*} & \frac{\beta ak N^* S^*}{(a+kM^*)^2} \\ \frac{\beta a S^*}{(a+kM^*)} - \delta S^* - \alpha - \delta S^* & \frac{\alpha(1-N^*)}{S^*} & -\frac{\beta ak N^* S^*}{(a+kM^*)^2} - \mu S^* \\ 0 & \varepsilon & -\varepsilon_0 \end{pmatrix}.$$

The characteristic equation for J_{E^*} is given by the following polynomial

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0,$$

where,

$$A_1 = \frac{\beta a S^*}{a+kM^*} + \delta S^* + \omega + \epsilon_0 + \frac{\alpha(1-N^*)}{S^*} \;,$$

$$A_2 = \left(\omega + \frac{\beta a S^*}{a+kM^*}\right)\left(\delta S^* + \frac{\alpha(1-N^*)}{S^*}\right) + \epsilon\left(\frac{\beta ak N^* S^*}{(a+kM^*)^2} + \mu S^*\right) + \epsilon\left(\delta S^* + \omega + \frac{\alpha(1-N^*)}{S^*} + \frac{\beta a S^*}{a+kM^*}\right) + \frac{\beta a N^*}{a+kM^*}\left(\frac{\beta a S^*}{a+kM^*} - \delta S^* - \alpha\right),$$

$$A_3 = \epsilon_0\left(\frac{\beta a S^*}{a+kM^*} + \omega\right)\left(\delta S^* + \frac{\alpha(1-N^*)}{S^*}\right) + \frac{\beta ak\epsilon N^* S^*}{(a+kM^*)^2}(\omega + \alpha + \delta S^*) + \frac{\beta a\mu\epsilon\, {S^*}^2}{a+kM^*} + \frac{\beta a\epsilon_0 N^*}{a+kM^*}\left(\frac{\beta a S^*}{a+kM^*} - \delta S^* - \alpha\right).$$

It is noted that the coefficient A_3 is positive provided $\left(\frac{\beta a S^*}{a+kM^*} - \delta S^* - \alpha\right) > 0.$

Further, some simple calculations yields that A_1A_2-A_3 is also positive, provided the same condition. Hence with the help of Routh-Hurwitz criterion, we claim that the endemic equlibrium is locally asymptotically stable under given condition.

4.2.3 Global stability of smoking free equilibrium

In this subsection, we will prove the global stability of smoking free equilibrium with the help of method given in [2]. Since,

$\frac{dM(t)}{dt}$= εS-$\varepsilon_0(M-M_0) \leq \varepsilon$ -$\varepsilon_0(M-M_0)$ as $S<1$ (constant total population). Then $M \to \frac{\epsilon}{\epsilon_0} + M_0$ as $t \to \infty$.Therefore taking M in limiting case, the model system (4.1) reduces to the following

$$\frac{dN}{dt} = \omega - \frac{\beta a\epsilon_0}{\epsilon_0(a+kM_0)+\epsilon k}NS - \omega N,$$

(4.2)

$$\frac{dS}{dt} = \frac{\beta a\epsilon_0}{\epsilon_0(a+kM_0)+\epsilon k}NS - \mu\left(\frac{\epsilon}{\epsilon_0} + M_0\right)S + (\delta S + \alpha)(1 - N - S) - \omega S.$$

Let X=(N) and Z=(S). Here $U^0 (X^0, Z^0)$, where $X^0 = (1)$ and $Z^0 = (0)$. At $Z=Z^0$,

$G(X, 0) = (1)$. We have,

$$\frac{dN}{dt} = \omega - \frac{\beta a\epsilon_0}{\epsilon_0(a+kM_0)+\epsilon k}NS - \omega N \leq \omega - \omega N. \qquad (4.3)$$

This system (4.3) is globally asymptotically stable i.e. as t→ ∞, N→ N^0.

Thus the condition (H1) of [2] is satisfied. Now, from the remaining infected compartment of (4.2), we get

$$\frac{dZ}{dt} = G(X, Z) = AZ - \bar{B}(X, Z),$$

where,

$$A = \left(\frac{\beta a \epsilon_0}{\epsilon_0(a + kM_0) + \epsilon k}\right) - \mu\left(\frac{\epsilon}{\epsilon_0} + M_0\right) - (\mu + \sigma + \rho S))$$

and$\bar{B}(X, Z) = 0$.

Clearly, A is an M-matrix (off-diagonal elements of A are non-negative, which is absent andhence trivial) and also$\bar{B}(X, Z) \geq 0$.Thus, both the conditions (H1) and (H2) of [2] are satisfied. Hence, the SFE is GAS if $\mathcal{R}_0$<1.

5 Numerical simulation and discussion

In this section, to justify our analytical findings, we perform some numerical simulations for the system (4.1) using MATLAB. The parameter values taken for the simulation are as follows:

$$\omega = 0.03\,, \beta = 0.52, a = 0.4, \mu = 0.002\,, k = 6\,, \delta = 0.34, \alpha = 0.8\,,$$
$$\epsilon = 0.5, \epsilon_0 = 0.14\,, M_0 = 1\,.$$

The corresponding equilibrium point are, N* = 0.9817, S^* = 0.0182, M^*= 1.0650, and the basic reproduction number$\mathcal{R}_0$>1.

If we choose

$$\omega = 0.13, \beta = 0.52, a = 0.4, \mu = 0.002\,, k = 6\,, \delta = 0.34, \alpha = 0.8\,, \epsilon = 0.5,$$
$$\epsilon_0 = 0.14\,, M_0 = 1\,.$$

then the system exhibit a smoking free equilibrium (1, 0, 1) with the basic reproduction number $\mathcal{R}_0$<1.

The local and global stability of SFE is shown in fig 1 and fig 2 in which all the solution trajectories approaches to its equilibrium point. Further, to show the local stability of smoking present equilibrium point for the above set of parameters, we plotted the solution trajectories in fig 3 with different initial conditions which shows that all the solution trajectories approaches towards smoking present equilibrium E^*inside the region of attraction.

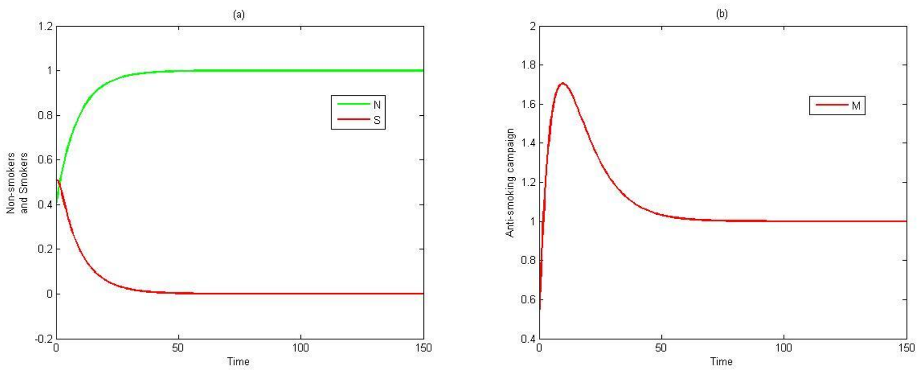

Fig. 1 Local Stability of SFE when $\mathcal{R}_0$<1.

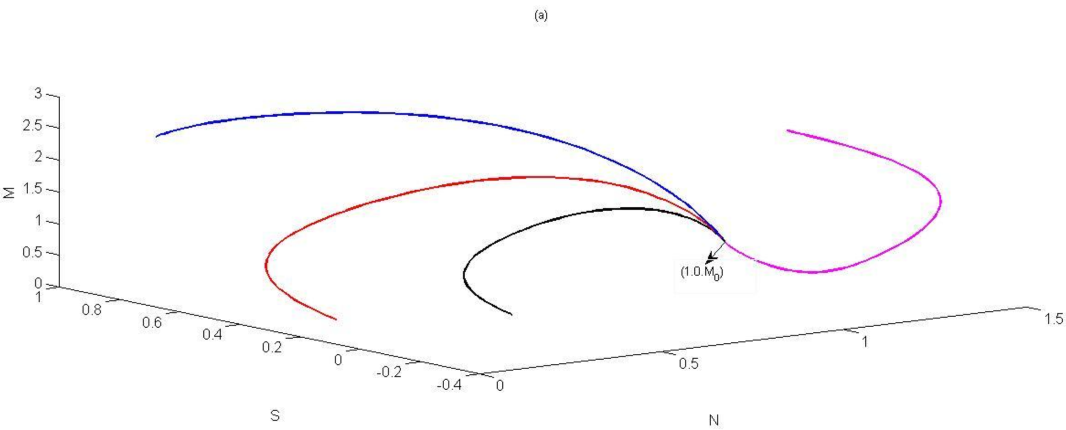

Fig. 2 Global Stability of SFE when $\mathcal{R}_0$<1.

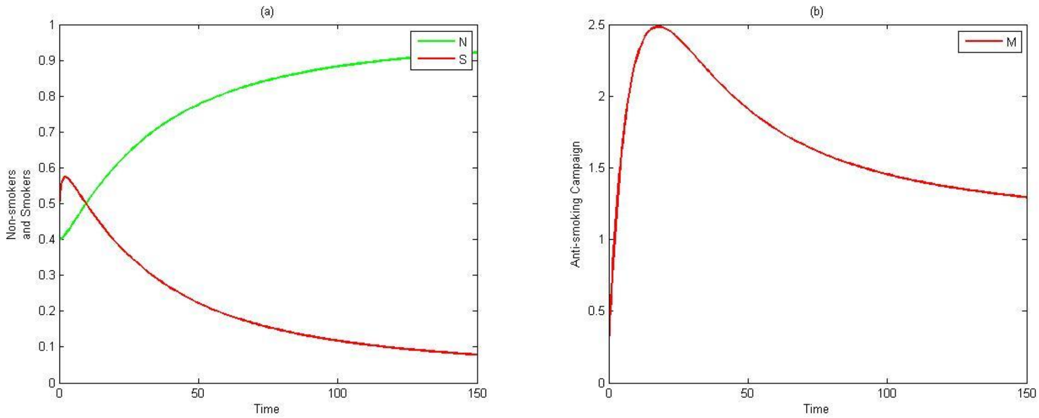

Fig. 3 Local Stability of smoking present equilibrium when $\mathcal{R}_0$<1.

6 Discussion and Conclusions

In this paper, A realistic and deterministic smoking cessation model has been proposed andanalyzed.We have incorporated the media awareness in the form of saturation (Holling type-II), which affect the non-smokers also by propagation of awareness about smoking related issues. In the analysis of mathematical model, firstly we find the region of attraction i.e. positivity andboundedness for the model. Further, we show the existence of two equilibria, which is smoking free equilibrium $E_0 = (1, 0, M_0)$, and E* which exist if $\mathcal{R}_0>1$.If the reproduction number $\mathcal{R}_0<1$. Then the SFE is locally asymptotically stable, whereasSFE becomes unstable if the value of$\mathcal{R}_0>1$. And smoking present equilibrium exists. This result implicates that number of smokers can be effectively controlled in a region by keeping the value of $\mathcal{R}_0$less than unity. Moreover, the local and global stability of SFE equilibrium has been shown by the Jacobian approach under some condition. Also, the local stability of smoking present equilibrium is shown by the Jacobian approach under certain condition, where as the global stability of smoking present equilibrium is shown numerically. Furthermore, some numerical simulation has been performed with assumed parametric values in justification of our analytical findings.

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