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Static Fluid Spheres with Equation of State $\boldsymbol{\rho = ap}$

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ABSTRACT

The present paper provides an exact static spherically symmetric solution of Einstein's field equation for the perfect fluid with ρ=ap.

1. INTRODUCTION

In fact the rise of interest in the theory of General Relativity as a tool for studying the evolution and behaviour of various cosmological models has been rapid and extensive. One reason for the prominence of modern relativity is its success in predicting the behaviour of large scale phenomena where gravitation plays a dominant role. Schwarzschild [18] considered the perfect fluid spheres with homogenous density and isotropic pressure in general relativity and obtained the solutions of relativistic field equations. Tolman [24] developed a mathematical method for solving Einstein's field equations applied to static fluid spheres in such a manner as to provide explicit solutions in terms of known analytic functions.

These new solutions were used by Oppenheimer and Volkoff [17] in the study of massive neutron cores. Krori [11] obtained exact solutions for some dense massive spheres and pointed out their astrophysical implications. Mehra et al [16] have obtained general solutions of the field equations for a composite sphere having a number of Shell's of different densities. Durgapal and Gehlot [5] have obtained exact internal solutions for dense massive stars in which the central pressure and density are infinitely large. Durgapal and Gehlot [6, 7] have further obtained exact solutions for a massive sphere with two different density distributions. Static and non-static solutions of Eintein's field equation have been extensively discussed by Leibovitz {[12], [13]} for the spherical distribution.

Sing and Abdussattar [19] obtained an exact solution of Einstein's field equation for a homogenous perfect fluid core surrounded by a frozen photon field. Teixeira et al [23] obtained and exact solutions of an unbounded plane symmetric distribution of disorded radiation. Davidson [4] had presented a solutions that provides a non- stationary analog to the static case when $\rho = 3p$ again depending only on algebraic functions of the space- co- ordinate τ. It is interpreted as an expanding perfect fluid cylinder of infinite radius.

Solutions with simple equation of state have been found in various cases, e.g for $\rho = p$ (Letelier [14], Letelier and Tabensky [15]), $\rho + 3p =$ constant (Whittakar [27]), $\rho = 3p$ (Klien [10], Sing and Abdussattar [19], Fenstien and Senovilla [18]), for p= $\rho +$ constant (Buchdahl and Land [3]), and for $\rho = (1 + a)\sqrt{p} - ap$ (Buchdahl [2]). But if one takes e.g polytropic fluid sphere $p = a\rho^{1+\frac{1}{n}}$ (Klien [9], Tooper [25] or a mixture of ideal gas and radiation (Suhonen [21]), one soon has to use numerical method. Sing and Yadav [20] have also studied the static fluid sphere with the equation state p= ρ (i.e Zeldovich fluid). Further study in this line has been done by Yadav and Saini [28], Yadav and Purushottam [29] and Acharya et. al. [1].

To overcome the difficulty of infinite density at the centre, it is assumed that the distribution has a core of radius r_o and constant density ρ_o which is surrounded by the fluid with $\rho = ap$. For different values of a, we get many previously reported results. Hence our solution may be considered more general**.**

2. THE FIELD EQUATIONS AND THEIR SOLUTIONS

We take the line element in the form

(2.1) $$ds^2 = -exp(\lambda)dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + exp(\nu)dt^2$$

Where λ and ν are functions of r only.

The field equations

(2.2) $$R_j^i - \frac{1}{2}R\delta_j^i = -8\pi T_j^i$$

For the metric (2.1) are (Toleman [24])

(2.3) $$-8\pi T_1^1 = exp(-\lambda)\left(\frac{V'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

(2.4) $$-8\pi T_2^2 = -8\pi T_3^3 = exp(-\lambda)\left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v' - \lambda'}{2r}\right),$$

(2.5) $$8\pi T_4^4 = exp(-\lambda)\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Where a prime denotes differentiation with respect to r. throughout investigation we set velocity of light C and gravitational constant K to be unity.

The energy momentum tensor is given by

(2.6) $$T_j^i = (\rho + p)v^i v_j - \delta_j^i p$$

We choose the equation of state as

(2.7) $$\rho = ap$$

Where a is positive constant.

In this case we find that

$$T_j^i = 0 (i \neq j)$$

We use commoving co-ordinates so that

$$u^1 = u^2 = u^3 = 0 \; and \; u^4 = \exp\left({-u}/{2}\right)$$

The non- vanishing components of energy momentum tensor are

$$T_1^1 = T_2^2 = T_3^3 = -p \; and \; T_4^4 = \rho$$

We can then write

(2.8) $$8\pi p = exp(-\lambda)\left(\frac{v'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

(2.9) $$8\pi p = exp(-\lambda)\left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r}\right)$$

(2.10) $$8\pi p = exp(-\lambda)\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Using equations (2.7), (2.8) and (2.10) we have

(2.11) $$a \, exp(-\lambda)\left(\frac{v'}{r} + \frac{1}{r^2}\right) - \frac{a}{r^2} = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

We choose $e^{\lambda} = k_1$ (a constant) which reduces (2.11) to the form

(2.12) $$v' + \frac{1}{r}\left[(1 - k_1)\left(1 + \frac{1}{a}\right)\right] = 0$$

Integrating we get

(2.13) $$e^{v} = k_2 r^{(k_1 - 1)(1 + \frac{1}{a})}$$

Where k_2 is a costant. Now (2.8) and (2.9) lead to $k_1 = 2 \; so \; that$

(2.14) $$e^{v} = k_2 r^{\left(1 + \frac{1}{a}\right)}$$

Consequently the metric (2.1) can be put into the form

(2.15) $$ds^2 = -2dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + k_2 r^{\left(1 + \frac{1}{a}\right)} dt^2$$

Absorbing the constant $k_2$in the co-ordinate differential dt the metric (2.15) is reduced to the form.

(2.16) $$ds^2 = -2dr^2 - r^2(d\theta^2 + sin^2\theta \phi^2) + r^{\left(1 + \frac{1}{a}\right)_{dt^2}}$$

The non-zero components of Riemann- Christoffel curvature tensor R_{hijk} for the metric (2.16) are

(2.17) $$R_{2424} sin^2\theta = \frac{R_{2323}}{2}\left(1 + \frac{1}{a}\right) r^{\left(\frac{1}{a} - 1\right)}$$

$$R_{3434} = \left(-\frac{1}{4}\right)\left(1 + \frac{1}{a}\right) r^{\left(1 + \frac{1}{a}\right)} sin^2\theta$$

and $$R_{1414} = \frac{1}{4}\left(1 - \frac{1}{a^2}\right) r^{\left(\frac{1}{a} - 1\right)}$$

Choosing the orthonormal tetrad $\lambda_{(j)}^{-i}$ as,

(2.18)
$$\left[\begin{array}{l} \lambda^{-i}_{(1)} = \left(\frac{1}{\sqrt{2}}, 0,\ 0,\ 0\right) \\ \lambda^{-i}_{(2)} = \left(0,\ \frac{1}{r},\ 0,\ 0\right) \\ \lambda^{-i}_{(3)} = \left(0,\ 0,\ \frac{1}{r\ sin\theta}, 0\right) \\ \lambda^{-i}_{(4)} = \left(0,\ 0,\ 0,\ \frac{1}{r^{\left(\frac{a+1}{2a}\right)}}\right) \end{array}\right.$$

The physical components $R_{(abcd\,)}$ of the curvature tensor defined by

$$R_{(abcd\,)} = \lambda^{-h}_{(a)} \quad \lambda^{-i}_{(b)} \quad \lambda^{-j}_{(c)} \quad \lambda^{-k}_{(d)} R_{hijk}$$

Are

(2.19)
$$R_{2424} = R_{3434} = \left(\frac{a+1}{2}\right) R_{2323} = -\frac{(a+1)}{4ar^2}$$

Since a is finite positive constant, we see that

$$R_{(abcd\,)} \to 0\ as\ r \to \infty$$

Hence it follow that the space time is asymptotically homaloidal. For the metric (2.16) the fluid velocity v^i is given by

(2.20)
$$v^1 = v^2 = v^3 = 0, v^4 = \frac{1}{r^{\left(\frac{a+1}{2a}\right)}}, v_4 = r^{\frac{a+1}{2a}}$$

The scalar of expansion $\theta = v^i_{;i}$ is identically zero. The non- vanishing components of the tensor of rotation w_{ij} defined by

(2.21)
$$w_{ij} = v_{i;j} - v_{j\,;i}$$

are

(2.22)
$$w_{14} = -w_{41} = -\left(\frac{a+1}{2a}\right) r^{\left(\frac{1-a}{2a}\right)}$$

The components of the shear tensor σ_{ij} defined by

(2.23)
$$\sigma_{ij} = \frac{1}{2}\left(v_{i;j} + v_{j\,;i}\right) - \frac{1}{3}\, \Theta hij$$

With the projection tensor

(2.24)
$$h_{ij} = g_{ij} - v_i v_j$$

are

(2.25)
$$\sigma_{14} = \sigma_{41} = \frac{1}{2}, \frac{a+1}{2a}\ r^{\frac{1-a}{2a}}$$

While other components are zero.

For the particular values of parameter a, several previously known solutions are contained herein. When a= 1 the solutions of this section reduces to the solutions obtained by Singh and Yadav [20] for Zeldovich fluid distribution. Also in this case the relative mass m of a particle in the gravitational field of (2.16) is related to its proper mass m_0 through

(2.26)
$$\frac{m}{m_0} = \frac{K^2}{r^2}$$

K being a constant, As the particle moves towards the origin, m increase and as r→ ∞, $m \to 0$ i.e the relative mass goes on decreasing continuously.

The case when a= 3 gives the distribution of disordered radiation obtained by singh and Abdussattar. [19]

3. SOLUTION FOR THE PERFECT FLUID CORE:

Pressure and density for the metric (2.16) (when a =1) are

(2.27)
$$8\pi p = 8\pi\rho = \frac{1}{2r^2}$$

It follows from (2.7) that the density of the distribution tends to infinity as r tends to zero and it varies inversely as the square of the radial distance from the centre of symmetry. In order to get rid of the singularity in the density we visualize that the distribution has a core of radius r_0 and constantρ_0. The field inside the core is given by the Schwarschild internal solution.

$$e^{-\lambda} = 1 - \frac{r^2}{R^2} \tag{2.28}$$

$$e^{\nu} = \left[A - B\left(1 - \frac{r^2}{R^2}\right)^{1/2}\right]$$

$$8\pi p = \frac{1}{r^2}\left[\frac{3B\left(1-\frac{r^2}{R^2}\right)^{1/2} - A}{A - B\left(1-\frac{r^2}{R^2}\right)^{1/2}}\right]$$

where A and B are constant and

$$R^2 = \frac{3}{8\pi\rho_0}$$

The constants appearing in the solution can be evaluated by the continuity conditions for the metric (2.16) and (2.28) at the boundary r=r_0.

4. DISCUSSION:

Our solutions of the field equations obtained in this chapter using equation of state $\rho = ap$ include various previously known solutions e.g a=1, gives the solutions due to Singh and Yadav [20] and a=3 gives the solutions obtained by singh and Abdussattar [19]. Again when a=1, the fluid has the equations of state p= ρ which describe several important cases, e.g radiation, relativistic degenerate Fermi gas and probably very dense baryon matter (Zeldovich and Novikov [30], Walecka [26]

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