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An Exact Analytical Solution for Anisotropic Fluid Sphere

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ABSTRACT

The present paper provides an exact analytical solution of Einstein's Field equations for static anisotropic fluid sphere assuming that space-time is conformally flat and by taking a judicious choice of energy densityρ. The solution is physically reasonable, singularity free and various physical parameters have been found.

1. INTRODUCTION

There are number of interesting solutions that have provided into the effects of anisotropy on star parameters [5.6]. However, many of these solutions have a limited applicability to astrophysical situations since they do not satisfy certain physical restriction usually imposed upon density and pressure, Viz. that the pressure should not exceed the energy density (dominant energy condition), and that the (adiabatic) derivatives of the pressure with respect to the density should be less than or equal to unity [4] (macro causality condition). Exact analytical solutions of Einstein's Field equations are of much value in general relativity. These solutions are generally obtained by using different conditions and assumptions. One of the assumptions made for obtaining the solution is that the space time be conformably flat. This assumption has been widely used in relativity theory [1, 3,7,8,9 and 10].

Here in this paper we have obtained two exact analytical solutions of Einstein's field equations for static anisotropic fluid spheres by assuming that space-time is conformally flat. In first case we have used a judicious choice of energy density ρ and in the second model we have chosen a suitable form of metric potential g_{11}. Both the models are physically reasonable and free from singularities energy density ρradial and tangential pressures have been calculated for both the models. It is seen that densities for these models drop continuously from their maximum values at the centre to the values which are positive at the boundary.

2. The field Equations and their solutions.

Consider the line element in the form

(2.1) $$ds^2 = e^{\beta} dt^2 - e^{\alpha} dr^2 - r^2(d\theta^2 + \sin 2\,\theta d\emptyset^2)$$

Where α and β are functions of r only.

The Einstein's field equations in general relativity.

(2.2) $$R_j^i - \frac{1}{2} R\delta_j^i = -8\pi\, T_j^i$$

For the metric (2.1) are

(2.3) $$-8\pi T_1^1 = e^{-\alpha}\left[\frac{\beta}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}$$

(2.4) $$-8\pi T_2^2 = -8\pi T_3^3 - e^{-\alpha}\left[\frac{\beta''}{2} + \frac{\alpha'\beta'}{4} + \frac{\beta'2}{4} + \frac{(\beta' - \alpha')}{2r}\right]$$

(2.5) $8\pi T_4^4 = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$

Where a prime denotes differentiation with respect to r. Throughout the investigation we set velocity of light c and gravitational constant K to be unity. The energy momentum tensor is given by

`(2.6) $T_j^i = (\rho + p)u^i u_j - p\delta_j^i$

The above field equations for anisotropic fluid sphere provides us

(2.7) $e^{-\alpha}\left(\frac{1}{r^2} - \frac{\alpha'}{r}\right) - \frac{1}{r^2} = -8\pi\rho$

(2.8) $\frac{1}{r^2} - e^{-\alpha}\left(\frac{1}{r^2} + \frac{\alpha'}{r}\right) = -8\pi pr$

(2.9) $e^{-\alpha}\left[\frac{1}{4}\alpha'\beta' - \frac{1}{4}\beta'^2 - \frac{1}{2}\beta'' - \frac{1}{2}\left(\frac{\beta' - \alpha'}{r}\right)\right] = -8\pi p_1$

Where ρ is energy density and P_r and P_1 are the radial tangential "pressure" respectively.

For the spherically symmetric metric (2.1) non vanishing components of the Weyl tensor.

(2.10) $C_{1212} = \frac{1}{12}r\beta' + \frac{1}{12}r\alpha' - \frac{1}{6}e^{\alpha} + \frac{1}{6} - \frac{1}{24}r^2\alpha'\beta' - \frac{1}{24}r^2\beta^2 + \frac{1}{12}r^2\beta''$

$C_{1313} = \sin^2\theta\, C_{1212},$

$C_{1010} = 2\frac{e^{\beta}}{r^2}C_{1212}$

$C_{2323} = -sin^2\theta\, e^{-\alpha}r^2 C_{1212,}$

$C_{2020} = e^{\beta-\alpha}C_{1212}$

$C_{3030} = -sin^2\theta\, e^{\beta-\alpha}C_{1212,}$

We suppose that the space-time is conformally flat for which vanishing of Weyl tensor give

(2.11) $\frac{e^{\alpha}}{r^2} - \frac{1}{r^2} - \frac{\beta'^2}{4} + \frac{\beta'\alpha'}{4} - \frac{\beta''}{2} - \frac{1}{2r}(\alpha' - \beta') = 0$

Now we use the transformations.

(2.12)

$e^{-\alpha} = \tau$

(2.13) $\beta = 2logy$

(2.14) $r^2 = z$

So that equations (2.8), (2.9) and (2.11) may be combined to give

(2.15) $z\tau,z + 1 - \tau - 4\pi z(p_1 - p_\perp) = 0$

(2.16) $(4\tau z^2)y,zz + (2z^2c,z - \tau + 1)y = 0$

Where the subscript Z following a comma denotes differentiation with respect to z. Integrations of equations (2.15) and (2.16) give

(2.17) $\tau = e^{-\alpha} = 1 + \lambda r^2 + 8\pi r^2 \int_0^r \left[\frac{P_r - P_1}{r}\right] dr$

(2.18) $y^2 = e^{\beta} = r^2 \left[Ce^{\xi(r)} + De^{-\xi(r)}\right]^2$

Where λ , C and D are integration constants and

(2.19) $\xi(r) = \int \frac{e^{\alpha/2}}{r} dr$

The constant λ, C and D can be fixed by matching the metric functions (2.17) and (2.18) to the exterior Schwarzschild solution for a mass M radius r_0 given by

(2.20)

$$C = \frac{e^{\xi(r_0)/2}}{2r_0}\left(\frac{3M}{r_0} - 1 + \frac{(1-2M)^{3/2})}{r_0}\right)$$

(2.21)

$$D = \frac{e^{\xi(r_0)/2}}{2r_0}\left(\frac{1-3M}{r_0} - 1 + \frac{(1-2M)^{3/2})}{r_0}\right)$$

(2.22)

$$e^{-\alpha(r_0)} = \left(\frac{1-2M}{r_0}\right)$$

We see that equations (2.7)-(2.9) and (2.17)-(2.19) are actually three equations in four unknowns ρ P_r, P_1 and $\xi(r)$ and thus the system is intermediate. To make the system determinate we require one more relations or conditions. For this we choose the energy density ρ As

(2.23)

$$8\pi\rho = \frac{3\lambda}{(1+\lambda r^2)}\left(\frac{1}{1+\lambda r^2} + \frac{1}{2}\right)$$

Where λ is a constant to be fixed up by boundary conditions.

The distribution has been already considered by Durgapal and Banerjee [2] for the perfect fluid solutions. Now using (2.23) into (2.28) we get.

(2.24) $e^{\alpha} = \frac{2(1+\mu x)}{2-\mu x}$ where

(2.25) $x = \frac{r^2}{r_0^2}$

(2.26) $\mu = \frac{4M/r_0}{3-4M/r_0}$

and M and r_0 are the mass and radius of the sphere.

Also the function is given by

(2.27) $e^{-\xi} r_0 2 = \exp\left[\sqrt{2} sin^{-1}\left(\frac{1-2\mu x}{3}\right)\right]\left[\frac{4+\mu x + 2\sqrt{2}n}{x}\right]$ with

(2.28) $n^2 = 2 + \mu x - \mu^2 x^2$

Putting this into equations (2.18) and using C and D from (2.20) and (2.21) we can finde^{β}. The density, radial pressure and tangential pressure are obtained as.

(2.29)

$$8\pi\rho r_0^2 = \frac{\mu}{1+\mu x}\left(\frac{1}{1+\mu x} + \frac{1}{2}\right)$$

(2.30)

$$8\pi p_r r_0^2 = \frac{Dx/ce^{-\xi} r_0^2 (4-5\mu x - 2\sqrt{2}n+) + 4x(4-5\mu x + 2\sqrt{2}n)}{2x(1+\mu x)(\mu x + Dx/ce^{-\xi} r_0^2)}$$

(2.31)

$$8\pi p_1 r_0^2 = \frac{3\mu^2 x}{(1+\mu x)^2}\left[\frac{Dx/ce^{-\xi} r_0^2 4-5\mu x - 2\sqrt{2}n + \mu x(4-5\mu x + 2\sqrt{2}n)}{2x(\mu x + D\,x/c)(\mu x + D\,x/c\,e^{-\xi} r_0^2)(1+\mu x)}\right]$$

3. REMARKS

Here we see that solutions obtained in this paper are free from singularities and density of the fluid sphere drop continuously from their maximum values at the centre to the values which are positive at the boundary.

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