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Static Fluid Sphere with Disordered Radiation in General Relativity

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ABSTRACT

In this paper we have obtained solutions of Einstein's field equations for static fluid sphere with disordered radiation using a judicius choice of metric potential g44. Various physical and geometrical properties have been studied.

1. INTRODUCTION

The theories of modern physics generally involve a mathematical model defined by a certain set of differential equations and supplemented by a set of rules for translating the mathematical results into meaningful statements about the physical world. In the case of theories of gravitation, it is generally accepted that the most successful is Einstein theory of general relativity. In fact the rise of interest in the theory of general relativity as a tool for studying to evolution and behaviour of various cosmological model has been rapids and extensive. Since the early 1920's to the present, the Einstein is theory of relativity has been used extensively as a tool in the predicition and modeling of the cosmos. One reason for the prominence of modern relativity is its success in predicting the behaviour of large scale phenomena where gravitation plays a dominant role.

Solution with a simple equation of state have been found in various cases, e.g. of $\rho = p$ (Letrelier [17] and Tabensky [18], $\rho + 3p$ = constant (Wittakar [31], $\rho = 3p$ (Klein [11], Singh and Abdussattar [23], Fenstein and Senovilla [90]), for $p = \rho$ + constant (Buchdah and Land [14]) and for $\rho - (1+a)\sqrt{p} - ap$ (Buchdah [3]. But if one takes e.g. polytropic fluid sphere $\rho = ap^{1+1/n}$ (Klein [10], Topper [28]) or a mixture of ideal gas and radiation (Suhonen [25]), one soon has to use numerical method. Singh and Yadav [24] have also studied the static fluid sphere with the equation of state $p = \rho$ (i.e., zeldovich fluid). Further study in this line has been doen by Yadav and Saini [32] Yadav and Purushottam [33] and Acharya, Yadav Purushottam [1].

The solutions with equation of state $p = 1/3\ \rho$ obtained by Feinstein and Senovilla [9] is not the same as that for the case $\gamma = 1/3$ derived by Wainwright and Goode [30] although in both solutions the g_{ij} depends on simple hyperbolic functions of a space co-ordinate and a time co-ordinate. Some other workers in this line can be seen in references [5, 10, 12-16, 21-23, 26].

In this paper we have obtained an exact, static spherically solution of Einstein's field equations using the equation of state $\rho = 3p$ and also with a judicious choice of metric potential g_{44}. To overcome the difficulty of infinite density at the centre, it is assumed that distribution has a core of radius r_0 which is surrounded by the fluid with $\rho = 3p$. Various physical and Geometrical properties of the model have been discussed.

2. THE FIELD EQUATIONS AND THEIR SOLUTION

We take the line element in the form

(2.1) $$ds^2 = e^v dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

Where v and λ are functions of r only the field equations.

(2.2) $$R^i_j \frac{1}{2} R\delta^i_j = -8\pi T^i_j$$

For the metric (2.1) are (Tolman [29])

(2.3) $$-8\pi T_1^1 = e^{\lambda}\left(\frac{v'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}$$

(2.4) $$-8\pi T_2^2 = -8\pi T_3^3 = e^{-\lambda}\left(\frac{v''}{2}-\frac{\lambda' v'}{4}+\frac{v'^2}{4}+\frac{v'-\lambda'}{2r}\right)$$

(2.5) $$8\pi T_4^4 = e^{-\lambda}\left(\frac{v'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}$$

Where a prime denotes differentiation with respect to r. Throughout the investigation we set velocity of light C and gravitational constant K to be unit. Disordered distribution of radiation can be regarded as a perfect fluid having the energy momentum tensor.

(2.6) $$T_j^i(\rho+p)u'u_j-\delta_j^i p$$

Characterized by the equation of state

(2.7) $$\rho = 3p$$

We use commoving co-ordinates so that

$$U^1 = u^2 = u^3 = 0 \text{ and } u^4 = e^{-v/2}$$

The non-vanishing components of the energy momentum tensor are

$$T_j^i = T_2^2 = T_3^3 = p \text{ and } T_4^4 = \rho$$

We can then write the field equation

(2.8) $$8\pi p = e^{-\lambda}\left(\frac{v'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}$$

(2.9) $$8\pi p = e^{-\lambda}\left(\frac{v''}{2}-\frac{\lambda' v'}{4}+\frac{v'^2}{4}+\frac{v'-\lambda'}{2r}\right)$$

(2.10) $$8\pi\rho = e^{-\lambda}\left(\frac{\lambda'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}$$

Using equation (2.7), (2.8) and (2.10) we have

(2.11) $$3e^{-\lambda}\left(\frac{v'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2} = e^{-\lambda}\left(\frac{\lambda'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}$$

From (2.11) we see that if v is known, λ can be obtained.

So we choose

(2.12) $$e^{v} = Dr^2$$

where D is constant.

Equation (2.13) reduces (2.11) to the form

(2.13) $$e^{-\lambda}\lambda' r - 10e^{-\lambda} + 4 = 0$$

We put $y = e^{-\lambda}$ so that Equation (2.13)

is Changed into the form

(2.14) $$\frac{dy}{dr}+\frac{10y}{r}=\frac{4}{r}$$

which is a linear differential equation whose solution is

(2.15) $$y=\frac{2}{5}+\frac{c}{r^{10}}$$

Therefore we get

(2.16) $$e^{-\lambda}=\frac{2}{5}+\frac{c}{r^{10}}$$

where e is constant.

Consequently the metric (2.1) can be put into the form

(2.17) $$ds^2=Ar^2dr^2-\frac{3}{2}r^{-3}dr^2-r^2(d\theta^2+\sin^2\theta d\phi 2)$$

and $\frac{2}{3}$ co-ordinate differential dr.

Absorbing the constant D in co-ordinate differential at the metric (2.17) goes to the form.

(2.18) $$ds^2=rdt^2-r^{-3},dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)$$

The non vanishing component of Riemann-Christoffel. Curvature tensor R_{hijk} for the metric (2.18) are.

(2.19) $$R_{1212}=\frac{100c}{2r^{10}+5c}$$

$$R_{2424}=\frac{5r^{12}}{(2r^{10}+5c)}$$

$$R_{1313}=\frac{25\sin^2\theta}{2r^{10}+c}$$

$$R_{1414}=\frac{25c}{r(2r^{10}+5c)}$$

$$R_{3434}=\frac{5\sin^2\theta r^2}{(2r^{10}+5c)}$$

$$R_{2323}=\frac{5r^{12}\sin^2\theta}{2r^{10}+5c}$$

Choosing the orthonormal tetrad $\bar{\lambda}^i_{(1)}$

(2.20) $$\bar{\lambda}^i_{(1)}-\left(\frac{5r^{10}}{2r^{10}+5c}\right)^{1/2}0,0,0$$

$$\bar{\lambda}^i_{(2)}=\left(0,\frac{1}{r}0,0\right)$$

$$\bar{\lambda}^{i}_{(3)} = \left(0, 0, \frac{1}{r\sin\theta} 0\right)$$

$$\bar{\lambda}^{i}_{(4)} = \left(0,\ 0, 0\frac{1}{r}\right)$$

The physical components $R_{(abcd)}$ of the curvature tens or defined by

(2.21) $$R_{(abcd)} = \bar{\lambda}^{i}_{(a)}\bar{\lambda}^{j}_{(b)}\bar{\lambda}^{k}_{(c)}\bar{\lambda}^{l}_{(d)}R_{ijkl}$$

(2.22) $$R_{(1212)} = \frac{500Cr^{8}}{(2r^{10}+5c)^{2}}$$

$$R_{(2424)} = \frac{5r^{5}}{2(2r^{10}+5c)}$$

$$R_{(3131)} = \frac{125r^{8}}{(2r^{10}+5c)^{2}}$$

$$R_{(1414)} = \frac{125cr^{2}}{2(2r^{10}+5c)^{2}}$$

$$R_{(3434)} = \frac{5}{2r^{4}(2r^{10}+5c)}$$

$$R_{(2323)} = \frac{2r^{8}}{2r^{10}+5c}$$

We see that $R_{(abcd)} \rightarrow 0 \text{ as } r \rightarrow \infty$. It follows that the space time is asymptotically homoloidal. Also for the metric (2.18) the fluid velocity u’ is given by

(2.23) $$u^{1} = u^{2} = u^{3} = u_{1} = u_{2} = u_{3} = 0$$

and $$u^{4} = \frac{1}{r}, u_{4} = r$$

The scalar or expansion $\theta = u^{i}, i$ is identically zero.

The non vanishing components of the tensor of rotation w_{ij} defined by

(2.24) $$w_{ij} = u_{ij} - u_{j,1}$$

are

(2.25) $$w_{14} = w_{41} = -1$$

The components of the shear tensor o_{ij} defined by

(2.26) $$\sigma_{ij} = \frac{1}{2}(u_{i,j} + u_{ij})\frac{1}{3}\theta h_{ij}$$

With the projection tensor

$$h_{ij} = g_{ij} - u_{1}u_{1}$$

are

$$\sigma_{14} = \sigma_{41} = -\frac{2}{5}$$ (2.27)

The other components being zero.

3. Solution for the perfect fluid core

Pressure and density for metric (2.18) are

$$8\pi\rho = \frac{8\pi\rho}{3} = \frac{r^{10} + 15c}{5r^{12}}$$ (3.1)

It follows from (3.1) that the density of the distribution tends to infinity as r tends to zero. In order to get rid of the singularity at r = 0. In the density we visualize that the distribution has a core of radius r_0 and constant density ρ_0. The field inside the core is given by the Schwarzschild internal solution.

$$e^{-\lambda} = 1 - \frac{r^2}{R^2}$$ (3.2)

$$e = \left[A - B\left(1 - \frac{r^2}{R^2}\right)^{1/2} \right]^2$$

$$3B\left(1 - \frac{r^2}{R^3}\right)^{1/2} - A$$

$$8\pi p = \frac{1}{R^2}\left[\frac{R^2}{r^2}\right]$$

$$A - B\left(\frac{r^2}{R^2}\right)^{1/2}$$

Where A and B are constants and

$$R^2 = \frac{3}{8\pi\rho_0}$$

The constant appearing in the solution can be evaluated by the continuity conditions for the metric.

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