RESEARCH REVIEW JOURNALS
Volume-03
Issue-12
December -2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

Some Static Solutions Compatible with Conformal Motion in General Relativity

Dr. Kamlesh Kumar

+ 2 B.N. High School Agandha, Shahpur, Belaganj (Gaya)

ARTICLE DETAILS

Article History
Published Online: 10 December 2018

Keywords
Ferro fluid, comoving co-ordinates pressure, density, constraints.

ABSTRACT

In the present paper we have found some solutions for anisotropic Ferro fluid systems in commoving co-ordinates admitting one parameter group of conformal motions using different conditions in different cases.

1. Introduction

The complicated nature of gravitational field equations shows that there are many supplementary constrains like infinitesimal point transformations which lead to conformal motion. These are usually imposed on system equations to make it easily integrable. Hence the present investigations manly depend on these basic motivating aspects.

The condition of conformal symmetry is essentially geometric, yet it provides two interesting physical aspects. Firstly it provides a generalization of self similarity in hydrodynamics. Secondly, it is a generalization of the property of incompressible Schawartzschild interior solution, it has eleven independent conformal symmetries besides the four Killing symmetries, since it is conformally flat (Kramer et al. [18]. Further the Schwartzchild interior solution is the most realistic known exact static stellar model. All in all, the restriction of conformal symmetry creats the possibility of making evaluation of exact solutions of field equations, which are generalization of Schwartzschild interior solution. It is to be noted that these models will be of physical interest.

Some exact analytical solutions of the static Einstein-Maxwell equations are investigated by Herrera et al. [6] and [7] for perfect and anisotropic distributions. These are under the assumption of spherical symmetry and the existence of one parameter group of conformal motions. According to them these solutions are the generalization of static spherically symmetric models obtained by Bayin [2] and Humi et al. [5].

The possibility of finding acceptable solutions with the condition of conformal motion is probed into its dependence cruicially on the form of the conformal Killing vector field ξ. Herrera et al. [6] and [7] they have assumed that ξ is not only spherically symmetric but also static and have presented a wide range of solutions. Each of these solution is found to be singular. To avoid a singularity in all these solutions one has to assume ξ to he non-static with the static conformal factor ψ. This will provide the possibility of solutions regular at steller centre. It is found by Maartens and Maharaj [9] that these are regular solutions for charged spheres firstly by considering the anisotropic perfect fluid but later have put measure of anisotropy equal to zero in all cases. Fluid space-time admitting a conformal Killing vector are studied by Coley and Tupper [3]. Coley and Tupper [4] have made a rigorous analysis of space-tiome which admits a covariantly constant vector and which satisfyies the Einstein field equations for an anisotropic fluid.

In this paper the attempt is made to examine the anisotropic ferrofluid space-times admitting one parameter groups of conformal motions with a view to solve the system equations. By using the conditions of conformal motions and constraints of commoving system the field equations are formulated in explicit form in section 2 for he anisotropic ferrofluid system. The next part includes varied classes of solutions of Einstein-Maxwell field equations under additional constraints on physical variables. Of those solutions some are spherically symmetric static dust models and model with uniform matter density.

2.THE FIELD SYSTEM EQUATIONS

In Schwartzchild coordinates the line element for the static spherically symmetric space time is described through the metric.

(2.1) $$ds^2 = e^{v}dt^2 - e^{\lambda}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi\right)$$

where $\lambda = \lambda(r)$ and $v = v(r)$.

For the choice of comoving coordinate system, the four velocity of the fluid v^i and the unit space like magnetic field vector H^i are chosen as

RRIJM 2015, All Rights Reserved

(2.2) $v^i = (0,0,0,e^{-v/2})$

(2.3) $H^i = (e^{-\lambda/2},0,0,0)$

The magnetic field equations are then solved by taking into account the above choice to get the values

(2.4) $h^2 = f^4/r^4$

and

(2.5) $m = \phi^2 f^2/2r^4$

in order to perform the componentwise evaluations of field equation the relation (2.1) to (2.5) are employed to yield

(2.6) $K\left[P_R - \left(\frac{1}{2}\right)\phi^2 f^2 r^{-4}\right] = r^{-2}e^{-\lambda}\left[rv+1\right] - r^{-2}$

(2.7) $K\left[P_T + \left(\frac{1}{2}\right)\theta^2 t^2 r^{-4}\right] = \left(\frac{1}{4}\right)e^{-\lambda}\left[2v'' + r^{-1}(v-\lambda)(2+rv)\right]$

(2.8) $K\left(\rho + \left(\frac{1}{2}\right)\alpha^2 f^2 r^{-4}\right) = r^{-2}e^{-\lambda}\left[r\lambda - 1\right] + r^{-2}$

One method to solve the field equations (2.6) to (2.8) is to assume that the fluid space time is mapped conformally onto itself along the direction ζ. This means that

(2.9) $L_{\zeta k} g_{ij} = 2\psi g_{ij}$

This can explicitly be expressed as

(2.10) $g_{ij,k}\zeta^k + g_{kj}\zeta^k_i + g_{ik}\zeta^k_{+j} = 2\psi g_{ij}$

Let us write the vector field $\overline{\zeta}$ in a specific form in terms of function $\alpha = \alpha(r,t)$ and $\beta = (r,t)$ such that

(2.11) $\overline{\zeta} = \alpha\partial r + \beta\partial t$

Further the assumption that the conformal factor is static imply

(2.12) $\psi = \psi(r)$

The utilization of equations (2.1), (2.1) and (2.12) in (2.10) procures Maartens and Maharaj[9]

(2.13) $\alpha = \left(\frac{1}{2}\right)Are^{-\lambda 2}.$

(2.14) $\beta = B + \left(\frac{1}{2}\right)ct.$

(2.15) $\psi = Ar^{-\lambda/2}$

(2.16) $e^v = D^2 r^2 \exp.\left[-(2c/A)\int\left(e^{\lambda/2}/r\right)dr\right].$

Where A, B, C and D are constants.

Thus by virtue of equations (2.13) to (2.15) the equation (2.11) is written as

(2.17) $\vec{\zeta} = \left(\frac{1}{2}\right)r\psi\partial r + \left(B + \left(\frac{1}{2}\right)kt\right)\partial t$

This gives a generalization of Minkowaskian isotropic conformal vector. Consequently the expression for e^λ and e^v are written in terms of conformal factor ψ as follows

(2.18) $e^\lambda = A^2/\psi^2$

(2.19) $e^v = D^2 r^2 \exp.\left(-2k\int dr/r\psi\right)$

RRIJM 2015, All Rights Reserved

The new version of differential equations is obtained by feeding equations (2.18) and (2.19) back into the field equations (2.6) to (2.8) as

(2.20) $K\rho = -(2\psi\psi'/A^2r) - \left(\psi^2/A^2r^2\right) + \left(1/r^2\right) - \left(K\phi^2f^2/2r^4\right)$

(2.21) $KP_R = (3\psi^2/A^2r^2) - \left(2\psi C/A^2r^2\right) - \left(1/r^2\right) + \left(K\phi^2f^2/2r^4\right)$

(2.22) $KP_T = (\psi^2/A^2r^2) + \left(2\psi\psi'/A^2r\right) - \left(2\psi C/A^2r^2\right) + (C^2/A^2r^2) - \left(K\phi^2f^2/2r^4\right)$

The equation (2.21) and (2.22) give rise to a result

(2.23) $K(P_R - P_T) = -\left(2\psi\psi/A^2r\right) + \left(2\psi^2/A^2r^2\right) - (C^2/A^2r^2) + \left(K_{ij}r^2f^2/r^4\right)$

The inspection (2.23) gives the conclusion that the measures of anisotropic factor $\Delta = P_R - P_T$ depends explicitly on the conformal factor ψ and its radial derivative. These equations generate the metric expression involving arbitrary faction ψ in the form.

(2.24) $ds^2 = \left(-A^2/\psi^2\right)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2 +$

$D^2r^2\exp.\left[-2k\int dr/r\psi\right]dt^2$

For different choices of the function ψ this metric form will lead to a class of static spheres admitting conformal motion with a source as anisotropic ferrofluid.

3. A CLASS OF SOLUTIONS WITH PHYSICAL RESTRACTIONS

Here we take into account the impact of various constraints imposed on dynamical variables. The discussion is followed in different cases.

Case I : Restriction of Anisotrop Δ.

For the sake of simplicity we consider

(3.1) $K(P_R - P_T) = rt$

Where t = t (r) is arbitrary function of r.

The equations (2.23) and (3.1) imply

(3.2) $2\psi\psi' - (2/r)\psi^2 = -A^2r^2t' - (C^2 + A^2)/r + (KA^2\phi^2t^2)/r^3$

This differential equations is solved under standard method to obtain the value of ψ^2 as

(3.3) $\psi^2 = -A^2r^2t + k_1r^2 + \left(\frac{1}{2}\right)(k^2 + A^2) + KA^2\phi^2r^2\int f^2r^{-5}dr$

Further if we restrict the value of f^2 with the condition

(3.3) $f' = r^5l$

with l = l (r),

then the integral (3.3) gets reduced to a simpler version

(3.3) $\psi^2 = -A^2tr^2 - k_1r^2 + \left(\frac{1}{2}\right)\left(k^2 + A^2\right) + KA^2\phi^2l$

where k_1 is a constant of integration.

Thus the line element (2.24) using equation (3.3) takes the form

(3.4) $ds^2 = -A^2\left[-A^2tr^2 + k_1r^2 + \left(\frac{1}{2}\right)\left(k^2 + A^2\right) + KA^2\phi^2r^2\right]^{-1}dr^2$

$-r^2\left(d\theta^2 + \sin^2\phi d\phi^2\right) + D^2r^2\exp.$

RRIJM 2015, All Rights Reserved

$$\left[(-2k)\int l/r\left\{-A^2tr^2+k_1r^2-\left(\frac{1}{2}\right)\left(k^2+A^2\right)+KA^2\phi^2r^2l\right\}^{1/2}dr\right]dr^2$$

This represents a class of static spherically symmetric space time models for the universe filled with the anisotropic ferrofluid admitting conformal motion compatible with the measure of anisotropy specified as by (3.1). Some particular sub-cases of the solution (3.4)

(a) Homothetic Spheres

For A = 0 = k_1, the value of ψ is

(3.5) $$\psi=\left(1/\sqrt{2k}\right)$$

Which reduces the solution (3.4) to the form

(3.6) $$ds^2=r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+D^2r^{(2-2\sqrt{2})}dt^2$$

This model (3.6) represents a static space time model corresponding to homothetic spheres with anisotropy in the specific form (3.1)

(b) Locally isotropic spheres ($P_R = P_T$)

The selection t = 0 gives the value of ψ as

(3.7) $$\psi^2=k_1r^2+\left(\frac{1}{2}\right)(C^2+A^2)+(K\phi^2r^2A^2l)$$

[vide (3.3)]

and the metric (3.4) takes the

(3.8) $$ds^2=-A^2\left[k_1r^2+\left(\frac{1}{2}\right)\left(C^2+A^2\right)+KA^2\phi^2r^2l\right]^{-1}dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+$$

$$D^2r^2\exp.\left[-2k\int(1/r)\left\{k_1r^2+\left(\frac{1}{2}\right)(C^2+A^2)+KA^2\phi r^2t\right\}^{-1/2}dr\right]dt^2$$

This model describes static spherical symmetric solution admitting conformal motion compatible with locally isotropic ferrofluid.

In particular for the choice A = 1 and k = 0 the space-time metric (3.8) becomes

(3.9) $$ds^2=-\left[k_1r^2+\left(\frac{1}{2}\right)+K\phi^2r^2l\right]^{-1}dr^2-$$

$$r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+D^2r^2dt^2$$

This is spherically symmetric static model for the universe filled with isotropic magnetofluid (Asgekar and Aherkar, [1]).

(c) Spheres with uniform anisotropy

The choice of the functional value

(3.10) $$t=k_2\log r+k_3$$

(where k_1 and k_3 are constants) generates the case of uniform anisotropy. Here the value of the conformal factor ψ is given by

(3.11) $$\psi^2=-A^2r^2\left\{k_2\log r+k_3+(k_1/A^2)\right\}+\left(\frac{1}{2}\right)(C^2+A^2)+KA^2\phi^2r^2l.$$

Thus the transformed version of metric (3.4) is given by

(3.12) $$ds^2=A^2\left[-A^2r^2\left\{k_2\log r+k_3+(k_1/A^2)\right\}+\left(\frac{1}{2}\right)(C^2+A^2)+KA^2\phi^2r^2l\right]^1$$

$$dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+$$

RRIJM 2015, All Rights Reserved

$$D^2r^2\exp.\left[-2k\int\left(l_r\int -A^2r^2\left\{k_2\log r+k_3+(k_1/A^2)\right\}\right)+\left(\frac{1}{2}\right)\left(C^2+A^2\right)+KA^2\phi^2r^2l\right]^{1/2}dr\Bigg]dr^2$$

This provides a class of static anisotropic ferrofluid with uniform anisotropy.

Case II. UNIFORM ENERGY DENSITY

The simplest form of matter distribution throughout the interior is the uniform density. Hence substituting ρ = Constant = k_5 in this case equation (2.20) implies

(3.13) $$2\psi\psi'+(1/r)\psi^2=(A^2/r)-(KA^2\phi^2f^2/2r^3)-k_5A^2r$$

Which on integration yields with the constant of integration k_6

(3.14) $$\psi^2=A^2-(1/2)(KA^2\phi^2/r)\int f^2r^{-2}dr-(1/3)(k_5A^2r^2)+k_6/r.$$

Thus the metric (2.24) becomes

(3.15) $$ds^2=-A^2\left[A^2-(1/2)(KA^2\phi^2/r)\int^2 r^{-2}dr-(1/3)(k_5A^2r^2)+k_6/r\right]^{-1}$$

$$dr^2-\left(d\theta^2+\sin^2\theta d\phi^2\right)+$$

$$D^2r^2\exp.\left[2k\int(1/r)\left\{A^2-(1/2)(KA^2\phi^2/r)\int f^2r^{-2}dr-(1/3)(k_5A^2r^2)+k_6/r\right\}^{-1/2}dr\right]dt^2$$

This provides a class of incompressible static anisotropic ferrospheres.

Case III. VANISHING TANGENTIAL PRESSURE

The equation (2.22) under the case $P_T = 0$ provides

(3.16) $$2\psi\psi'+(1/r)\psi^2-(2C\psi/r)=(KA^2\phi^2f^2/2r^3)-(C^2/r)$$

This under the condition C = 0 yields an easy integral in the form

(3.17) $$\psi^2=(1/2)KA^2\phi^2/r)\int f^2r^{-2}dr+k_4/r$$

Thus the metric (2.24) provides

(3.18) $$ds^2=-A^2\left[(1/2)(KA^2\phi^2/r)\int f^2r^{-2}dr+k_4/r\right]^{-1}$$

$$dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+D^2r^2dt^2$$

This represents static spherically symmetric universe filled with anisotropic ferrofluid where the tangential pressure is zero.

Case IV : EQUATION OF STATE

The well know equation of state governing the matter density ρ and pressure p is

(3.19) $$p=(\gamma-1)\rho,$$

where

(3.20) $$p=(1/3)P_R+(2/3)P_T$$

Note : By supposing C = 0 and specifying the choice of γ different solutions are generated with the utilization of equation (2.21) and (2.22). These subcases are discussed below :

(i) Dust filled universe

By equating the value of p to zero it is obtained that

(3.21) $$2\psi\psi'+(5/2)(\psi^2/r)=(A^2/2r)+(KA^2\phi^2f^2/4r^3)$$

Thus the value of ψ for dust filled universe is given by

(3.22) $$\psi^2=(A^2/5)+(1/4)(KA^2\phi^2/r^{5/2})\int f^2r^{-1/2}dr+k_7/r^{5/2}$$

where k_7 is a constant of integration.

From the metric (2.24) it follows that

(3.23) $$ds^2=-A^2\left[(A^2/5)+(1/4(KA^2\phi^2/r^{5/2})\int f^2r^{12}dr+(k_2/r^{5/2})\right]^1$$

$$dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+D^2r^2dt^2$$

RRIJM 2015, All Rights Reserved

This exhibits the static anisotropic ferrodust universes.

(ii) Radiating universe :

For radiating universe (ρ = 3p) one obtains.

(3.24) $2\psi\psi' + (2\psi^2/r) = -(2A^2/3r)$

which implies

(3.25) $\psi^2 = -(A^2/3) + k_8.r^2$

where k_8 is a constant of integration.

The models (2.24) will be seen in the form

(3.26) $ds^2 = -A^2\left[-(A^2/3) + k_8/r^2\right]^{-1} dr^2 -$

$r^2(d\theta^2 + \sin^2\theta d\phi^2) + D^2r^2dt^2$

REFERENCE

1. Aherkar, S.M. and Asgekar, G.G. (1990), IL Nuovo Cinento, 105 B(6), P. 639.
2. Bayin, S. (1982), Phys. Rev. D, 26, P. 1262.
3. Coley, A.A. and Tupper, B.O.J., (1990a), GRG, 22(3), P. 241.
4. Coley, A.A. and Tupper, B.O.J., (1991), GRG, 23(10). P. 1113.
5. Humi, M. and Mansour, J. (1984), Phys. Rev. D., 29, P. 1076.
6. Herrera, L., Jimenez, J., Lea, L., Ponce De Leon J. Esculpi, M. and Galina, V. (1984), J. Math Phys., 25(11), P. 3274.
7. Herrera, L. and Ponce De Leon, J. (1985), J. Math. Phys. 26(9). P. 2302.
8. Karmmer, D. Stephani, H. Herli, E. and Maccallum (1980), Exact olutons of Einstein Field Equations, Cambridge University Press, Cambridge.
9. Maartens, R. and Maharaj, M.S. (1990), J. Math. Phys., 31(1), P. 151.
10. Wainwright. J. (1985). In Anisotropic system, edited by M.A.H. Maccalum, Cambridge University Press, Cambridge.

RRIJM 2015, All Rights Reserved