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Slowly Rotating Universes with Radiating Perfect Fluid Distribution Coupled with a Scalar Field

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ABSTRACT

The present paper provides some solutions for slowly rotating universes with radiating perfect fluid distribution coupled with scalar field.

1. INTRODUCTION

Lanczos [1] was the first to give a model of rotating matter distribution with explicitly solved field equations. Thereafter many physicists investigatied on the rotational motion of the cosmological objects. Brill and Cohen [2], Hawking [3], Krasinsk [4], Bayin and Cooperstock [5], Bayin [6-8], Krori et. al. [9]. Van den Bergh and Wils [10], Islam [11], Tiwari et al. [12], Maniharsingh and Bhamra [13], and Manihar Singh [14, 15] have studied the rotating fluid distribution under different conditions in order to understand the structure and equilibrium, the nature and role of rotating astrophysical objects in this universe.

In the centre of the stars where the gravitational field is strong, a scalar field may have some effects on stellar configurations. Such an effect becomes important only when the general relativistic effect itself becomes important. It is, therefore, expected that the stellar configuration with be appreciable affected by its own scalar field in the case of a neutron star and more particularly in the case of the pulsar. Zero-mass scalar fields have acquired particular importance since Weinberg [16] and Wilczek [17] proposed the existence of a low-mass ($\leq$ 1eV) scalar boson, the so-called axion. Such particles with explain the absence of charge conjugation and parity (CP) non conservation is strong interactions as pointed out by Pecci and Quinn [18]. Any light particle has a potential for playing a major role in stellar energy loss, so there may exist a cosmic background of these particles. Thus the results of our investigations here will be applicable in the exploration of the behaviour and characteristics and the properties and nature of cosmic axion field in this universe, and many more conclusions may be drawn. For our problem here we choose the field equation of the scalar field as defined in eq. (5), which was used by Krori et al. [9]. Some other workers in this line are Manihar Singh [19-22] and Vaidya [23].

In this paper we study slowly rotating radiating fluid distribution coupled with a zero mass scalar field as the object of our study as it will be very stimulating to investigate such models.

2. DERIVATION OF FIELD EQUATIONS

For this problem we consider the metric

$$ds^2 = \exp(\psi)dt^2 - \exp[h(r) + k(t)dr^2 \tag{2.1}$$

$$-\exp[k(t)](r^2 d\Theta^2 + r^2 \sin\Theta d\phi^2)$$

$$+2r^2 \exp[k(t)\sin^2\Theta\Omega d\Phi dt$$

Where ψ is an arbitrary function of r and t, h r and arbitrary function of r, k(t) an arbitrary function of time t, and Ω (r, t) is the metric retation function which is related to the local dragging of inertial frames.

The energy-momentum tensor $T_{\mu\nu}$ is here taken to be

(2.2) $$T_{\mu v} = P_{\mu v} + N_{\mu v} + S_{\mu v}$$

Where $P_{\mu v}$ is the energy-momentum tensor due to perfect fluid and takes the form

$$P_{\mu v} = (p + \rho)u_\mu u_v - pg_{\mu v},$$

P being the isotropic pressure, ρ the fluid density and u_μ the four-flow vector satisfying the relation.

(2.3) $$g_{\mu v} u^\mu u^v = 1$$

$N_{\mu v}$ is is the energy-momentum tensor for the radiation field and takes the from $N_{\mu v} = \varepsilon z_\mu z_v$

where ε is the source density of the radiation field and z_μ are the components of radiation satisfying the relation.

(2.4) $$z_1 z^1 = 0,\ \ z_1 \neq 0,\ z_2 = 0,\ z_3 \neq 0,\ z_4 \neq 0$$

$S_{\mu v}$ is the energy-momentum tensor due to zero-mass sclar field [9] and takes the form

$$S_{\mu v} = \frac{1}{\phi^2}\left(\phi_\mu \phi_v - \frac{1}{2} g_{\mu v}, \phi^l \phi_l\right)$$

where ϕ is the scalar field which satisfies the relation

(2.5) $$\frac{\partial}{\partial x}[\phi_\alpha (-g)^{1/2} g^{\mu v}] - \frac{\phi_\alpha \phi_v}{\phi}(-g)^{1/2} g^{\mu v} = 0$$

Here we have

(2.6) $$U^0 = [0, 0\omega \exp(-\psi/2), \exp(-\psi/2)]$$

where $\omega = d\phi/dt$ is the angular velocity of matter.

We define the null vector $z^i{}_{\text{as dx}}{}^i / d\tau$ [23], where $d_T = \exp(\psi/2)dt.$

This gives

(2.7) $$z^4 = \exp(-\psi/2),\ z_4 = \exp(-\psi/2)$$

and

(2.8) $$z^3 = \omega \exp(-\psi/2)\ z_3 = r^2 \exp(k + \psi/2)(\Omega - \omega)\sin^2\theta$$

Now, because of the complexity of the problem, in the following we consider only the cases when

(2.9) $$\psi = 0$$

Then from (2.4); (2.7) and (2.8) we obtain

(2.10) $$z^1 = \exp\left(-\frac{h}{2} - \frac{k}{2}\right),\ z_1 = -\exp\left(\frac{h}{2} + \frac{k}{2}\right)$$

Thus Einstein's field equations

$$R_{\mu v} = \frac{1}{8\pi G}\left(T_{\mu v} - \frac{1}{2} g_{\mu v} T\right)$$

give

(2.11) $$\frac{\bar{k}}{2}\exp(h + k) + \frac{h'}{r} + \frac{3}{4}k^2 \exp(h + k)$$

$$=8\pi G\left[\frac{1}{2}(\rho-p)\exp(h+k)+z_1z_1+\frac{\phi'^2}{\phi^2}\right]$$

(2.12) $$1+\frac{1}{2}rh'\exp(-h)+\frac{1}{2}r^2k\exp(k)+\frac{3}{4}r^2k^2\exp(k)-\exp(-h)$$

$$=8\pi G\left[\frac{1}{2}(\rho-p)r^2\exp(k)\right]$$

(2.13) $$\frac{3}{2}k+\frac{3}{4}k^2=-8\pi G\left[\frac{1}{2}(\rho+3p)+\in z_4z_4+\frac{\phi^2}{\phi^2}\right]$$

(2.14) $$0=\in z_1z_4+\frac{\phi'\phi}{\phi^2}$$

(2.15) $$\frac{3}{4}k\Omega'+\frac{1}{2}\Omega'=0$$

(2.16) $$\left[\left(\frac{r}{2}h'-1\right)\exp(-h)\Omega+\frac{1}{2}r^2\left(k+\frac{3}{2}k^2\right)\exp(k)\Omega+\Omega\right.$$

$$\left.+\left(\frac{1}{4}r^2h'-2r\right)\exp(-h)\Omega'-\frac{1}{2}r^2\exp(-h)\Omega''\right]\sin^2\theta$$

$$=-8\pi G\left[\left(\frac{\Omega}{2}-\omega\right)\rho r^2\exp(k)\sin^2\theta-\omega pr^2\exp(k)\sin^2\theta\right.$$

$$\left.+\frac{3}{2}p\Omega r^2\exp(k)\sin^2\theta+\in z_1z_4\right.$$

The dot and prime respectively denote differentiations with respect to t and r, and a semicolon followed by a subscript denotes covariant differentiation.

3. SOLUTIONS OF THE FIELD EQUAITONS

From relation (2.5) we get

(3.17) $$\left(\phi+\frac{3}{2}k\phi-\frac{\phi^2}{\phi}\right)\exp(k)-\left(\phi''-\frac{(\phi')^2}{\phi}+\frac{2}{r}\phi'-\frac{h'}{2}\phi\right)\exp(-h)=0$$

which gives

(3.18) $$\frac{\phi}{\phi}=a\exp\left(\frac{-3}{2}k\right)$$

and

(3.19) $$\frac{\phi'}{\phi}=\frac{b}{r^2}\exp\left(\frac{h}{2}\right)$$

Where a and b are arbitrary constants.

Now using (2.7), (2.10), (3.18) and (3.19) in (2.14)

$$\in = \frac{ab}{r^2}\exp(-2k) \tag{3.20}$$

Again from (2.11) and (2.12) we have, making use of the relations (2.10) and (3.18) – (3.20).

$$1+\frac{3}{2}r\exp(-h)h' + r^2\exp(k)\vec{k} + \frac{3}{2}r^2\exp(-h)k^2 - \exp(-h) \tag{3.21}$$

$$= 8\pi G[(\rho - p)r^2\exp(k) + ab\exp(-k) + b^2r^{-2}]$$

Thus now from (3.21) and (2.13) we get

$$p = \frac{1}{16\pi G}\left[\frac{1}{2}r^{-2}\exp(-h-k) + 4\pi G b^2 r^{-4}\exp(-k) - 2k \right. \tag{3.22}$$

$$-\frac{3}{2}k^2 - \frac{1}{2}r^{-2}\exp(-k) - \frac{3}{4}r^{-1}h'\exp(-h-k)$$

$$\left. -4\pi G abr^{-2}\exp(-2k) - 8\pi G a^2\exp(-3k)\right]$$

and

$$p = \frac{1}{16\pi G}\left[\frac{3}{2}r^{-2}\exp(-k)\frac{9}{4}r^{-1}h'\exp(-h-k) \right. \tag{3.23}$$

$$+\frac{3}{2}k^2 - \frac{3}{2}r^{-2}\exp(-h-k) - 20\pi G abr^{-2}\exp(-2k)$$

$$\left. -12\pi G b^2 r^{-4}\exp(-k) - 8\pi G a^2\exp(-3k)\right]$$

Then making use of the relations (2.7) (2.8), (3.22) and (2.16) becomes.

$$\frac{1}{2}r^2\exp(-h)\Omega'' - \left(\frac{1}{4}r^2h' - 2r\right)\exp(-h)\Omega' \tag{3.24}$$

$$= (\Omega - \omega\left[\exp(-h) - 1 - \frac{r}{2}\exp(-h)h'\right.$$

$$\left. + r^2\exp(k)\bar{k} + 8\pi G a^2 r^2\exp(-2k)\right]$$

Also from (2.15) we get

$$\Omega(r,t) = M(r)\exp\left(-\frac{3}{2}k\right) + N(t) \tag{3.25}$$

Where M(r) is an arbitrary function of r and N (t) is an arbitrary function of time.

Thus (3.24) and (3.25) together given

$$\frac{1}{2}\exp(-h)\frac{M''}{M} - \left(\frac{1}{4}r^2h' - 2r\right)\exp(-h)\frac{M'}{M} \tag{3.26}$$

$$= \left[\exp(-h) - 1 - \frac{r}{2}h'\exp(-h) + 8\pi G a^2 r^2\exp(-2k)\right.$$

$$+r^2\exp(k)\bar{k}\Big]\left[1+\frac{N(t)\exp(3k/2)}{M(r)}-\frac{\omega\exp(3k/2)}{M(r)}\right]$$

Here we see that the left hand side is a function of r only. Therefore the right hand side must be a function of either r only or t only, since it cannot be a mixture of both, for in that case it cannot be equal to the left hand side. Thus here we take the right hand side to be a function of t only so that each side can be set equal to a constant.

Then we see that the form and value of ω will be restricted according to this condition. Now we take up the following cases.

Case I :

In this case we assume ω = N(t), N(t) being an arbitrary function of time.

Then equation (3.26) takes the form

(3.27)
$$\frac{1}{2}\exp(-h)\frac{M''}{m}-\left(\frac{1}{4}h'-\frac{2}{r}\right)\exp(-h)\frac{M'}{M}$$

$$=\left[r^{-2}\exp(-h)-r^{-2}-\frac{r^{-1}}{2}h'\exp(-h)\right.$$

$$\left.+\exp(k)\ddot{k}+8\pi Ga^2\exp(-2k)\right]$$

Thus here in order that the right hand side may be a function of t only we assume

$$r^{-2}\exp(-h)-r^{-2}-\frac{1}{2}r^{-1}h'\exp(-h)=c$$

where c is an arbitrary constant. This gives

(3.28)
$$h=-\log\left(1+\frac{c}{2}r^2\right)$$

Now using relation (3.28) in equation (3.27) we obtain

$$\left(1+\frac{c}{2}r^2\right)\frac{M''}{M}+\left(\frac{5}{2}cr+\frac{4}{r}\right)\frac{M'}{M}$$

$$=2[c+\exp(k)\bar{k}+8\pi a^2\exp(-2k)]$$

which can be separated into

(3.29)
$$\left(1+\frac{c}{2}r^2\right)\frac{M''}{M}+\left(\frac{5}{2}cr-\frac{4}{r}\right)\frac{M'}{M}=z$$

(3.30)
$$c+\exp(k)\ddot{k}+8\pi Ga^2\exp(-2k)=\frac{z}{2}$$

where z is a separation constant.

Here if we sole eq. (3.30), we get only an approximate and series solution for k. Therefore to get an exact solution we assume some relation between the constant, say, here c = z/2. Then in this case we get from (3.30)

(3.31)
$$k^2=\frac{16\pi Ga^2}{3}\exp(-3k)+s_1$$

Where s_1 is a positive arbitrary constant. Thus (3.31) gives

$$k=\frac{2}{3}\log[k_1\sinh(k_2+k_3)]$$ (3.32)

Where

$$k_1=4a\left(\frac{\pi G}{3s_1}\right)^{1/2},k_2=\frac{3}{2}(s_1)^{1/2}$$

and k_3 is an arbitrary constant. Then in the case we obtain

$$p=\frac{1}{16\pi G}[c\{k_1\sinh(k_2t+k_3)\}1/2$$ (3.33)

$$+4\pi Gb^2r^{-4}\{k_1\sinh(k_2t+k_3)\}^{-2/3}$$

$$-4\pi Gabr^{-2}\{k_1\sinh(k_2t+k_1)\}^{-4/3}$$

$$\left.-16\pi Ga^2\{k_1\sinh(k_2t+k_3)\}^{-2}-\frac{2}{3}k_2^{-2}\right]$$

$$\rho=\frac{r^{-4}}{16\pi G}\{k_1\sinh(k_2l+k_3)\}^{-4/3}\left[\frac{2}{3}k_2^2r^4\{k_1\sinh(k_2t+k_1)\}^{4/3}\right.$$ (3.34)

$$-(3cr^4+12\pi Gb^2)\{k_1\sinh(k_2t+k_3)\}^{3/1}-20\pi Gabr^2]$$

$$\in=abr^{-2}[k_1\sinh(k_2l+k_3)]^{-4/3}$$ (3.35)

$$z^1=\left(1+\frac{c}{2}r^2\right)^{1/2}[k_1\sinh(k_2t+k_3)]^{-4/3}$$ (3.36)

$$\phi=\left[\tan\left\{\frac{1}{2}(k_2t+k_3\right\}\right]^{ak_1k_2}\exp\left[-\left(\frac{c}{2}\right)^{1/2}br^{-1}\left(r^2+\frac{2}{c}\right)^{1/2}\right]$$ (3.37)

Again using the substitution y= –(c/2)r^2 in (3.29) we get

$$y(1-y)\frac{d^2M}{dy^2}+\left(\frac{5}{2}-3y\right)\frac{dM}{dy}-\frac{z}{2c}M=0$$ (3.38)

Here we see that eq. (3.38) is similar to the hypergeometric equation

$$y(1-y)\frac{d^2F}{dy^2}+[\gamma-(1+\alpha+\beta)y]\frac{dF}{dy}-\alpha\beta F=0$$ (3.39)

of which the general solution is given by

$$F=A_0F(\alpha,\beta;\gamma;y)+A_1y^{-1\gamma}F(1-\gamma+\alpha,1-\gamma+\beta;2-\gamma;y)$$

Where A_0 and A_1 are arbitrary constant and

$$F(\alpha,\beta;\gamma;y)=\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{\angle n(\gamma)_n}$$

Thus we get the general solution of (3.38) as.

$$F(r) = A_0 \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{\angle n (5/2)_n} y^n + A_1 y^{-3/2} \sum_{n=0}^{\infty} \frac{(\alpha - 3/2)_n (\beta - 3/2)_n}{\angle n (-1/2)_n}$$
(3.40)

Since the second term is not regular at y = 0 we take $A_1 = 0$. Then we get. $$M(r) = A_0 \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{\angle n (5/2)_n} y^n$$

or

$$M(r) = A_0 (1-y)^{(5/2)-\alpha-\beta} \sum_{n=0}^{\infty} \frac{(5/2-\alpha)_n (5/2-\beta)_n}{\angle n (5/2)_n} y^n$$
(3.41)

Thus here according to the different values of α and β we get different values of M(r) and thereby different values of Ω. For example, if $\alpha = -1$, $\beta = 3$ then we have

$$M(r) = A_0 \left(1 + \frac{3}{5} c r^2 \right)$$

In that case

$$\Omega(r,t) = A_0 \left(1 + \frac{3}{5} c r^2 \right) \exp\left(-\frac{3}{2} k \right) + N(t)$$
(3.42)

Now the solution for M (r) obtained in (3.41) is a series solution and hence is approximate. Thus get an exact solution we give particular values to c and z, and the solve eq. (3.29). In the following we take up some different cases in which we get different interesting types of solutions.

Case Ia

Here taking c = 2 we obtain three different expressions for Ω(r,t) corresponding to three different values of z.

If we taken z = 3 then from (2.29) we get

$$M(r) = \frac{c_1}{2} r^{-2} (1 - r^2)^{1/2} - \frac{c_1}{2} r^{-3} \sinh^{-1} r + c_2 r^{-3}$$
(3.43)

Where, c_1 and c_2 are arbitrary constants. Therefore,

$$\Omega(r,t) = \left[\frac{f_1}{2} r^{-2} (1 - r^2)^{1/2} - \frac{c_1}{2} r^{-3} \sinh^{-1} r \right.$$
(3.44)

$$\left. + c_2 r^{-3} \right] \exp\left(-\frac{3}{2} k \right) + N(t)$$

Again if z = 5 then we get

$$M(r) = c_3 (1 + r^2)^{1/2} - \frac{c_4}{3} (8r + 4r^{-1} - r^{-3})$$

Where c_3 and c_4 are arbitrary constants. Therefore,

$$\Omega(r,t) = \left[c_3 (1 + r^2)^{1/2} - \frac{c_4}{3} (8r + 4r^{-1} - r^{-3}) \right] \exp\left(-\frac{3}{2} k \right) + N(t)$$
(3.45)

When z = – 4 we obtain

$$M(r) = r^{-3} [(c_5 r - c_5 (1 + r^2)^{1/2} \sin^{-1} r + c_6 (1 + r^2)^{1/2}]$$

and

(3.46) $$\Omega(r,t) = [r^{-3}\{c_5 r - c_5(1+r^2)^{1/2}\sin^{-1} r + c_6(1+r^2)^{1/2}\}]$$

$$\times \exp\left(-\frac{3}{2}k\right) + N(t)$$

Case Ib

In this case taking c = –2 we obtain different values of M(r) and thereby different values for Ω(r, t) by giving different values to z.

If z = 3 then from (3.29) we get

$$M(r) = \frac{d_1}{2} r^{-3}\sin^{-1} r - \frac{d_1}{2} r^{-2}(1+r^2)^{1/2} + d_2 r^{-3}$$

Where d_1 and d_2 are arbitrary constants. Thus

(3.47) $$\Omega(r,t) = \left[\frac{d_1}{2} r^{-3}\sin^{-1} r - \frac{d_1}{2} r^{-2}(1+r^2)^{1/2} + d_2 r^{-3}\right]$$

$$\times \exp\left(-\frac{3}{2}k\right) + N(t)$$

Again if z = 0 we have

$$M(r) = \frac{d_3}{2} r^{-2}(1-r^2)^{1/2} + \frac{d_3}{2}\log\tan\left[\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\{r^{-1} - r)^{1/2}\}\right]$$

$$-d_3 r^{-1}(1-r^2)^{1/2} - \frac{d_3}{3} r^{-3}(1-r^2)^{3/2} + d_4$$

where d_3 and d_4 are arbitrary constants. Thus

(3.48) $$\Omega(r,t) = \left[\frac{d_3}{2} r^{-2}(1-r^2)^{1/2} + \frac{d_3}{2}\log\tan\left\{\frac{\pi}{4} + \frac{1}{2}\tan^{-1}[r^{-1}(1-r)^{1/2}\right\}\right.$$

$$-d_3 r^{-1}(1-r^2)^{1/2} - \frac{d_3}{3} r^{-3}(1-r^2)^{3/2}$$

$$+d_4]\exp\left(-\frac{3}{2}k\right) + N(t)$$

When z = – 5 we obtain

$$M(r) = \frac{d_5}{3}(r^{-3} + 4r^{-4} - 8r) + d_6(1-r^2)^{1/2}$$

Where d_5 and d_0 are arbitrary constants. Thus

(3.49) $$\Omega(r,t) = \left[\frac{d_5}{3}(r^{-3} + 4r^{-1} - 8r)d_5(1-r^2)^{1/2}\exp\left(-\frac{3}{2}k\right) + N(t)\right.$$

Case II

Here we take $\omega = \Omega$ which corresponds to the case of "perfect dragging". Then from (3.24) we get.

(3.50) $$\Omega'' - \left(\frac{1}{2}h' - 4r^{-1}\right)\Omega' = 0$$

Now using (3.25) in eq. (3.50) we have

$$M^n - \left(\frac{1}{2}h' - 4r^{-1}\right)M' = 0$$

Which gives

$$M(r) = \left[\int m_1 r^{-4} \exp\left(\frac{h}{2}\right)dr + n_1\right] \quad (3.51)$$

Where m_1 and m_1 are arbitrary constant. Thus in this case

$$\Omega(r, t)\left[\int m_1 r^{-1} \exp\left(\frac{h}{2}\right)dr + m_1\right]\exp\left(-\frac{3}{2}k\right) + N(t) \quad (3.52)$$

Reference

1. Lanczos, K. (1924). Z. Physick. 21, 73.
2. Brill, D.R., and Cohen., J.M. (1966). Phys. Rev. 143, 1011.
3. hawking, S. (1969), Mon. Not. R. Astr. Soc. 142, 129.
4. Krasinski, A. (1978), Acta Cosmologica 27, 119.
5. Bayin, S.S. and Cooperstock, F.I. (1980), Phys. Rev. D22, 2317.
6. Bayin, S.S. (1981), Phys. Rev. D24, 2056.
7. Bayin, S.S. (1981), Phys. Rev. D27, 3030.
8. Bayin, S.S. (1985). Can. J. Phys. 63, 1320.
9. Krori, K.D. Sarmah, J.C., and Goswami, D. (1983), Can. J. Phys. 61, 744.
10. Van den Bergh, N., and Wils, P. (1984). In Classical General Relativity, W.B. Bonnor, et al., eds. (Cambridge University Press, Cambridge).
11. Islam, J.N. (1985). Rotating Fields in General Relativity (Cambridge University Press, Cambridge)
12. Tiwari, R.N., Rao, J.R. and Kanakamedala, R.R. (986). Phys. Rev. D34, 327.
13. Manihar Singh, KJ., and Bhamra, K.S. (1986). Pramana – J Phys. 26, 117.
14. Manihar Singh, KJ. (1988). Astrophys. Space Sci., 143, 137.
15. Manihar Singh, KJ. (1991), Astrophys, Space Sci., 180, 253.
16. Weinberg, S. (1978), Phys., Rev. Lett., 40, 279.
17. Wilczek, F. (1978), Phys. Rev. Lett. 40, 279.
18. Pecci, R.D. and Quinn, H. (1979). Phys. Rev., D16, 1791.
19. Maniharsingh, KJ. (1988) Int. J. theor, Phys., 26, 1975.
20. Manihar Singh, KJ. (1989), Astrophys. Space Sci., 162, 129.
21. Manihar Singh, KJ. (1990). Astrophys. Space Sci., 164, 153.
22. Manihar Singh KJ. (1993), Astrophys. Space Sci., 205, 259.
23. Vaidya, P.C. (1966) , Astrophys. J., 144, 943.