

A Numerical Solutions of Nonlinear Fredholm Integral Equations of Second Kind by B-Spline Wavelet Method

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ABSTRACT

In this paper, nonlinear integral conditions have been fathomed numerically by utilizing B-spline wavelet method. Compactly upheld semi-orthogonal linear B-spline scaling and wavelet works along with their dual capacities are applied to surmise the arrangement of nonlinear Fredholm integral conditions of second kind. A few models are introduced to compare the precision of linear B-spline wavelet method & Variational Iteration Method with their accurate arrangements.

INTRODUCTION

Wavelets allow the precise portrayal of an assortment of functions and administrators. Besides, wavelets build up an association with quick numerical calculations. Wavelets can be isolated into two particular sorts, orthogonal and semi-orthogonal. A computational way to deal with light up vital conditions is a fundamental work in logical research. Integralequation has been one of the essential devices for various zone of associated science. Basic conditions happen typically in various fields of science and engineering Wavelets speculation is a by and large new and rising domain in logical examination. It has been associated in a broad assortment of planning controls; particularly, wavelets are successfully used in banner examination for waveform depictions and divisions, time repeat assessment, and snappy computations for basic execution. Wavelets permit the exact depiction of a variety of capacities and managers. Moreover, wavelets develop a relationship with fast mathematical figuring. Wavelets can be disconnected into two specific sorts, symmetrical and semi-symmetrical. The research works available in open composition on integral equations have exhibited a checked tendency for orthogonal wavelets. This is likely because the main wavelets, which were commonly used for banner dealing with, were basically symmetrical. In banner dealing with applications, rather than vital condition strategies, the wavelet itself is never evolved since simply its scaling capacity and coefficients are required. Regardless, symmetrical wavelets either have huge assistance or a non-symmetric, and now and again fractal, nature. These properties can make them a helpless choice for depiction of a capacity. Curiously, the semi-symmetrical wavelets have restricted assistance, both even and odd evenness, and fundamental insightful explanations, ideal qualities of a reason work.

In this paper, we apply moderate partner maintained direct semi-orthogonal B-spline wavelets, outstandingly developed for the restricted interval to rough the dark capacity present in integral equations. Semi-orthogonal wavelets utilizing B-spline uniquely created for the restricted interval & these wavelets can be addressed in a shut casting. Semi-orthogonal wavelets outline the reason in space $L^2(\mathbb{R})$. Utilizing this reason, an arbitrary capacity in $L^2(\mathbb{R})$ can be imparted as wavelet arrangement. For the restricted interval $[0, 1]$, the wavelet plan can't be completely presented by using this reason. This is because sponsorships of some reason are shortened at the left or right end motivations behind the interval. Hence a remarkable reason must be brought into the wavelet improvement on the restricted interval. These capacities are implied as the cutoff scaling capacities and breaking point wavelet capacities. B-spline wavelet strategy has been associated with understand straight and nonlinear basic conditions and their frameworks. The above strategy lessens the essential conditions to frameworks of mathematical conditions and a while later these frameworks can be lit up by any standard mathematical strategies. Here, we have associated Newton's technique with appropriate beginning hypothesis for tackling these frameworks.

APPLICATION OF B-SPLINE WAVELET METHOD FOR SOLVING NONLINEAR FREDHOLM INTEGRAL EQUATIONS OF SECOND KIND

Here, one consider the second kind nonlinear Fredholm integral condition of the going with structure

$$u(x) = f(x) + \int_0^1 K(x, t)Ftu(t)dt, \quad 0 \leq x \leq 1 \quad (1)$$

where $K(x, t)$ is the piece of integral equation, $f(x)$ and $K(x, t)$ are known capacities furthermore, $u(x)$ is the dark capacity that is to be resolved.

Most importantly, we accept

$$\mathcal{Y}(x) = F(x, u) \quad 0 \leq x \leq 1 \quad (2)$$

we can estimate the functions $u(x)$ & $y(x)$ as

$$u(x) = A^T \Psi(x), \quad y(x) = B^T \Psi(x) \tag{3}$$

A & B are $(2^{M+1} + 1) \times 1$ column vectors like C defined in eq. (3).

Again by using dual of the wavelet functions $f(x)$ and $K(x, t)$ as pursues

$$f(x) = D^T \tilde{\psi}(x), \quad K(x, t) = \tilde{\psi}^T(t) \theta \tilde{\psi}(x) \tag{4}$$

Where,

$$\theta_{(i,j)} = \int_0^1 \left[\int_0^1 K(x, t) \psi_i(t) dt \right] \psi_j(x) dx.$$

Equations (2)-(4), we get

$$\int_0^1 K(x, t) F(t, u(t)) dt = \int_0^1 B^T \psi(t) \tilde{\psi}^T(t) \theta \tilde{\psi}(x) dt = B^T \theta \tilde{\psi}(x), \tag{5}$$

$$\int_0^1 \psi(t) \tilde{\psi}^T(t) dt = I$$

using (2)-(5) in eq. (1), then

$$A^T \psi(x) = D^T \tilde{\psi}(x) + B^T \theta \tilde{\psi}(x) \tag{6}$$

$$A^T P - D^T + B^T \theta = 0, \tag{7}$$

Where P is a $(2^{M+1} + 1) \times (2^{M+1} + 1)$ square matrix given by

$$P = \int_0^1 \Psi(x) \Psi^T(x) dx = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \text{ and } \int_0^1 \tilde{\Psi}(x) \Psi^T(x) dx = I.$$

Eq. (5) gives a framework of $(2M+1 + 1)$ algebraic conditions with $2(2M+1 + 1)$ questions for A & B vectors given in eq. (3).

$$F(x, A^T \Psi(x)) = B^T \Psi(x), \tag{8}$$

with collocation points $x_i = \frac{i-1}{2^{m+1}}$, where $i = 1, 2, \dots, 2^{M+1} + 1$.

Eq. (6) gives an arrangement of $(2^{M+1} + 1)$ algebraic conditions with $2(2^{M+1} + 1)$ questions for A & B vectors given in eq. (3).

Joining equations (5) & (6), one have a whole of $2(2^{M+1} + 1)$ algebraic conditions with same number of questions for A and B. Fathoming those equations for dark coefficients in vectors An and B, one can get the arrangement $u(x) = A^T \psi(x)$. The gotten outcomes have been differentiated and the outcomes procured by VIM.

• **VIM method for eq.**

For tackling eq. (1) by VIM, first one has to take partial derivative of equations (1) concerning x.

$$u'(x) = f'(x) + \int_0^1 \frac{\partial K(x, t)}{\partial x} F(t, u(t)) dt. \tag{9}$$

We apply VIM for the equation (9). As demonstrated by this technique, Correction utilitarian can be characterized as

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(\xi) \left(u'_n(\xi) - f'(\xi) - \int_a^b \frac{\partial K(\xi, t)}{\partial \xi} F(t, \tilde{u}_n(t)) dt \right) d\xi, \tag{10}$$

where $\lambda(\xi)$ is an overall Lagrange multiplier which can be recognized in a perfect world by the variational theory, subscript n implies the nth solicitation guess and \tilde{u}_n considered as a restricted variation, i.e., $\delta \tilde{u}_n = 0$. The progressive approximations $u_n(x), n \geq 1$ for the arrangement $u(x)$ can be expeditiously gotten resulting to choosing the Lagrange multiplier and picking an appropriate starting capacity $u_0(x)$. Thusly, approximate solution may be gained by utilizing $u(x) = \lim_{n \rightarrow \infty} u_n(x)$.

To make the above revision useful fixed, we have

$$\begin{aligned} \delta u_{n+1}(x) &= \delta u_n(x) + \delta \int_0^x \lambda(\xi) \left(u'_n(\xi) - f'(\xi) - \int_a^b \frac{\partial K(\xi, t)}{\partial \xi} F(t, \tilde{u}_n(t)) dt \right) d\xi \\ &= \delta u_n(x) + \lambda \delta u_n|_{\xi=x} - \int_0^x \lambda'(\xi) \delta u_n(\xi) d\xi \end{aligned} \tag{11}$$

Under stationary condition, $\delta u_{n+1} = 0$,

$$\lambda'(\xi) = 0, \tag{12}$$

using boundary condition

$$1 + \lambda(\xi)|_{\xi=x} = 0(13)$$

Solving boundary condition (13), one get general Lagrange Multiplier $\lambda = -1$.

Substituting recognized Lagrange multiplier into eq. (13),

$$u_{n+1}(x) = u_n(x) - \int_0^x \left(u'_n(\xi) - f'(\xi) - \int_a^b K'(\xi, t)F(t, \tilde{u}_n(t))dt \right) d\xi, \quad n \geq 0. \tag{14}$$

By starting with initial approximate function $u_0(x = f(x)$ (say), one can decide approximate solution $u(x)$.

Examples

Example 1: Consider the equation

$$u(x) = -\frac{x}{9} - \frac{x^2}{8} + x^3 + \int_0^1 (x^2t + xt^2)u^2(t)dt, \quad 0 \leq x \leq 1,$$

with the right arrangement $u(x) = x^3$. The surmised arrangement is obtained by the B-spline wavelet strategy for $M = 2$ and $M = 4$ and moreover by VIM. The going with Table 1 alludes to the mathematical arrangements gained by B-spline technique and VIM accomplished with relating positive arrangements and the outright blunders. Figures 1-2 and Figures 3-4 present the assessment graphically between the mathematical arrangements procured by B-spline wavelet technique with right arrangements and VIM arrangements independently.

Table 1: Comparison of numerical results by BWM and VIM

X	Exact	BWM solution		Error in BWM		VIM solution	Error in VIM
		M = 2	M = 4	M = 2	M = 4		
0.0	0	-2.496E-4	-3.617E-6	0.0002495	3.61701E-6	0	0
0.1	0.001	0.00166976	0.00105608	0.0006695	0.0000560	0.000965618	0.0000343
0.2	0.008	0.0105762	0.00816602	0.0025762	0.0001660	0.00792412	0.0000758
0.3	0.027	0.0313409	0.0272625	0.0043409	0.0002625	0.0268755	0.0001245
0.4	0.064	0.068719	0.0642781	0.0047190	0.0002781	0.0638198	0.0001802
0.5	0.125	0.12735	0.125145	0.0023496	0.0001454	0.124757	0.0002430
0.6	0.216	0.223711	0.216493	0.0077114	0.0004932	0.215687	0.0003130
0.7	0.343	0.355417	0.343778	0.0124172	0.0007776	0.34261	0.0003901
0.8	0.512	0.527106	0.512931	0.0151063	0.0009313	0.511526	0.0004743
0.9	0.729	0.743534	0.729887	0.0145338	0.0008869	0.728434	0.0005656
1.0	1	1.00957	1.00058	0.0095707	0.0005807	0.999336	0.0006641

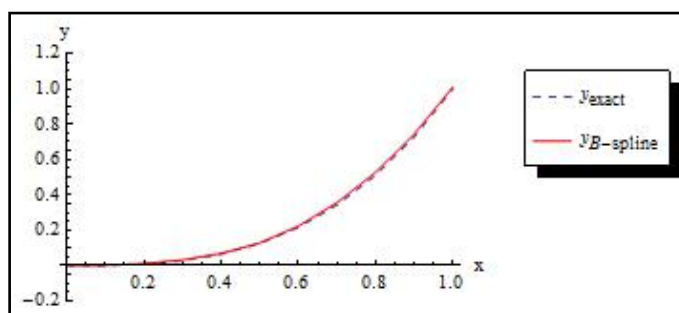


Figure 1: Comparison of numerical solution obtain by B-spline (M= 2) with exact solution

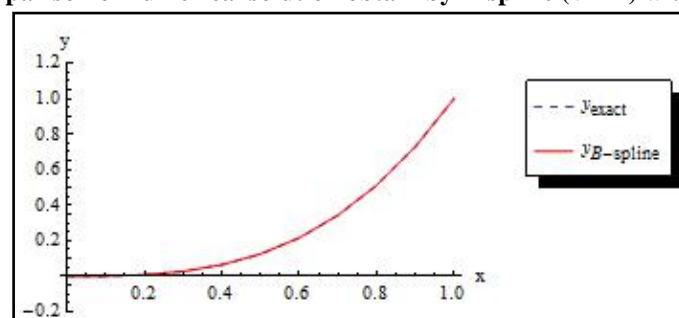


Figure 2: Comparison of numerical solution obtains by B-spline (M= 4) with exact solution

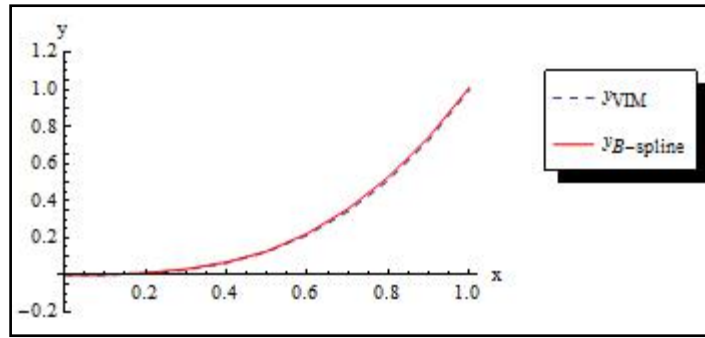


Figure 3: Comparison of numerical solution obtains by B-spline (M= 2) with VIM solution

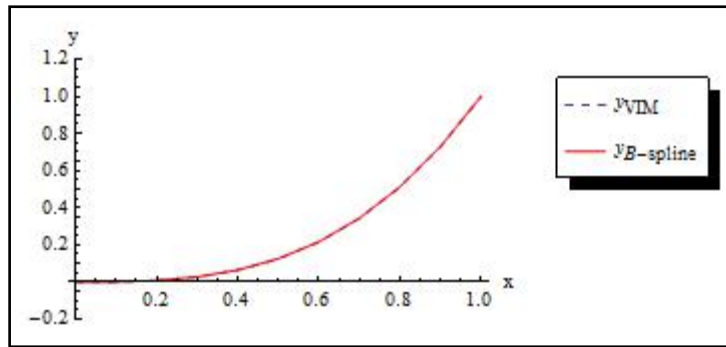


Figure 4: Comparison of numerical solution obtains by B-spline (M= 4) with VIM solution

Example 2: Consider the equation

$$u(x) = \cos x - \frac{1}{2}x \sin(2) + \int_0^1 x(u^2(t) - \sin^2(t))dt, \quad 0 \leq x \leq 1,$$

with the right arrangement $u(x) = \cos x$. The rough arrangement is gotten by the B-spline wavelet strategy for $M = 2$ and $M = 4$ and besides by VIM. The going with Table 2 alludes to the mathematical arrangements gotten by B-spline technique and VIM accomplished with relating cautious arrangements and the outright blunders. Figures 5-6 and Figures 7-8 present the relationship graphically between the mathematical arrangements obtained by B-spline wavelet technique with right arrangements and VIM arrangements independently.

Table 2: Comparison of numerical results by BWM and VIM

x	Exact	BWM solution		Error in BWM		VIM solution	Error in VIM
		M = 2	M = 4	M = 2	M = 4		
0.0	1	1.0013	1.00008	0.0013027	0.0000813	1	0
0.1	0.995004	0.994854	0.994995	0.0001501	9.23203e-6	0.994897	0.0001069
0.2	0.980067	0.979103	0.980006	0.0009632	0.0000600	0.979853	0.0002139
0.3	0.955336	0.954196	0.955265	0.0011401	0.0000718	0.955016	0.0003209
0.4	0.921061	0.920326	0.921014	0.0007350	0.0000472	0.920633	0.0004279
0.5	0.877583	0.877732	0.877591	0.0001495	8.82354E-6	0.877048	0.0005348
0.6	0.825336	0.824168	0.825263	0.0011679	0.0000721	0.824694	0.0006418
0.7	0.764842	0.763003	0.764727	0.0018390	0.0001148	0.764093	0.0007488
0.8	0.696707	0.694726	0.696581	0.0019810	0.0001252	0.695851	0.0008558
0.9	0.62161	0.619877	0.621499	0.0017320	0.0001110	0.620647	0.0009628
1.0	0.540302	0.53905	0.540222	0.0012520	0.0000807	0.539233	0.0010697

CONCLUSION: -

In present paper, semi-orthogonal minimally upheld linear B-spline wavelets have been associated with locate the numerical arrangement of nonlinear Fredholm integral conditionsof second kind and their frameworks. The double wavelets for these B-spline wavelets have been moreover shown. Considering semi-orthogonally, diminished assistance and vanishing moment’s properties of B-spline wavelets, the matrices are very scanty. Using this method, the integral equations and their system have been decreased to fathom systems of algebraic equations. The illustrative examples have been incorporated to exhibit the validity and applicability of procedure. These examples in like manner show the exactness and capability of the present method.

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