RESEARCH REVIEW JOURNALS
Volume-02
Issue-06
June-2017

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com

Fixed Point Through Rational Inequality

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ARTICLE DETAILS

Article History
Received: 10 June 2017
Accepted: 15 June 2017
Published Online: 18 June 2017

Key words
Fixed point through Rational Inequality

ABSTRACT

In this paper we have established fixed point through, Rational Inequality for expansion type of mappings in 2-metric spaces. Our results improve and extend the results of Som (2003), T. Seki (1975).

1. INTRODUCTION

Generalization of fixed point theorems for 2-metric space has been done by many authors in many ways. We know that the contraction mappings are uniformly continuous but converse is not is not true. Kanna [Some results of fixed points, Bull. Cal. Math Soc. (68), (1968)] established that there exist maps possessing discontinuity in their domain but still admitting fixed point. Later on Jaggi& Das [41] extended this idea naming it fixed point through rational expression in a metric space.

In this paper, we have extended this idea to 2-metic space. We have proved common fixed point theorems for discontinuous mappings satisfying relatively more general type of rational inequality our results extends the result of Fisher [10], Kanan and other in moe general form.

Theorem : Let S and T be two self-mappings of a complete 2-metric space (X,d) into itself such that for all x, y, a in X either :

(i) $$d(Sx,Ty,a) \le \alpha_1 \frac{[d(Sx,y,a)]^2 + [d(Ty,x,a)]^2}{d(Sx,y,a) + d(Ty,x,a)}$$
$$+\alpha_2[d(Sx,x,a) + d(ty,y,a)] + \alpha_3 d(x,y,a)$$

if $d(Sx,y,a) + d(Ty,x,a) \neq 0, a_1 \ge 0 (i = 1,2,3)$ with atleast one of a_1 is non-zero and $2\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$ or ,

(ii) $d(Sx,Ty,a) = 0$ if $d(Sx,y,a) + d(Ty,x,a) = 0$

Then S and T have a unique common fixed point.

Proof : Let x_0 be any arbitrary point in x and (x_n) be a sequence defined as $x_0x_1, = Sx_0; x_2 = Tx_1; x_3 = S_{x2}; \ldots\ldots\ldots\ldots x_{2n-1} = Sx_{2n-2}; x_{2n} = Tx_{2n-1,} \ldots\ldots\ldots$and so on

We may assume that $x_n \neq x_{n+1}$

First we show that $d(x_{2n}x_{2n+1}, x_{2n+2}) = 0$

$$d(x_{2n}x_{2n+1}, x_{2n+2}) = d(x_{2n}Sx_{2n}Tx_{2n+1}) = d(Sx_{2n}, Tx_{2n+1}, x_{2n})$$
$$\le \alpha_1 \frac{[d(Sx_2, x_{2n+1}, x_{2n})]^2 + [d(Tx_{2n+1}, x_{2n}, x_{2n})]^2}{d(Sx_{2n}, x_{2n}, x_{2n}) + d(Tx_{2n+1}, x_{2n}, x_{2n})}$$
$$+\alpha_2[d(Sx_{2n}, x_{2n+1}, x_{2n}) + d(Tx_{2n+1}, x_{2n+1}, x_{2n})] + \alpha_3 d(x_{2n}, x_{2n+1}x_{2n})$$
$$= \alpha_1 \frac{[d(x_{2n+1}, x_{2n+1}, x_{2n})]^2 + [dx_{2n+2}, x_{2n}, x_{2n})]^2}{d(x_{2n+1}, x_{2n+1}, x_{2n}) + d(x_{2n+2}, x_{2n}, x_{2n})}$$
$$+\alpha_2[d(x_{2n}+1, x_{2n}+1, x_{2n}) + d(x_{2n}+2, x_{2n}x_{2n})] + \alpha_3 d(x_{2n}, x_{2n+1}, x_{2n}) = 0$$

i.e. $d(x_{2n}, x_{2n+1,} x_{2n+2}) \le 0$, which gives $d(x_{2n}, x_{2n+1}, x_{2n+2}) = 0$

Now,

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$$d(x_{2n+1}, x_{2n+2}, a) = d(Sx_{2n}, Tx_{2n+1}, a)$$
$$\leq \alpha_1 \frac{[d(Sx_{2n}, x_{2n+1}, a)]^2 + [d(Tx_{2n+1}, x_{2n}, a)]^2}{d(Sx_{2n}, x_{2n+1}, a) + (Tx_{2n+1}, x_{2n}, a)}$$
$$+\alpha_2[d(Sx_{2n}, x_{2n}, a) + d(Tx_{2n+1}, x_{2n+1}, a)] + a_3 d(x_{2n}, x_{2n+1}, a)$$
$$= \alpha_1 \frac{[d(x_{2n+1}, x_{2n+1}, a)]^2 + [d(x_{2n+1}, x_{2n}, a)]^2}{d(x_{2n+1}, x_{2n+1}, a) + d(x_{2n+2}, x_{2n}, a)}$$
$$+\alpha_2[d(x_{2n+1}, x_{2n}, a) + d(x_{2n+2}, x_{2n-1}, a)] + a_3 d(x_{2n}, x_{2n+1}, a)$$
$$= \alpha_1, d(x_{2n+2}, x_{2n}, a) + a_2[d(x_{2n+1}, x_{2n}, a) + d(x_{2n+2}, x_{2n+1}, a)] +$$
$$\alpha_3 d(x_{2n}, x_{2n+1}, a)$$
$$\leq \alpha_1[d(x_{2n+2}, x_{2n}, x_{2n-1}) + d(x_{2n+2,} x_{2n+1}, a) + d(x_{2n+1}, x_{2n}, a)$$
$$+\alpha_2[d(x_{2n+1}, x_{2n}, a) + d(x_{2n+2}, x_{2n+1}, a)] + a_3 d(x_{2n}, x_{2n+1}, a)$$
$$= \alpha_1[d(x_{2n+2}, x_{2n+1}, a) + d(x_{2n+1}, x_{2n}, a)]$$
$$+\alpha_2[d(x_{2n+1}, x_{2n}, a) + d(x_{2n+2}, x_{2n+1}, a)] + a_3 d(x_{2n}, x_{2n+1}, a)$$
$$+(\alpha_1 + \alpha_2) d(x_{2n+1}, x_{2n+2}, a) + (a_1 + a_2 + a_3) d(x_{2n}, x_{2n+1}, a)$$
or, $(1 - a_1 - a_2) d(x_{2n+1}, x_{2n+2}, a) \leq (a_1 + a_2 + a_3) d(x_{2n}, x_{2n+1}, a)$

or, $d(x_{2n+1}, x_{2n+2}, a) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{(1 - a_1 - a_3)} d(x_{2n}, x_{2n+2}, \alpha)$

Again,

$$\alpha(x_{2n+2}, x_{2n+3,} \alpha) = d(Tx_{2n+1}, Sx_{2n+2}, \alpha)$$
$$= d(Sx_{2n+2}, Tx_{2n+1}, \alpha)$$
$$\leq a_1 \frac{[d(Sx_{2n+2}, x_{2n+1}, a)]^2 [d(Tx_{2n+1}, x_{2n+2}, a)]^2}{d(Sx_{2n+2}, x_{2n+1}, a) + d(Tx_{2n+1}, x_{2n+2}, a)}$$
$$+a_2[d(Sx_{2n+2}, x_{2n+2}, a) + d(Tx_{2n+1}, x_{2n+1}, a)] + a_3 d(x_{2n+2}, x_{2n+1}, a)$$
$$= a_1 \frac{[d(x_{2n+3}, x_{2n+1}, a)]^2 [d(x_{2n+2}, x_{2n+2}, a)]^2}{d(x_{2n+3}, x_{2n+1}, a) + d(x_{2n+2}, x_{2n+2}, a)}$$
$$+a_2[d(x_{2n+3}, x_{2n+2}, a) + d(x_{2n+2}, x_{2n+2}, a)] + a_3 d(x_{2n+2}, x_{2n+1}, a)$$
$$= a_1 d(x_{2n+3}, x_{2n+1}, a) + a_2 d(x_{2n+3}, x_{2n+2}, a) + a_2 d(x_{2n+1}, x_{2n+2}, a)$$
$$\leq a_1[d(x_{2n+3}, x_{2n+2}, a) + d(x_{2n+2}, x_{2n+1}, a) + a_2 d(x_{2n+3}, x_{2n+2}, a) +$$
$$a_3 d(x_{2n+1}, x_{2n+2}, a)$$
$$= (a_1 + a_2) d(x_{2n+3}, x_{2n+2}, a) + a(_1 + a_3) d(x_{2n+1}, x_{2n+2}, a)$$
$$\text{or, } (1 - a_1 - a_2) d(x_{2n+3}, x_{2n+2}, a) \leq (a_1 + a_3) d(x_{2n+1}, x_{2n+2}, a)$$
$$\text{or, } d(x_{2n+3}, x_{2n+2}, a) \leq \frac{a_1 + a_3}{(1 - a_1 - a_2)} d(x_{2n+1}, x_{2n+2}, a)$$
$$\leq \left(\frac{a_1 + a_3}{1 - a_1 + a_2}\right)\left(\frac{a_1 + a_2 + a_3}{1 - a_1 + a_2}\right) d(x_{2n}, x_{2n+1}, a)$$

Containing in this way we observe that $\{X_{2n}\}$ is a Cauchy sequence. Lim X_1 is complete 2-metric space, so $\{X_{2n}\}$ converges to z(Say) i.e. $\lim_{n\to\infty} x_n = z;$

Now,

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$$d(Sz,z,a) \le d(Sz,z,x_{2n}) + d(Sz,x_{2n},a) + d(x_{2n},z,a).$$
$$= d(Sz,z,x_{2n}) + d(x_{2n}z,a) + d(Sz,Tx_{2n-1},a)$$
$$\le d(Sz,z,x_{2n}) + d(x_{2n},z,a) + a_1 \frac{[d(Sz,x_{2n-1},a)]^2 + (d(tx_{2n-1},z,a)]^2}{d(Sz,x_{2n-1},a) + d(Tx_{2n-1},z,a)}$$
$$+a_2[d(Sz,z,a) + d(Tx_{2n-1},x_{2n-1},a)] + a_3 d(z,x_{3n-1},a)$$

When $n \to \infty, x_{2n} \to z$ and $x_{2n-1} \to z$

$$d(Sz,z,a) \le 0 + 0 + a_1 d(Sz,z,a) + a_2 d(Sz,z,a) + a_3, 0$$
$$ord(Sz,z,a) \le (a_1 + a_2) d(Sz,z,a)$$

Thus, Sz = z, Similarly we can prove that Tz = z. So z is a common fixed point of sand T'. Now we claim that z is the common fixed point of S and T. If possible let w is the another common fixed point of S and T such that $z \ne w, S(w) = T(w) = w$ Then,

D(z,w,a) = d(Sz, Tw, a)

$$\le \alpha_1 \frac{[d(Sz,w,a)]^2 + [d(Tw,z,a)]^2}{d(Sz,w,a) + d(Tw,z,a)}$$
$$+\alpha_2[d(Sz,z,a) + d(Tw,w,a)] + a_3 d(z,w,a)$$
$$= \alpha_1 \frac{[d(z,w,a)]^2 + [d(w,z,a)]^2}{d(z,w,a) + d(z,w,a)} + \alpha_2[d(z,z,a) + d(z,x,a)] + \alpha_3 d(z,w,a)$$
$$= (\alpha_2 + \alpha_3) d(z,w,a)$$

i.e. d (z, w, a) $\le (a_1 + a_3)$ d (z, w, a), which is not possible and hence our supposition was wrong. So d(z, w, a) = 0. i.e. z = w. i.e. S and T have a unique common fixed point.

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