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A Non-Static Magnetohydrodynamic Universe in General Relativity

Dr. Sunil Kumar

Assistant Teacher Rashtriya Inter School, Daudnagar, Aurangabad (Bihar)

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ABSTRACT

In this paper we have discussed Bianchi type-I cosmological model with perfect fluid and electromagnetic, field. Taking a suitable metric we have also calculated various physical and geometrical properties e.g. pressure, density, scalar of expansion and components of shear tensor. We have also discussed behaviour of a test particle and Doppler's Effect in the model.

1. Introduction

The behaviour of the magnetic field of a star was investigated by Cowling [12] and Wrubel [20]. Shikin [14] also constructed a uniform axially symmetric solution (model) of Einstein-Maxwell equations in the case of propagation by an ideal fluid in the presence of magnetic field directed along the axis of symmetry. Magnetic field in stellar bodies was also discussed by Monoghan [11]. Gravitational collapse of a magnetic star was studied by Ginzburg [4]. Seymour [13] also derived some models of the galactic magnetic field. Jacobs [7, 8] has studied the behaviour of the general Bianchi-type I cosmological model in the presence of a spatially homogeneous magnetic field. This problem has been studied again by De [3] with a different approach. This work has been further extended by Tupper [18] to include Einstein-Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI cosmologies with electromagnetic field (Tupper [19]). Roy and Prakash [12] taking the cylindrically symmetric metric of Marder [10] have constructed a spatially homogenous cosmological model in the presence of an incident magnetic field which is also anisotropic and non-degenerate Petrov type – I Bali et al. [21 a, b] have studied magnetized cylindrically symmetric universe in general relativity. Cosmological model in geneal relativity have been also studied by Paul [12 (a)]. Singh and Yadav [15] have constructed a spatially homogeneous cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. Some other worker's in this time are Zeldovich [22, 25] Theone [16] and Yadav and Purushothem [24].

In this paper we have discussed Bianchi type-I cosmological model with perfect fluid and electromagnetic, field. Taking a suitable metric we have also calculated various physical and geometrical properties e.g. pressure, density, scalar of expansion and components of shear tensor. We have also discussed behaviour of a test particle and Doppler's Effect in the model.

2. The Field Equations and Their Solution

We start with the metric [Marder [10])

(2.1) $$ds^2 = \alpha^2\left(dt^2 - dx^2\right) - \beta^2 dy^2 - \gamma^2 dz^2$$

Where □, □ and □ are functions of t only. The distribution consists of a perfect fluid with an electromagnetic field. The energy momentum tensor of the composite field is assumed to be the sum of the corresponding energy momentum tensor. Thus

(2.2) $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \wedge g_{\mu\nu} = G_{\mu\nu} + \wedge g_{\mu\nu}$$

$$= -k\left[(\rho + p)\lambda_\mu \lambda_\mu - p g_{\mu\nu} + E_{\mu\nu}\right]$$

(2.3) $$E_{\mu\nu} = -g^{kl} F_{\mu k} F_{\nu l} + \frac{1}{4} g_{\mu\nu} F_{ab} F^{ab}$$

(2.4) $$F_{[\mu\nu;\sigma]} = 0$$

(2.5) $$F^{\mu\nu}_{;\nu} = J^{\mu}$$

Where p and □ are pressure and density respectively of the distribution, E□v is the electromagnetic energy momentum tensor, F□v is the electromagnetic field tensor, J□ is the current 4- vector, ^ is the cosmological constant and λ_μ is the flow vector satisfying

(2.6) $$g_{\mu\nu} \lambda^\mu \lambda^\nu = 1$$

The co-ordinates are chosen to be commoving so that

(2.7) $$\lambda^{\mu} = \left(0,0,0,\frac{1}{\alpha}\right)$$

And we label the co-ordinates $(x,y,z,t) = (x^1,x^2,x^3,x^4)$

We assume the electromagnetic field to be in the direction of x-axis so that F_{14} and F_{23} are the only non-vanishing components of the field tensor $F_{\mu v}$. We write

(2.8) $$F_{14}^2\alpha^{-4} + F_{23}^2\beta^{-2}\gamma^{-2} = M^2$$

The diagonal components of the equation (2.2) may be written as

(2.9) $$\frac{2}{\alpha^2}\left(\frac{\alpha_{44}}{\alpha} + \frac{\beta_{44}}{\beta} + \frac{\gamma_{44}}{\gamma} - \frac{\alpha_4\gamma_4}{\alpha\gamma} - \frac{\alpha_4\beta_4}{\alpha\beta} - \frac{\alpha_4^2}{\alpha^2}\right) - 2\wedge = -k\left(-M^2 + (\rho + 3p)\right)$$

(2.10) $$-\frac{2}{\alpha^2}\left[\frac{\alpha_{44}}{\alpha} + \frac{\alpha_4\beta_4}{\alpha\beta} + \frac{\alpha_4\gamma_4}{\alpha\gamma} - \frac{\alpha_4^2}{\alpha^2}\right] + 2\wedge = k(M^2 + (\rho + p))$$

(2.11) $$-\frac{2}{\alpha^2}\left[\frac{\beta_{44}}{\beta} + \frac{\beta_4\gamma_4}{\beta\gamma}\right] + 2\wedge = k(-M^2 + (\rho + p))$$

(2.12) $$-\frac{2}{\alpha^2}\left[\frac{\gamma_{44}}{\gamma} + \frac{\beta_4\gamma_4}{\beta\gamma}\right] + 2\wedge = -k(-M^2 + (\rho - p))$$

Where the suffix 4 after the symbols □, □, □ stands for ordinary differentiation with respect to time. It is evident from these equations that M^2, □ and p are each functions of time alone. From equation (2.4) and (2.8) it follows that F_{23} is a constant and F_{14} is a function of t only i.e.

(2.13) $$F_{23} = k$$

$$F_{14} = \pm\alpha^2\left(M^2 - k^2\beta^{-2}\gamma^{-2}\right)^{1/2}$$

where k is a constant.

The case when there is no electric field i.e., when $F_{14} = 0$, we have J□ = 0. It is the case considered by Roy and Prakash [12]. Hence wfe assume that $F_{14} \neq 0$ and find the only non-zero component of J□ to be

(2.14) $$J^{\mu} = \pm\frac{1}{\alpha^2\beta\gamma}\frac{\partial}{\partial t}\left[\beta\gamma\left(M^2 - k^2\beta^{-2}\gamma^{-2}\right)^{1/2}\right]$$

Equation (2.14) shows that J□ is space like, unless $M^2 = f\beta^{-2}\gamma^{-2}$ where f is a constant in which case $J^{\mu} = 0$.

The 4 – current J□ is in general the sum of the convectin current and conduction current (Lichnerowicz [9] and Greenburg [5]);

(2.15) $$J^{\mu} = \varepsilon_0\lambda^{\mu} + \xi\lambda_v F^{\mu v}$$

where □0 is the rest charge density and ξ is the conductivity. In the case considered here we have □0= 0 i.e. magnetohydrodynamics. Thus

(2.16) $$\xi = -\frac{1}{\alpha}I_4 I^{-1}$$

where $I = \beta\gamma\left(M^2 - k\beta^{-2}\gamma^{-2}\right)^{1/2}$

Finally we illustrate the situation described here by an example. Consider the space time with metric.

(2.17) $$ds^2 = t^{16h^2}(dx^2 - dt^2) + t^{4h}(dy^2 + dz^2)$$

Which is obtained from metric (2.1) when

$$\alpha = t^{8h^2}, \beta = \gamma = t^{2h}$$

where h being an arbitrary constant parameter.

Equations (2.9) – (2.12) lead to

(2.18) $M^2 = 2h(8h+1)(2h-1)t^{-16h^2-2}$

(2.19) $\rho = h\left(-16h^2+6h-1\right)t^{-16h^2-2} + \wedge$

(2.20) $P = h\left(16h^2+2h-3\right)t^{-16h^2-2} - \wedge$

Clearly M^2, □, P all are decreasing function of time.
From equation (2.18)

(2.21) $0 > h > -\frac{1}{8}$

Which implies that □> 0 and P > 0 We also find that

(2.22) $\frac{3}{2} < \frac{\rho}{P} < 3$

It is seen that the model satisfies the dominant energy condition (Hawking and Penrose [6]) and the fluid energy condition. □ + P > 0

The electromagnetic field components are

(2.23) $F_{23} = K$

$$F_{14} = \pm t^{16h^2} - 4h\left[t^{-16h^2+8h-2}2h(8h+1)(2h-1) - K^2\right]^{1/2}$$

And the magnitude of the magnetic field is restricted by

(2.24) $K^2(2h(2h-1)(8h+1)t^{-16h^2+8h-2}$

The non-zero component of the current four vector is

(2.25) $J = \pm h(2h-1)(8h+1)(16h^2-8h+2)t^{-32h^2+4h-3}$

$$\times\left[2h(2h-1)(8h+1)t^{-16h^2+8h-2} - K^2\right]^{1/2}$$

Also the conductivity for the model is given by

(2.26) $\xi = h(2h-1)(8h+1)\left(16h^2-8h+2\right)t^{-24h^2+8h-3}$

$$\times\left[2h(2h-1)(8h+1)t^{-16h^2+8h-2} - K\right]^{-1}$$

Which is positive and is zero for h = 0 and $h = -\frac{1}{8}$ respectively.

Scalar of expansion □ is giving.

(2.27) $\theta = 4ht^{-8h^2-1}[1-2h]$

Hence the electric field, the 4-current the fluid density, the pressure and scalar of expansion all start with infinite values at the initial singularity (t = 0) and tend to zero when $t \to \infty$.

The components of the shear tensor σ_{ij} are

(2.28) $\sigma_{11} = 4h[2h-1]t^{8h^2-1}$

$$\sigma_{22} = \sigma_{33} = 2h[2h-3]t^{-8h^2+4h-1}$$

$$\sigma_{44} = 4h[1-2h]t^{8h^2-1}$$

3. Behaviour of a Test Particle in the Model

The equation of deodesic viz.

(3.1) $$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\nu\lambda}\frac{dx^\lambda}{ds}\frac{dx^\nu}{ds}=0$$

For the metric (2.17) when □ = 1, 2, 3, 4 are given by

(3.2) $$\frac{d^2x}{ds^2}+8h^2t^{-1}\frac{dx}{ds}\frac{dt}{ds}=0$$

(3.3) $$\frac{d^2y}{ds^2}+2ht^{-1}\frac{dy}{ds}\frac{dt}{ds}=0$$

(3.4) $$\frac{d^2z}{ds^2}+2ht^{-1}\frac{dz}{ds}\frac{dt}{ds}=0$$

(3.5) $$\frac{d^2t}{ds^2}+8h^2t^{-1}\left(\frac{dx}{ds}\right)^2-2ht^{-16h^2+4h-1}$$

$$\left(\frac{dy}{ds}\right)^2-2ht^{-16h^2-4h-1}\left(\frac{dz}{ds}\right)^2-8h^2t^{-1}\left(\frac{dt}{ds}\right)^2=0$$

In a particle is initially at rest, that is, if

(3.6) $$\left(\frac{dx}{ds}\right)=\frac{dy}{ds}=\frac{dz}{ds}=0$$

And the particle would remain permanently at rest.

4. The Doppler Effect in the Model

The track of a light pulse in the model is obtained by setting ds^2= 0 i.e.

(4.1) $$\left(\frac{dx}{dt}\right)^2+t^{+2-16h^2}\left(\frac{dy}{dt}\right)^2+t^{+4h-16h^2}\left(\frac{dz}{dt}\right)^2=1$$

For the case when velocity is along z exis equation (4.1) gives

$$\left(\frac{dz}{dt}\right)=\pm t^{-2h+8h^2}=\pm t^{2h(4h-1)}=\pm\psi(t)$$

Hence the light pulse leaving a particle at (0, 0, z) at time t_1 would arrive at a latter time t_2 given by

(4.2) $$\int_{t_1}^{t_2}\psi(t)dt=\int_0^z dz$$

Hence

(4.3) $$\psi_2(t)\sigma t_2=\psi_1(t)\delta t_1+\frac{dz}{dt}dt_1=\psi_1(t)\delta t_1+U_z\delta t_1$$

Where $\frac{dz}{dt}U_z$ is the z – component of the velocity of the particle at the time of emission and $\psi_1(t)$ and $\psi_2(t)$ are the value of $\psi(t)$ for t = t_1 and t = t_2 respectively.

From the above equation, we get

(4.4) $$\delta t_2=\left\{\frac{\psi_1(t)+U_z}{\psi_2(t)}\right\}\delta t_1$$

The proper time interval δt_1^0 between the successive wave crests as measured by the local observer moving with the source is given by

(4.5) $$\delta t_1^0 = \left\{ + t^{16h^2} - t^{16h^2}\left(\frac{dx}{dt}\right)^2 - t^{4h}\left(\frac{dy}{dt}\right)^2 - t^{4h}\left(\frac{dz}{dt}\right)^2 \right\}^{1/2} dt_1$$

This can be written as

(2.6) $$\delta t_1^0 = \left[t^{16h^2} - U^2 \right]^{1/2} \delta t_1$$

Where U is the velocity of the source at the time of emission, similarly we may write

(4.7) $$\delta t_2^0 = t^{8h^2} \delta t_2$$

As the proper time interval between the reception of two successive ware creates by an observer at rest at the origin. Hence following Tolman (1960) [17] the red shift in this case given by

$$\delta t_2^0 = t^{8h^2} = t^{8h^2}\left[\frac{t_1^{2h} + U_z}{t_2^{2h}} \right]$$

(2.8) $$\frac{\lambda + \delta\lambda}{\lambda} = \frac{\delta t_2^0}{\delta t_1^0} = t^{8h^2} = \frac{t_1^{2h(4h-1)} + U_z}{t_2^{2h(4h-1)}\left(t^{16h^2} - U^2\right)^{1/2}}$$

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