

Static Charged Fluid Spheres with Conformal Flatness In Einstein-Maxwell Theory

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ABSTRACT

The present paper provides some solutions of Einstein-Maxwell field equations for static conformally flat charged perfect fluid sphere by considering uniform mass density. Pressure and density have been calculated for the distribution.

1. Introduction

Much attention has been shown in finding exact solutions of coupled Einstein-Maxwell equations for static spherical distribution of charged matter [1(a), 5, 6, 79, 13, 16, 15, 22, 22(a), 23, 24]. These distributions constitute possible sources for the Reissner-Nordstrom metric which uniquely describes the exterior field of a spherically symmetric charged distribution of matter. As the field equations do not completely determine the system, different solutions were obtained by many authors by using different conditions to supplement the field equations. The supplementary conditions were used partly specify the physical model and partly to simplify the mathematical work.

Krori and Barua [13] found some solutions for charged fluid distributions with spherical symmetry which are very regular. Chakraverti and De [7] studied the problem of static charged fluid distributions in the form of a spherical ball and presented some new solutions which are regular where and of course, the solutions could be matched with outside Reissner-Nordstrom metric. In all their solutions, the charge to mass ratio of the spherical ball was as expected, less than unity. Singh and Yadav [22] have also found some exact solutions of charged fluid sphere in general relativity.

Bonnor [2], Effinger [10], Kyle and Martin [14] have considered the interior solution for a static charged sphere. As the field equations do not completely determine the system, different solutions were obtained by Effinger [10] and Kyle and Martin [14] by using different conditions to supplement the field equations. A conformally flat spherically symmetric non-static internal solution was obtained by Singh and Abdussattar [21]. Later on Roy and Raj Bali [19] found a general solution representing conformally flat perfect fluid distribution of spherical symmetry. They have also discussed various physical properties of the model.

Gurses [12] has shown that the only static distribution of the fluid with positive density and pressure which would generate a conformally flat metric through the Einstein's equations without cosmological term is that described by the Schwarzschild interior solution. Burman [4] discussed the motion of the particles in Conformally flat space-time. Singh and Abdussattar [21] has obtained a non static generalization of the Schwarzschild interior solution which is conformal to flat space time. They have also shown that the model admits of distribution of discrete particles and disordered radiation. Zaleev [25] and Shikin [20] have obtained conformally flat non static solution in general relativity theory and scalar-tensor theories of gravitation. Callinson [8] has shown that every conformally flat axisymmetric stationary space-time is static. He has also proved that if the source is a perfect fluid the space-time is the interior Schwarzschild field. Gupta [11] has observed that if a conformally flat space-time describes a perfect fluid distribution of matter $p \neq 0$, then it is necessarily of embedding class one.

Rao and Reddy [17] have shown that there are no spherically symmetric static conformally flat solution of Nordvedt-Brakar field equations for perfect fluid distribution with disordered radiation obeying the equation of state $p = 3\rho$, except for the trivial empty flat space-time of Einstein's theory. Zhu-shi-chang [5, 6] has obtained some conformal flat interior solution of the Einstein-Maxwell equations for a charged stable static sphere which satisfy physical conditions inside the sphere. The metrics of the spheres of charged dust have been investigated by Bonner and Wichremasuriya [3] and Raychaudhuri [18].

In this paper we have presented some solutions of Einstein-Maxwell field equations for static conformally flat charged perfect fluid sphere by considering uniform mass density. Pressure and density have been calculated for the distribution.

2. The Field Equations

We use here the static spherically symmetric line element in the form.

$$(2.1) \quad ds^2 = e^\beta dt^2 - e^\alpha dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where β and α are functions of r only.

The Einstein – Maxwell equations for the charged perfect fluid distribution in general relativity are

$$(2.2) \quad R_{ij} = -8\pi T_{ij},$$

$$(2.3) \quad \left[(-g)^{1/2} F^{ij} \right]_{,j} = 4\pi J^i (-g)^{1/2},$$

$$(2.4) \quad F_{(i j,k)} = 0$$

Here T_{ij} is the energy momentum tensor, J^i is the current four vector R_{ij} is the Ricci tensor and R the curvature scalar. For the system under study the energy momentum tensor, T_j^i splits up into two parts viz. M_j^i for matter and charges respectively i.e.

$$(2.5) \quad T_j^i = M_j^i + E_j^i$$

where

$$(2.6) \quad M_j^i = (\rho + p_j u^i u_j - p \delta_j^i)$$

with

$$(2.7) \quad u^i u_i = 1$$

The non-vanishing components of M_j^i are

$$(2.8) \quad M_1^1 = M_2^2 = M_3^3 = -p, M_4^4 = \rho$$

This the Einstein-Maxwell field equations are

$$(2.9) \quad e^{-\alpha} \left(\frac{1}{r^2} - \frac{\alpha'}{r} \right) - \frac{1}{r^2} = -8\pi\rho - E^2$$

$$(2.10) \quad \frac{1}{r^2} e^{-\alpha} \left(\frac{1}{r^2} - \frac{\beta'}{r^2} \right) = -8\pi p + E^2$$

$$(2.11) \quad e^{-\alpha} \left[\frac{1}{4} \beta' \alpha' - \frac{1}{4} \beta'^2 - \frac{12}{2} B'' - \left(\frac{\beta' - \alpha}{r} \right) \right] = -8\pi p - E^2$$

where $Q(r)$ represents the total charge contained within the sphere of radius r , we have

$$(2.13) \quad Q(r) = 4\pi \int \rho_c r^2 dr$$

where ρ_c is the charge density.

3. Solution Of The Field Equation

We have five equations (2.1) – (2.4) and (2.9) in six variables $\alpha, \beta, Q(r), E^j, p$ and ρ . Hence the system is indeterminate. To make the system determinate.

We require one more relation. For this we choose uniform mass density in the form.

$$(3.1) \quad \rho = A$$

where A is constant. Now integrating equation (2.9) we get

$$(3.2) \quad c^{-\alpha} = 1 + \eta r^2 \xi(r)$$

where η is integration constants and $\xi(r)$ is

$$(3.3) \quad \xi(r) = 2r^2 \int E'/r dr$$

from (3.1), (3.2) and (2.2) we get

$$(3.4) \quad \xi'/r + \xi r^2 = -8\pi(A + Br^2) - E^2 - 3\eta$$

Now differentiating (3.4) we get

$$(3.5) \quad \frac{dE^2}{dr} + 2 \frac{E^2}{r} = 0$$

Solution of (3.5) we get

$$(3.6) \quad E^2 = \frac{\bar{A}}{r^2}$$

$$(3.7) \quad Q^2(r) = Ar^2$$

where \bar{A} is the integration constant.

Using (3.6) in to (3.3) we find

$$(3.8) \quad \xi(r) = -\bar{A}$$

From (3.8) and (3.2) we get

$$(3.9) \quad e^{-\alpha} = 1 - \bar{A} + \eta r^2$$

Inserting (3.9) and (3.2) into (3.2) we can prove

$$(3.10) \quad \eta = -\frac{8\pi}{3} - A = -\frac{1}{R^2}$$

so that

$$(3.11) \quad e^{-\alpha} = e - \frac{r^2}{R^2}$$

where $e = 1 - \bar{A}$

using equation (3.6) and (3.11) into eliminant of (2.10) and (2.11) we get.

$$(3.12) \quad \beta' + \frac{1}{2}\beta'2 - \left[\frac{1}{2} - \frac{-r/R^2}{C - r^2/R^2} \right] \beta' = \frac{2(c-1)}{r^2(c - r^2/R^2)}$$

By use of transformations

$$(3.13) \quad r^2 = x$$

and

$$(3.14) \quad x^{-1/2} e^{\beta/2} = z$$

equation (3.12) is transformed to

$$(3.15) \quad \frac{d^2z}{dx^2} + \left[x^{-1} \frac{1}{2(cR^2 - x)} \frac{dz}{dx} \right] = \frac{(c-1)}{2x^2 - c}$$

Again using the transformation

$$(3.16) \quad u = \frac{D}{R} \frac{1}{(c)^{1/2}} \left[\frac{(c - x/R^2)^{1/2}}{x} + \frac{c}{x} - \frac{1}{2\sqrt{e}} R^2 \right]$$

where D is constant, equation (3.15) is changed into

$$(3.17) \quad D^2 \frac{d^2z}{dx^2} - \frac{(2c-1)R^2}{4} z = 0$$

Thus final solution of (3.12) is (for $r^2 \geq cR^2$)

$$(3.18) \quad e^\beta = r^2 \{ c_1 \sin[1/\sqrt{c} \tan^{-1}(r^2/cR^2 - 1)^{1/2}] + c^2 \cos[1/\sqrt{c} \tan^{-1}(r^2/cR^2 - 1)^{1/2}] \}^2$$

where c_1 and c_2 are integration constants. Also pressure is given by

$$(3.19) \quad 8\pi\rho = 2c/r^2 - 3/r^2 - 2/\sqrt{c}(c/r^2 - 1/R^2)(r^2/cR^2 - 1)^{-1/2}$$

$$\frac{c_1 - c_2 \tan[1/\sqrt{c} \tan^{-1}(r^2 - cR^2 - 1)^{1/2}]}{c_1 \tan(1/\sqrt{c} \tan^{-1}(r^2/cR^2 - 1)^{1/2}) + c^2}$$

For $r^2 < cR^2$, the final solution of (3.12) is

$$(3.20) \quad e^\beta = r^2 \{c_1 \sinh[1/\sqrt{c} \tanh^4(1 - r^2/cR^2)^{1/2}]$$

$$+ c_2 \cosh[1/\sqrt{c} \tanh^4(1 - r^2/cR^2)^{1/2}]\}^2$$

and also

$$(3.21) \quad 8\pi\rho = 2c/r^2 - 3/R^2 - 2/\sqrt{c}(c/r^2 - 1/R^2)(1 - r^2/cR^2)^{1/2}$$

$$\times \frac{c_1 - c_2 \tanh[1/\sqrt{c} \tanh^{-1}(1 - r^2/cR^2)^{1/2}]}{c_1 \tanh(1/\sqrt{c} \tanh^{-1}(1 - r^2/cR^2)^{1/2}) + c^2}$$

If $\bar{A} \neq 0$ and $c \neq 0$ (3.6) and (3.21) are not regular, they will diverge we can not get a physically reasonable solution for charged perfect fluid with constant mass density. The constant appearing in the solution can be evaluated using boundary conditions by taking Recessner-Nordstrom metric out side the boundary.

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