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Fixed Point Theorems for Generalized Contraction Maps

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ABSTRACT

In this paper we have established fixed point theorem for Generalized contraction maps our results are extension of Jaggi& Das [3], Ahmad & Khan [4], Fisher [1] and others.

I. Introduction

In the theory of fixed point in 2-metric space was initiated by Iseki [38]. Later on Naidu and Prasad [17] introduced the concept of weak commutalivity. Further, Murthy et al. [68] introduced the concept of compatible maps in 2-metric space. T. Sam [Few common fixed point results for compatible mappings. Bull. Col. Math. Soc. 95, (4), 307-312 (2003)] proved the following result metric space using the motion of compatibility of mappings.

Definition (2.0.5) : Let (S,T) be a pair of self maps on a 2 metric space (X, d). (S, T) is said to be compatible $\lim_{n\to\infty} d(STx_n, TSx_n, a) = 0$ for all acX, whenever the sequence $\{X_n\}$ is a sequence in X such that $n \to \infty$.

Example (2.0.6) : Let $X = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}, \ldots\ldots.\right\}$. Define d : Xx, Xx, X $\to (0,\infty)$ by d(x, y, z) = 0. If x, y, z are distinct and $\left\{\frac{1}{n}, \frac{1}{n+1}\right\} \subset \{x, y, x\} = 1$ otherwise. Then (X, d) is a 2-metric space as shown in [68]. Let I be the identity on X and define a self-map S as follows :

$$S\left(\frac{1}{n}\right) = \frac{1}{n+2}, S(o) = 1 \text{ and } x_n = \frac{1}{n}.\text{Then}$$

$$\lim_{n\to\infty} d(lx_n, 0, a) = \lim_{n\to\infty} d(x_n, 0, a) = 0$$

$$\lim_{n\to\infty} d(Sx_n, 0, a) = \lim_{n\to\infty} d\left(\frac{1}{n+2}, 0, a\right) = 0$$

For all $a \in X$. Thus $\{x_n\}$ and $\{Sx_n\}$ converge to x = 0. Now the pair (I, S) is commutating. Hence it is compatible.

Theorem (2.1.1) : Let a be a continuous self mappings of a complete metric space (X, d) and Let S, T be two self-mappings of (X, d) such that :

(i) (A, T) and (A, S) are compatible.

(ii) $T(X) \subseteq A(x); S(X) \subseteq A(X)$

(iii) for all x, y, $a \in X$

ad(Sx, Ty, a) + bd (Sx, Ax, a) + cd(Ty.Ay.a)-
Min (d(Sx, Ay, a), d(Ty, Ax, a)} = qd(Ax, Ay, a).
where a, b, c > 0, q > 0 with a > q + l and a + b + c ? q.
Then S, T and A has a unique common fixed point.

Proof : Let x_0 be any arbitrary point in X. since $S(X) \subseteq A(X)$ we can choose a point x_i in Xs, t. $Ax_j = Sx_0$. Also $T(X) \subseteq A(X)$, we can choose a point x_2 such that $Ax_2 = Tx_1$. In general, $Ax_{2p+1} = Sx_{2p}$ and $Ax_{2p+2} = Tx_{2p+1}$, for p = 0, 1, 2, ...

Now putting $x = x_{2p+1}$ and $y = x_{2p+2}$ we have :

$$= ad(Sx_{2p+1}, Tx_{2p+2}, a) + bd(Sx_{2p+1}, Ax_{2p+1}, a) + cd(Tx_{2p+2}, Ax_{2p+2}, a)$$
$$- \min(d(Sx_{2p+1}, Ax_{2p+2}, a), d(Tx_{2p+2}, Ax_{2p+1}, a)\} = qd(Ax_{2p+1}, Ax_{2p+2}, a)$$
$$\text{i.e.}, ad(Ax_{2p+2}, Ax_{2p+3}, a) + bd(Ax_{2p+2}, Ax_{2p+1}, a) + cd(Ax_{2p+3}, Ax_{2p+2}, a)$$
$$- \min\{d(Ax_{2p+2}, Ax_{2p+2}, a), d(Ax_{2p+3}, Ax_{2p+1}, a)\} \le qd(Ax_{2p+1}, Ax_{2p+2}, a)$$
$$\text{or, } (a+c)d(Ax_{2p+2}, Ax_{2p+3}, a) + bd(Ax_{2p+2}, Ax_{2p+1}, a) \le qd(Ax_{2p+1}, Ax_{2p+2}, a).$$
$$\text{or, } (a+c)d(Ax_{2p+2}, Ax_{2p+3}, a) + bd(Ax_{2p+2}, Ax_{2p+1}, a) \le qd(Ax_{2p+1}, Ax_{2p+2}, a)$$
$$\text{or, } (a+c)d(Ax_{2p+2}, Ax_{2p+3}, a) \le (q-b)d(Ax_{2p+1}, Ax_{2p+2}, a).$$
$$\text{or, } d(Ax_{2p+2}, Ax_{2p+3}, a) \le \frac{q-b}{a+c} d(Ax_{2p+1}, Ax_{2p+2}, a)$$
$$\text{or, } d(Ax_{2p+2}, Ax_{2p+3}, a) \le pd(Ax_{2p+1}, Ax_{2p+2}, a)$$
$$\text{where } p = \frac{q-b}{a+c} < 1$$
$$\text{i.e. } d_{2n+2} \le pd_{2n+1} \le p^2 d_{2n} \le, \ldots\ldots\ldots \le p^{2n+1} d_0 \to 0 \text{ as } n \to \infty$$

Thus $\{Ax_{2p}\}$ is a Cauchy sequence in X and since X is complete, there exists a point u in X such that $\{Ax_{2n}\}$ converges to u. By continuity of A and compatibility of A with t and S we get :

$$\lim_{p\to\infty} TAx_{2p+2} = \lim_{p\to\infty} ATx_{2p+2} = Au \text{ and}$$
$$\lim_{p\to\infty} SAx_{2p+1} = \lim_{p\to\infty} ASx_{2p+1} = Au$$

Now putting x = x_{2p+2} and y = Ax_{2p+1} in (iii) we have

$$ad(Sx_{2p+1}, TAx_{2p+1}, a) + bd(Sx_{2p+1}, Ax_{2p+1}, a) + cd(TAx_{2p+1}, A^2x_{2p+1}, a)$$
$$- \min\{d(Sx_{2p+1}, A^2x_{2p+1}, a), d(TAx_{2p+1}, Ax_{2p+1}, a)\} \le qd(Ax_{2p+1}, A^2x_{2p+1}, a)$$

In limiting case we have $(a-1-q)(u, Au, a) \le 0$, but $(a-1-q) \le 0$ as a>1+q and also d(u, Au, a) < 0 is not possible. So d(u, Au, a) =0 i.e. Au = u. Thus u is a fixed point of A.

Again by considering x = u, y = x_{2p+2} in (iii) we have

$$ad(Su, Tx_{2p+2}, a) + bd(Su, Au, a) + cd(Tx_{2p+2}, Ax_{2p+2}, a)$$
$$- \min\{d(Su, Ax_{2p+2}, a), d(Tx_{2p+2}, Au, a)\} \le qd(Au, Ax_{2p+2}, a)$$

In limiting case, we have and putting Au = u. We have (a +b) d(Su, u, a) $\le$ 0, but (a + b) $\le$ 0 as a, b > 0 and also d(Su, u, a)< 0 is not possible. So d (Su, u, a) = 0 i.e. Su = u. thus u is a fixed point of S.

Now again putting x = x_{2n+1} and y = u in (iii) we have,

$$ad(Sx_{2p+1}, Tu, a) + bd(sx_{2p+1}, Ax_{2p+1}, a) + cd(Tu, Au, a)$$
$$- \min\{d(Sx_{2p-1}, Au, a), d(Tu, Ax_{2p+1}, a)\} \le qd(Au, Ax_{2p+1}, a)$$

In limiting case and using A_2 = u we have (a + c) d (Tu, u, a) $\le$ 0, but (a + c) $\le$ 0 as a, c > 0 and also d (tu, u, a) < 0 is not possible. So, d(Tu, u, a) = 0 i.e. tu = u. thus u is a fixed point of T.

Thus we prove that u is a common fixed point of A, T and S. Now we shall prove that u is a unique common fixed point of A, T and S. If possible let v is another common fixed point of A, T and S such that $u \ne v.$ then we have,

$$ad(Su, Tv, a) + bd(Su, Au, a) + cd(Tv, Av, a) - \min\{d(Su, Av, a), d(Tv, Au, a)\} \le qd(Au, Av, ma).$$
$$\text{or}, ad(u, v, a) + bd(u, u, a) + cd(v, v, a) - \min\{d(u, v, a), d(v, u, a)\} \le qd(u, v, a).$$
$$\text{or, } ad(u, v, a) - d(u, v, a) \le qd(u, v, a)$$
$$\text{or}, (a-1-q)d(u, v, a) \le 0, \text{ but} (a-1-q) \le 0 \text{ as } a > 1+q.$$

Also d(u,v,a) $\le$ 0. Thus d(u,v,a) = 0 i.e. u = v. thus, u is a unique common fixed point of A,T and S.

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