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Some Bianchi Type-IX Viscous Fluid Cosmological Models in General Relativity

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ABSTRACT

In this paper we have considered Bianchi Type – IX viscous fluid cosmological model in General Relativity and to solve the field equations, we have assumed judicious relations between metric potential A and B. Further we have taken $\eta \propto \theta$ where η is the co-efficient of shear viscosity and θ is the scalar of expansion in the model. We have also taken co-efficient of bulk viscosity (ζ) to be constant. Various physical geometrical properties of the model have been also discussed and it is found that the model does not approach isotropy for large values of T in the absence of viscosity.

1. Introduction

Many relativists have been taken interest in studying Bianchi type – IX universe because familiar solutions live Robertson walker universe with positive curvature, the de-Sitter universe, the Taub-NUT solutions e.t.c. are Bianchi type-IX space – times. In these models, neutrino viscosity does not guarantee isotropy at the present.

Further Bianchi Type – IX Cosmological models are interesting because these models allow not only expansion but also rotation and shear and in general are anisotropic. Viscosity is important in cosmology for a number of reason. Misner [19, 20] has studied the effect of viscosity on the evolution of cosmological models. Collins and Stewart [9] have studied the effect of viscosity on the formation of galaxies. Murphy [18] has studied the influence of viscosity on the formation of initial singularity. Weinberg [28] derived general formula for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. Heller and Klimek [12] have investigated viscous universe without initial singularity. They have shown that introduction of bulk viscosity removes the initial singularity. Roy and Prakash [22, 23] investigated viscous fluid cosmological models of petrov type-ID and non-degenerate petrov type - 1 in which co-efficient of viscosities are constant.

In fact to study the evolution of the universe, many workers have investigated cosmological models with a fluid containing viscosities. Mohanty and Pradhan [16] studied the problem of Murphy [18] for non-zero curvature of the Friedman model and derived solutions for their model. In the same paper they interpreted their result to explain the present status of the universe. Belinskii and Khalatickov [4a] investigating a Bianchi type 1 cosmological model under the influence of viscosity, found the important property that near the initial singularity, the gravitational field creates matter.Szydlowski and Heller [25] have constructed world models filled with interacting matter and radiation including bulk viscosity dissipation. They have shown the existence of stationary solutions in which the bulk viscosity term can be interpreted as phenomenologically describing the creation of matter and radiation. Santos et al. [24] obtained exact solutions of an isotropic homogeneous cosmology with general viscosity for open closed, and flate universe. Banerjee and Santos [23] obtained some exact solutions for a homogeneous anisotropic model using certain restrictions. Banerjee et al. [4] obtained some Bianchi type 1 solutions for the case of stiff matter by using the assumptions that shear viscosity co-efficients are power functions of the energy density. However the bulkviscosity co-efficients in the model are zero or constant. Recently Huang [13] presented exact solution of a Bianchi type I cosmological model with bulk viscosity without introducing shear viscosity – However, he adobted the restriction that the viscous co-efficients are constant or proportional to the energy density. Finally, Huang [13] studied various Physical aspects of the problem. Mohanty and Pattanik [17] have investigated an anisotropic spatially homogenious bulk viscous model without introducing shear viscosity by taking Bertotti-Robinson-type metric.

Krori et al. [14] and Wang [26, 27] studied the exact solutions of string cosmology for Bianchi type – II, VI_0, VIII and IX space – times. Pradhan et al. [21] have investigated the generation of Bianchi type – V cosmological models with varying cosmological term. Bali and Jain [7, 8] have obtained some expanding and shearing Bianchi Type – I viscous fluid cosmological models in which co-efficient of shear viscosity is proportional to the rate of expansion in the model and free gravitational field is Petrov Type – ID and non-degenerate. Bali et al. [6] have studied Bianchi type – IX viscous fluid cosmological models in general relativity. Robertson walker cosmological models with bulk viscosity and equation of state $p = (\gamma - 1)p, 0 \leq \gamma \leq 2$ is investigated by Mohanty and Pradhan [15]. Cademi and Fabri [10] have carried out the research on homogeneous viscous universe and investigated models of Bianchi Type – V, Type – VIII and Type – IX. Some other workers in this field are Bali et al. ([15, [16a]).

Here in this paper we have considered Bianchi Type – IX viscous fluid cosmological model in General Relativity and to solve the field equations, we have assumed judicious relations between metric potential A and B. Further we have taken $\eta \propto \theta$ where η is the co-efficient of shear viscosity and □ is the scalar of expansion in the model. We have also taken co-efficient of bulk viscosity (ζ) to be constant. Various physical geometrical properties of the model have been also discussed and it is found that the model does not approach isotropy for large values of T in the absence of viscosity.

2. The field equations

We consider Bianchi Type – IX metric in the form given by

(2.1) $$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2\sin^2 y + A^2\cos^2 y) \qquad dz^2 - 2A^2\cos y dx\, dz$$

where A = A(t) and B = B(t)

The energy momentum tensor T_μ^v for viscous fluid distribution is given by Landau and Lifshitz [14(a)]

(2.2) $$T_\alpha^B = (\rho + p)u_\alpha u^B + pg_\alpha^B + \eta\left(u_{\alpha;}^B + u_{;\alpha}^B + u^B u^\ell u_{;\ell}^B\right)$$
$$-\left(\zeta - \frac{2}{3}\eta\right)y_{;\ell}^\ell\left(g_\alpha^B + u_c u^B\right)$$

Here p is the isotropic pressure, □ the density, η and ζ are the co-efficient of viscosity u^α the flow vector satistying

(2.3) $$g_{\alpha\beta}u^\alpha u^\beta = -1$$

Here we use co-moving co-ordinates, so that

(2.4) $$u^1 = 0 = u^2 = u^3 \text{ and } u^4 = 1$$

The Einstein's field equation

(2.5) $R_\alpha^\beta - \frac{1}{2}Rg_\alpha^\beta + \wedge g_\alpha^\beta = -8\pi T_\alpha^\beta$ (c = 1, G = 1 in gravitational units) for on the line element (2.1) Leads to

(2.6) $$\left[\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4}\right] - 8\pi\left[p - 2\eta\frac{\dot{A}}{A}\left(\zeta - \frac{2}{3}\eta\right)\theta\right]$$

(2.7) $$\left[\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB}\frac{\ddot{B}}{B} + \frac{\dot{A}^2}{4B^4} + \wedge\right] = -8\pi\left[p - 2n\frac{\dot{B}}{B} - \left(\zeta - \frac{2}{3}\eta\right)\theta\right]$$

(2.8) $$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{A^2}{4B^4} + \frac{1}{B^2} + \wedge = 8\eta\rho$$

Here dot over A and b denotes ordinary differentiation with respect to t and expansion □ is given by

(2.9) $$\theta = u_{;\alpha}^\alpha$$

3. Solution of the field equations

Equation (2.6)-(2.8) are the three equations in Six unknowns $A, B\rho, p\zeta$ and η. To make the system determinate we need three more conditions. For this we take co-efficient of shearviscouty□ directly proportional to expansion □ and co-efficient of bulk viscosity ζ to be constant i.e.

(3.3.1a) $\eta\alpha\theta$ and

(3.3.1b) $\zeta =$ constant

We further assume judious relations between metric potentials A and B to solve the field equations

Case I : Here we assume

(3.3.2) $A^3 = k_1 B$

where k_1 is constant.

Equation (3.2.6) and (3.2.7) gives

$$\left[\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\ddot{A}}{A}-\frac{\dot{A}\dot{B}}{AB}-\frac{A^2}{B^4}+\frac{1}{B^2}\right]=16\pi\eta\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right) \quad (3.3.3)$$

Condition (3.3.1) leads to

$$\eta=\alpha\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right) \quad (3.3.4)$$

Equation (3.3.3) with use of (3.3.2) and (3.3.4) provides

$$B\ddot{B}+\mu\dot{B}^2=\frac{3}{2}B^{-\frac{4}{3}}-\frac{3}{2} \quad (3.3.5)$$

Where we have taken $k_1 = 1$ to avoid mathematical complexity

Here

$$\mu=\frac{4}{3}(1+28\pi\alpha) \quad (3.3.6)$$

Equation (3.3.5) leads to

$$\frac{d}{dB}(\phi^2)+\frac{2\mu}{B}(\phi^2)=-3B^{-7/3}+\frac{3}{B} \quad (3.3.7)$$

where

$$\dot{B}=\phi(B) \quad (3.3.8)$$

From equation (3.3.7), we have

$$\left(\frac{dB}{dt}\right)^2=\frac{9}{2}\frac{B^{-4/3}}{(3\mu-2)}+\frac{\lambda}{B^2\mu}-\frac{3\mu}{2} \quad (3.3.9)$$

where □ is constant of integration. Thus the metric (3.2.1) reduces to

$$ds^2=-\left(\frac{dt}{dB}\right)^2 dB^2+B^{2/3}dx^2+B^2dy^2 \quad (3.3.10)$$

$$+(B^2\sin^2 y+B^{2/3}\cos^2 y)dz^2-2B^{1/3}\cos y\,dx\,dz$$

where $A^3 = B$. After using (3.3.9) into the metric (3.3.10) we have

$$ds^2=-\frac{dT^2}{\left[\frac{9T^{-4/3}}{2(3\mu-2)}+\frac{\lambda}{T^{2\mu}}-\frac{3}{2\mu}\right]}+T^{2/3}dx^2+T^2dy^2 \quad (3.3.11)$$

$$+\left(T^2\sin^2 Y+T^{2/3}\cos^2 Y\right)dz^2-2T^{2/3}\cos Y\,dZ$$

where $B=T, x=X, y=Y, z=Z$

4. Some Physical and Geometrical Features

For the model (3.3.11) pressure and density are found to be

$$8\pi p=\left[\frac{40\pi\alpha+4-\frac{3\mu}{4}}{8(3\mu-2)}\right]T^{-10/3} \quad (4.1)$$

$$+\frac{\lambda}{3}\left[\frac{224\pi\alpha}{9}-\frac{1}{3}+4\mu\right]T^{-2(\mu+1)}+\frac{1}{9}\left(\frac{3-224\pi\alpha}{2\mu}\right)\frac{1}{T^2}$$

$$+\left[\frac{56}{3}\pi\zeta\left\{\frac{T^{-10/3}}{\frac{2}{9}(3\mu-2)}+\frac{\lambda}{T^{2\mu+2}}-\frac{1}{\left(\frac{2\mu}{3}\right)T^2}\right\}^{1/2}\right]-\wedge$$

$$8\pi\in=\frac{32-3\mu}{4(3\mu-2)}T^{-10/3}+\frac{5\lambda}{3}T^{-2(\mu+1)}+\left(\frac{2\mu-5}{2\mu}\right)\frac{1}{T^2}+\wedge$$

(4.2)

The energy conditions (Ellis [11]) are

(i) $\in+p>0$

$\in+3p>0$

The condition (i) leads to

$$\left[\frac{40\pi\alpha+68-\frac{27\mu}{4}}{8(3\mu-2)}\right]T^{-10/3}+\frac{\lambda}{3}\left[\frac{224\pi\alpha}{9}+\frac{14}{3}+4\mu\right]T^{-2(\mu+1)}$$

(4.3)

$$+\left(\frac{9\mu-21-112\pi\alpha}{9\mu}\right)\frac{1}{T^2}$$

$$+\left\{\frac{56\pi\zeta}{3}\left\{\frac{T^{-10/3}}{\frac{2}{9}(3\mu-2)}+\frac{\lambda}{T^{2\mu+2}}-\left(\frac{3}{2\mu}\right).\frac{1}{T^2}\right\}^{1/2}\right\}>0$$

and the condition (ii) leads to

$$\left[\frac{32-3\mu}{4(3\mu-2)}+\left\{\frac{40\pi\alpha+4-\frac{3\mu}{4}}{8(3\mu+2)}\right\}\right]T^{-10/3}$$

(4.4)

$$+\frac{\lambda}{3}\left[\frac{224\pi\alpha}{3}+4+12\mu\right]T^{-2(\mu+1)}+\frac{1}{3}\left(6-224\pi\alpha-\frac{15}{2\mu}\right)T^{-2}$$

$$+56\pi\zeta\left\{\frac{9T^{-10/3}}{2(3\mu-2)}+\frac{\lambda}{T^{2(\mu+1)}}-\frac{3T^{-2}}{2\mu}\right\}^{1/2}>2\wedge$$

which puts restrictions on $\wedge$

The expansion □ and shear □ for the model (3.11) are given by

(4.5) $$\theta = \frac{7}{2}\left[\frac{9T^{-10/3}}{2(3\mu-2)} + \frac{1}{T^{2(\mu+1)}}\left(\lambda - \frac{3T^{2\mu}}{2\mu}\right)\right]^{1/2}$$

and

(4.6) $$\sigma = \frac{2}{3}\sqrt{\frac{2}{3}}\left[\frac{9T^{-10/3}}{2(3\mu-2)} + \frac{1}{T^{2(\mu+1)}}\left(\lambda - \frac{3T^{2\mu}}{2\mu}\right)\right]^{1/2}$$

In the absence of viscosity i.e. $\alpha \to 0,\ \mu = \frac{4}{3}$, the metric (3.11) reduces to

(4.7) $$ds^2 = \frac{-dT^2}{\left(\frac{9T^{-4/3}}{4} + \frac{\lambda}{T^{8/3}} - \frac{9}{8}\right)} + T^{2/3}dx^2$$

$$+T^2dY^2 + \left(T^2\sin^2 Y + T^{2/3}\cos^2 Y\right)dxz^2 - 2T^{2/3}\cos y\,dx\,dZ$$

The expression for pressure p density $\in$, expansion □ and shear □ are given by

(4.8) $$8\pi p = \frac{3}{2}T^{-10/3} + \frac{5\lambda}{3}T^{-14/3} - \frac{1}{8}T^{-2} - \wedge$$

(4.9) $$8\pi \in = \frac{7}{2}T^{-10/3} + \frac{5\lambda}{3}T^{-14/3} - \frac{7}{8}.T^{-2} + \wedge$$

(4.10) $$\theta = \frac{7}{3}\left(\frac{9T^{-10/3}}{4} + \lambda T^{-14/3} + \frac{9}{8}T^{-2}\right)^{1/2}$$

(4.11) $$\sigma = \left(\frac{2}{3}\right)^{3/2}\left(\frac{9}{4}T^{-10/3} + \lambda T^{-14/3} + \frac{9}{8}T^{-2}\right)^{1/2}$$

The condition (i) $\in + p > 0$ and (ii) $\in + 3p > 0$ leads to

(4.12) $$5T^{-10/3} + \frac{10}{3}\lambda T^{-14/3} - T^{-2} > 0$$

(4.13) $$8T^{-10/3} + \frac{20}{3}\lambda T^{-14/3} - \frac{5}{4}T^{-2} > 2\wedge$$

Which gives condition on $\wedge$. IN the absence of viscosity, the expansion in the model starts with big bang at T = 0 and expansion in the model decreases with time. When $T \to 0$ then $\in \to \infty$ and $p \to \infty$ and when $T \to \infty$ Then $\in \to \wedge$ and $p \to -\wedge$. Since $\lim_{T\to\infty}(\sigma/\theta) \neq 0$. Hence the model does not approach isotropy for large values of T in the absence of viscosity in general.

5. Case II (Model II)

Here η and ζ are the same as in case I and

(5.1) $A^2 = k_2 B^3$

where k_2 is constant.

Now equation (2.6) and (2.7) gives

$$\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}-\frac{A^2}{B^4}+\frac{1}{B^2}=16\pi\eta\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right) \tag{5.2}$$

$$\eta=\alpha_1\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right) \tag{5.3}$$

where □$_1$ is constant

Equations (5.2), (5.1) and (5.3) provide

$$B\ddot{B}+\bar{\mu}\dot{B}^2=-2B+2 \tag{5.4}$$

Here we have taken k_2 = 1, to avoid mathematical complexity and

$$\bar{\mu}=\frac{5}{2}+56\pi\alpha_1 \tag{5.5}$$

Equation (5.4) leads to

$$\frac{d}{dB}\left(\phi_1^2\right)+\frac{2\bar{\mu}}{B}\left(\phi_1^2\right)=-4+\frac{4}{B} \tag{5.6}$$

where

$$\dot{B}=\phi_1(B) \tag{5.7}$$

Equation (5.4) gives

$$\left(\frac{dB}{dt}\right)^2=\frac{4B}{1-2\bar{\mu}}+\frac{\lambda_1}{B^2\bar{\mu}}+\frac{2}{\bar{\mu}} \tag{5.8}$$

where □$_1$ is constant of integration

Now metric (2.1) goes to the form

$$ds^2=-\left(\frac{dt}{dB}\right)^2 dB^2+\left(B^3\right)dx^2+B^2dy^2 \tag{5.9}$$

$$+(B^2\sin^2 y+B^3\cos^2 y)dz^2-2B^{3/2}\cos y\,dx\,dz$$

Use of (5.8), reduces (3.11) to the form

$$ds^2=\frac{dT^2}{\left[\frac{4T}{2\bar{\mu}+1}+\frac{\lambda_1}{r^2\bar{\mu}}+\frac{2}{\bar{\mu}}\right]}+T^3dx^2+T^2dY^2 \tag{5.10}$$

$$+\left(T^2\sin^2 Y+T^3\cos^2 Y\right)dz^2-2T^3\cos Y\,dx\,dz$$

Where $B=T, x=X, y=Y, z=Z$

6. Physical and Geomatrical Features

Pressure and density for the model (5.10) can calculated as in case I. Further energy conditions (i) $\in +p>0$ and (ii) $\in +3p>0$ can be also discussed as in previous case.

Also □ and □fro metric (5.10) are given by

$$\theta=\frac{7}{2}\left[\frac{-4T^{-1}}{(2\bar{\mu}-1)}+\frac{1}{T^2\bar{\mu}+2}\left(\lambda_1+\frac{2T^{2\bar{\mu}}}{\bar{\mu}}\right)\right]^{1/2} \tag{6.1}$$

(6.2) $$\sigma = \sqrt{\frac{3}{2}}\left[\frac{-4T^{-1}}{2\bar{\mu}-1} + \frac{1}{T^{2\bar{\mu}+2}}\left(\lambda_1 + 2\frac{T^{2\bar{\mu}}}{\bar{\mu}}\right)\right]^{1/2}$$

In the absence of viscosity i.e. $\bar{\mu} \to 0$, the metric (5.10) reduces to he form

(6.3) $$ds^2 = \frac{dT^2}{\left[\frac{2T}{-3} + \frac{\lambda_1}{T5} + \frac{4}{5}\right]} + T^3dx^2 + T^2dy^2$$
$$+(T^2\sin^2 y + T^3\cos^2 y)dz^2 - 2T^3\cos y\, dx\, dz$$

From equation (5.3)

(6.4) $$\bar{\mu} = \left[\frac{5}{2} + 5B\pi\alpha_1\right]$$

In the absence viscosity $\bar{\mu} = \frac{5}{2}$ and $\alpha_1 = 0$. The expressions for pressure, density, Expansion (□) and shear (□) are given by

(6.5) $$8\pi p = \frac{17}{12}T^{-1} + 4\lambda_1 T^{-7} + \frac{9}{5}T^{-2} - \wedge$$

(6.6) $$8\pi \in = -\frac{35}{12}T^{-1} + 4\lambda_1 T^{-7} + \frac{21}{5}T^{-2} + \wedge$$

(6.7) $$\theta = \frac{7}{2}\left[\frac{-2T^{-1}}{3} + \lambda_1 T^{-7} - \frac{4}{5}T^{-2}\right]^{1/2}$$

(6.8) $$\sigma = \frac{1}{\sqrt{6}}\left[-\frac{2T^{-1}}{3} + \lambda_1 T^{-7} - \frac{4}{5}T^{-2}\right]^{1/2}$$

(6.9) $$\left[-4T^{-1} + 8\lambda_1 T^{-7} - 6T^{-2}\right] > 0$$

and

(6.10) $$\left[-\frac{47}{6}T^{-1} + 16\lambda_1 T^{-7} + \frac{3}{5}T^{-2}\right] > 2\wedge$$

Which gives condition on $\wedge$. In the absence of viscosity, the expansion in the model stars with a big band at T = 0 and expansion in the model decreases with time. When $T \to 0$ then $\in \to \infty, p \to \infty$ when $T \to \infty$ Then $\rho \to \wedge$ and $p \to -A$. Since $\mathrm{Lim}\ T \to \infty (\sigma/\theta) \neq 0$. Hence this model also does not approach isotropy for large values of T in the absence of viscosity in general.

REFERENCE

1. Banerjee, A., Sunyal A.K. Chkraborty S, (1990) Pramana – J. Phys. 34.
2. Banerjee, A. and Santosh, N.O. (1983) Journal of Mathematical Physics, 24, 2089.
3. Banerjee, A. and Santosh, N.O. (1984) General Relativity and Gravitation 16, 217.
4. Banerjee, A. Dattachoudhury, S.B., and Sanyal A.K. (1985), Journal of Mathematical Physics, 2611.

4.(a) Belinskii, V. A : and Khalatikov, I.M. (1976), Soviet physics JETP 42. 205.

5. Bali R., Electron. J. (2008) Theor. Phys. 5, 105.
6. Bali R. and Anjali 2006 Astro phys. Space. Sci 302, 201.

6.(a) Bali R., and Yadav M.K.(2005) Pramana J. Phys. 64, 187.

7. Bali R. and Jain DR. Astro Phys. Space Sci, 139, 178.

8. Bali R and Jain DR, Astro Phys. (1988) space science. 141, 207.
9. Collins CB and Stewart J.M. (1971) Mon. Not. R. Astron. Sco., 153. 419.
10. Caderni, N and Fabri R. (1979) Phys. Rev., D20, 1251.
11. Ellis GFR. (1971) General relativity and cosmology edited by sachs R.K. P. 117.
12. Heller M. and Klimek E. (1975) Astro Phys. Space. Sci., 23(2), 37
13. Huang, W.H. (1988). Physics letters A. 129, 429.
14. Krori, K.D. Chodhury, I. Mahanta, G.R. and Mazumder a, Gen. Relativ.

14.(a) Landau, L.D. and lifshitz, E.M. (1963) Fluid mechanics 6, 503.

15. Mohanty G. and Pradhan B.D. (1989). Astro physics and space science.
16. Mohanty G. and Pradhan B.D. (1989). Asto Physics and space science.
17. Mohanty and Pattanaki (1991), Int. J. Theo. Phys., 30, 239.
18. Murphy, G.L. (1973). Physical Review, D8, 4231.
19. Misner C.W. (1967) Nature, 214, 40.
20. Misner C.W. 1968) Astro Phys. 1. 151, 431.
21. Pradhan A, Yadav A.K. and Yadav L., (2005) C.Z. etch. J. Phys., 55, 503.
22. Roy S.R. and Prakash S. (1976) J. Phys. A. Mathematical and General, 9(2), 261.
23. Roy S.R. and Praksh S. (1977). Indian J. Pure Appl. Math, 8(6), 723 (1977).
24. Santos, N.O., Dias, R.S. and Banerjee, A. (1985) Journal of Mathematical Physics, 26, 878.
25. Szydlowski, M., and Heller, M. (1983) Act. Physics Polonica, B.14 303.
26. Wang xx, 2003 Chin Phys., Lett. 20, 615.
27. Wang xx, 2003 Chin, Phys. Lett, 20, 1205.
28. Weinberg, S. (1971). Astrophys J. 168, 175.

