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Some Static Charged Fluid Spheres In General Relativity

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ABSTRACT
The paper presents some solutions some of Einstein-Maxwell field equations for static fluid sphere using different assumptions. We have also discussed boundary conditions and have found that these solutions satisfy physical conditions. The pressure, matter density, electric field and charge density for the distribution have been also obtained.

1. Introduction

A pretty number of relativists have focused their mind towards the problem of determination of exact solution of coupled Einstein-Maxwell equations for static spherical distributions of charged matter [3, 7, 10-14, 25]. Spherically symmetric charged dust distribution have been investigated by Papapeetrous [19], Bonnor and Wickramasuriya [3] and Ray Choudhuri [21]. It is known that the pressure less charged distribution in equilibrium will have the absolute value of the charge to mass ratio as unity in relativistic units (De and Ray Chaudhury [6]).

For a spherically symmetric charge distribution the unique exterior metric was obtained by Rissner [21] and Nordstrom [17]. This is a straight forward generalization of the Schwarschild metric, which represents the exterior field for a body with no net charge. Here, as in other area of physics, the property of spherical symmetry greatly simplifies the analysis. The relative simplicity has enabled various investigators to find exacts static interior solutions for the Reissner-Nordstrom metric. However, often it is mathematical rather than physical consideration which directs the nature and approach to the solution Cooperstock and De La Cruz [5] found some solutions which are motivated by physical consideration.

Some conformal flat interior solutions of the Einstein-Maxwell equations for a charged stable static sphere has been obtained by Shi-Chang. [1982]. These solutions satisfy physical conditions inside the sphere. Some exact static solutions of Einstein-Maxwell equations representing a charge fluid sphere were obtained by Singh and Yadav [22] Xingxiang [27] obtained an exact solution by specifying matter distribution and charge distribution. The metric is regular and can be matched to the Reissner-Nordstrom metric and pressure is finite. In the limit of vanishing charge, the solution can be reduced to the interior solution of an uncharged sphere. Buchdahl [4] has also considered some regular general relativistic charged fluid spheres. Some other cases of the interior solutions for charged fluid sphere have been presented by Glazer [9], Bekenstein [2], Srivastava [24], Baliyn [1], Whitman and Brurch [26], Nduka [15, 16] Yadav et al. [28] and Yadav and Purushottam [29].

Here in this paper, we have obtained some solutions some of Einstein-Maxwell field equations for static fluid sphere using different assumptions. We have also discussed boundary conditions and have found that these solutions satisfy physical conditions. The pressure, matter density, electric field and charge density for the distribution have been also obtained.

2. The Field Equations

We consider the line element in the form

(2.1) $$ds^2 = e^{v}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where □ and v are functions of r only.

The Einstein-Maxwell field equations for the charged perfect fluid distribution in general relativity are

(2.2) $$R_{\alpha\beta} - \frac{1}{2}R_{g_{\alpha\beta}} = -8\pi T_{\alpha\beta}$$

(2.3) $$F^{\alpha\beta}_{;\beta} = 4\pi j^{\alpha} = 4\pi\sigma u^{\alpha}$$

(2.4) $$F_{[\alpha\beta;\gamma]} = 0$$

Where T□□ is the energy momentum tensor, j□ is the charged current four vector, R□□ is the Ricci tensor and R the scalar of curvature tensor.

For the system under study the energy momentum tensor T^{α}_{β} splits up in to two part viz. $\bar{T}^{\alpha}_{\beta}$ and E^{α}_{β} for matter and charges respectively.

(2.5) $$T^{\alpha}_{\beta} = \bar{T}^{\alpha}_{\beta} + E^{\alpha}_{\beta}$$

where

(2.6) $$\bar{T}^{\alpha}_{\beta} = \left[(\rho + p)u^{\alpha}u_{\beta} - p.\delta^{\alpha}_{\beta}\right]$$

with

(2.7) $$u^{\alpha}u_{\alpha} = 1$$

The non-vanishing component of $\bar{T}^{\alpha}_{\beta}$ are

(2.8) $$\bar{T}^{1}_{1} = \bar{T}^{2}_{2} = \bar{T}^{3}_{3} = -p \text{ and } \bar{T}^{4}_{4} = \rho$$

Here p is internal pressure, □ and □ are densities of matter and charges respectively, u□ is the velocity vector of the matter.

The static condition is given by

(2.9) $$u^{1} = u^{2} = u^{3} = 0 \text{ and } u^{4} = (g_{44})^{-1/2}$$

i.e. $u^{4} = e^{-v/2}$

The electromagnetic energy momentum tensor E^{α}_{β} is given by

(2.10) $$E^{\alpha}_{\beta} = -F_{\beta\gamma}F^{\alpha\gamma} + \frac{1}{4}\delta^{\alpha}_{\beta}F_{1m}F^{1m}$$

We assume the field to be purely electrostatic i.e. F□ □ = 0 and

$F_{4\gamma} = \phi,\gamma = \phi_{\gamma}$ where ϕ is the electrostatic potential.

Thus the Einstein – Maxwell field equations are reduced into the form

(2.11) $$e^{-\lambda}\left(\frac{1}{r^{2}} - \frac{\lambda}{r}\right) - 1/r^{2} = -8\pi\rho - E$$

(2.12) $$\frac{1}{r^{2}} - e^{-\lambda}\left(\frac{1}{r^{2}} + \frac{v}{r}\right) = -8\pi p + E$$

(2.13) $$e^{-\lambda}\left[\frac{1}{4}v''\lambda' - \frac{1}{4}v'^{2} - \frac{1}{2}v'' - \frac{1}{2}\left[\frac{v'\lambda'}{r}\right]\right] = -8\pi p - E$$

where

(2.14) $$E = -F^{41}F_{41}$$

and

(2.15) $$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{v' + \lambda'}{2}F^{41}\right]e^{v/2}$$

By the use of equation s (2.11) – (2.13), we get the expressions for p, □ and E as

(2.16) $$8\pi p = \frac{e^{-\lambda}}{2}\left[\frac{3v'}{2r} + \frac{v''}{2} - \frac{\lambda'v'}{4} + \frac{v'^{2}}{8} - \frac{v'}{2r} + \frac{1}{r^{2}}\right] - \frac{1}{2r^{2}}$$

(2.17) $$8\pi p = e^{-\lambda}\left[\frac{5\lambda'}{4r} - \frac{v''}{4} + \frac{\lambda'v'}{8} - \frac{v'^{2}}{8} + \frac{v'}{4r} - \frac{1}{2r^{2}}\right] + \frac{1}{2r^{2}}$$

(2.18) $$2E = e^{-\lambda}\left[\frac{v''}{2} - \frac{\lambda'v'}{4} + \frac{v'^{2}}{4} - \frac{v'}{2r} - \frac{\lambda'}{2r} - \frac{1}{r^{2}}\right] + \frac{1}{2r^{2}}$$

3. Solution Of The Field Equations

We have four equations (2.11) – (2.13) and (2.15) in six variables (□, E, p, □, v, □). Thus the system is indeterminate. For complete determination of the system we require two more relations. For this we take □ and v as two free variables.

We choose

(3.1) $$e^{-\lambda} = \frac{Ar^6 + Br^2 + C}{r^6 + C}$$

(3.2) $$e^{v} = Lr^6 + Mr^2 + C$$

where A, B, C, L and M are arbitrary constant.

Using equations (3.1) and (3.2) in equations (2.15) – (2.18) we get

(3.3) $$8\pi p = \frac{r^6 + C}{2(Ar^6 + Br^2 + C)} \left[\frac{180L^2r^{11} + 270LMr^{10} + 80CLr^5 + 5M^2r^5 + 6CMr^4}{4r(Lr^6 + Mr + C)} - \frac{6r^{14}C(a-1)(4Lr^{10} + 3Mr^{10} + 3Mr^5 + 2Cr^4)}{2(r^6 + C)(Ar^6 + Br^2 + C)(Lr^6 + Mr + C)} + \frac{1}{r^2} \right] \frac{1}{2r^2}$$

(3.4) $$8\pi\rho = \frac{r^6 + C}{2(Ar^6 + Br^2 + C)} \left[\frac{6r^9C(A-1) + (12Lr^{10} + 11Mr^{10} + 3Mr^5 + 10Cr^4}{2(r^6 + C)(Ar^6 + Br^2 + C)(Lr^6 + Mr + C)} + \frac{4L^2r^{11} + 6LMr^{10} + 3M^2r^5 + 2CMr^4}{4r(Lr^6 + Mr + C)} - \frac{1}{r^2} \right] + \frac{1}{2r^2}$$

(3.5) $$E = \frac{r^6 + C}{2(Ar^6 + Br^2 + C)} \left[\frac{9(4L^2r^{11} + 6LMr^{10} + 3M^2r^5 + 2CMr^4)}{4r(Lr^6 + Mr + C)^2} \frac{6r^{14}C(A-1)(Lr^{10} + Mr^{11} + 2Lr^5 + Mr^4 + Cr^4)}{(r^6 + C)(Ar^6 + Br^2 + C)(Lr^6 + Mr + C)} \frac{1}{r^2} \right] + \frac{1}{2r^2}$$

(3.6) $$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\frac{6r^5(CA - br^2 - C) + 6r^7 + 2CBr}{(6^6 + C)(Ar^6 + Br^2 + C)} + \frac{6r^5L + M}{Lr^6 + Mr + C}}{2} F^{41} \right]$$

$$\left(Lr^6 + Mr + C\right)^{1/2}]$$

Boundary Conditions :

We impose the following boundary conditions :

(1) $e^{-\lambda}$ is constnuous across the boundary ($r = r_b$) of the fluid space.
(2) The function e^v is continuous across the boundary ($r = r_b$) of the fluid sphere.
(3) The function $\frac{de^v}{dr}$ is the continuous across the boundary of the fluid sphere.

The exterior metric (i.e. for $r > r_b$) is given by Reissner-Nordstrom metric which is

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q_b^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{Q_b^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3.7)$$

where $Q_b = Q_{(r_b)}$ and $\bar{M}$ is the total mass of the sphere given by

$$\bar{M} = 4\pi\int_0^{r_b} \rho(r).r^2 dr \quad (3.8)$$

The constants appearing in the solution are fixed by the following equations :

$$\frac{r_b^6 + C}{Ar_b^6 + Br_b^2 + C} = \left(1 - \frac{2\bar{M}}{r_b} + \frac{Q_b^2}{r_b^2}\right) \quad (3.9)$$

$$Lr_b^6 + Mr_b + C = \left(1 - \frac{2\bar{M}}{r_b} + \frac{Q_b^2}{r_b^2}\right) \quad (3.10)$$

Further continuity of $\frac{\partial g_{44}}{\partial r_b}$ implies

$$\frac{6Lr_b^5 + M}{2} = \left(\frac{\bar{M}}{r_b} - \frac{Q_b^2}{r_b^3}\right) \quad (3.11)$$

Now we consider the following different cases :
Case I : When A = 0
(3.1) and (3.2) become

$$e^{\lambda} = \frac{Br^2 + C}{r^6 + C} \quad (3.12)$$

$$e^v = Lr^6 + Mr^2 + C \quad (3.13)$$

On (3.11) and (3.12) in equations (3.7). (3.10) we get

$$8\pi p = \frac{r^6 + C}{2(Br^2 + C)} \left[\frac{180L^2r^{11} + 270LMr^{10} + 80CLr^5 + 5M^2r^5 + 6CMr^4}{4r(Lr^6 + Mr + C)} \right.$$
$$\left. \frac{6r^{14}C(-1)(4Lr^{10} + 3Mr^{10} + 3Mr^5 + 2Cr^4}{2(r^6 + C)(Br^2 + C)(Lr^6 + Mr + C)} + \frac{1}{r^2}\right] - \frac{1}{2r^2} \quad (3.14)$$

$$8\pi\rho = \frac{r^6 + C}{2(Br^2 + C)}\left[\frac{6r^9C(-1) + (12Lr^{10} + 11Mr^5 + 10Cr^4)}{2r(r^6 + C)(Br^2 + C)(Lr^6 + Mr + C)} + \right.$$
$$\left. \frac{4L^2r^{11} + 6LMr^{10} + 3M^2r^5 + 2CMr^4}{4r(Lr^6 + Mr + C)^2} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (3.15)$$

(3.16) $$E=\frac{r^6+C}{2(Br^2+C)}\left[\frac{9(4L^2r^{11}+6LMr^{10}+3M^2r^5+2CMr^4)}{4r(Lr^6+Mr+C)^2}\right]$$

$$-\left[\frac{6r^{14}C(-1)(Lr^{10}+Mr^{11}+2Lr^{2m-1}+Mr^4+Cr^4)}{(r^{2n}+C)(Br^2+C)(Lr^{2m}+Mr+C)}-\frac{1}{r^2}\right]+\frac{1}{2r^2}$$

(3.17) $$4\pi\sigma=\left[\frac{\partial F^{41}}{\partial r}+\frac{2}{r}F^{41}+\frac{\frac{6r^5(Br^2-C)+6r^7+2CBr}{(r^6+C)(Br^2+C)}+\frac{6r^5L+M}{Lr^6+Mr+C}}{2}F^{41}\right.$$

$$\left.(Lr^6+Mr+C)^{1/2}\right]$$

Now using the above boundary conditions, we have

(3.18) $$\frac{r_b^6+C}{Ar_b^6+C}=1-\frac{2\bar{M}}{r_b}+\frac{Q_b^2}{r_b^2}$$

(3.19) $$Lr_b^6+C=1-\frac{2\bar{M}}{r_b}+\frac{Q_b^2}{r_b^2}$$

(3.20) $$6Lr_b^5=\frac{\bar{M}}{r_b^2}-\frac{Q_b^2}{r_b^3}$$

Case II : When L = 0

(3.1) and (3.2) become

(3.21) $$e^{\lambda}=\frac{Ar^6+Br^2+C}{r^6+C}$$

(3.22) $$e^{v}=Mr^2+C$$

p, ρ, E, σ and constants can be found as in case I.

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