Volume-03
Issue-12
December -2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

Examination of Various Studied in Strict Convexity and Riesz Representation Theorem

[1]Monika and [2]Dr. Pardeep Goel

[1]Research Scholar, Department of Mathematics, OPJS University, Churu, Rajasthan
[2]Professors, Department of Mathematics, OPJS University, Churu, Rajasthan

ARTICLE DETAILS

Article History
Published Online: 10 December 2018

Keywords
Strict Convexity, Riesz

ABSTRACT

The present paper investigation of some comparable conditions for the strict convexity of a normed straight space X by utilizing properties and methods of a summed up semi-inward item good with the standard of X. We demonstrate that in the event that X is homogeneous g.s.i.p space, solid assembly is identical to powerless combination in the principal contention consistently on the unit circle of X. The riesz portrayal hypothesis in a summed up semi-internal item space is demonstrated under conditions more fragile than those utilized by Milicie. Further, we snow that for each legitimate shut subspace of a reflexive Banach space, Э a non zero vector symmetrical to it. We demonstrate a disintegration hypothesis for a smooth g.s.i.p which is entirely raised and reflexive. Additionally we demonstrate that the conjugate space of the immediate entirety of entirely raised reflexive g.s.i.p spaces is isometrically isomorphic to the immediate whole of their conjugate spaces.

1. Introduction

In this part we examine some identical conditions for the strict convexity of a normed straight space X by utilizing the properties and systems of g.s.i.p on X good with the standard of X. We demonstrate that on the off chance that X is homogeneous g.s.i.p space, solid combination is comparable to feeble intermingling in the main contention consistently on the unit circle of X. We demonstrate a decay hypothesis for a ceaseless g.s.i.p space which is entirely arched and reflexive. We build up the Riesz portrayal hypothesis in reflexive banach space X by utilizing the systems of steady g.s.i.p on X, and we reason an outcome that for each legitimate shut subspace of X there exists a non zero vector symmetrical to it. Also it is demonstrated that the conjugate space of the immediate aggregate of entirely raised reflexive g.s.i.p spaces is isometrically isomorphic to the immediate whole of their conjugate spaces. The riesz portrayal hypothesis in a summed up semi-internal item space is demonstrated under conditions more fragile than those utilized by Milicie [39]. It is seen that the portrayal hypothesis demonstrated by Giles [19], Husain and Malviya [24], and Faulkner [11] are altogether contained in our hypothesis. Further, we snow that for each legitimate shut subspace of a reflexive Banach space, there exists a non zero vector symmetrical to it.

2. Strictly convex q.s.i.p spaces

A normed direct space is entirely raised if each purpose for the unit circle is an unprecedented motivation behind unit ball. Strict convexity has ended up being exceptionally helpful in the investigations of the geometry of Branch spaces. By utilizing the properties of a g.s.i.p, one set up the comparability of a few strict convexity states of a normed straight space. In like manner we exhibit that in a homogeneous g.s.i.p space, the strong association is proportionate to weak blending in the primary conflict reliably on the unit circle.

First we demonstrate the accompanying:

Lemma 1.1 Let X be a normed direct space & [.,..] be any g.s.i.p good with the standard of X. If x, yEX to such an extent that ‖ y+x‖ = ‖ y‖ +‖ x‖ , then the standard of X. In the event that x, yEX with the end goal that ‖ y+x‖ = ‖ y‖ +‖ x‖ , that point

[x, y+x] = ‖x‖ ‖y+x‖ p-1 = ‖y‖ ‖y+x‖p-1, (1<p<).

Evidence. We have [x, y+x] + [y, y+x] = ‖y+x‖ p

=‖y+x‖p-1 (‖y‖ + ‖x‖).

Henceforth,

(a) (‖x‖ ‖y+x‖p-1 – Re ([x, y+x])) + (‖y‖ ‖y+x‖p-1 – Re ([y, y+x])) =0.

However, we have

(b) Re ([x, y+x]) < □[x, y+x]□ ⊲‖y‖ ‖x‖p-1 also,

(c) Re ([y, y+x]) < □[y, y+x]□ ⊲‖y‖ ‖y+x‖p-1

Since disparities (b) and (c) infer that each term in (an) is non negative, one have

Re ([x, y+x]) = ‖x‖ ‖y+x‖p-1 also,

Re ([y, y+x]) = ‖y‖ ‖y+x‖p-1

Presently these balances together with (b) and (c) infer the ideal outcomes.

Presently we will demonstrate the fundamental aftereffect of this area.

Hypothesis 1.1. Give X a chance to be a normed direct space and [.,.] be any g.s.i.p. reliable with the standard of X. At that point the accompanying conditions are proportionate:

(1) X' is entirely arched

(2) If M is shut subspace of X, at that point each point in X has atmost one closest point in M

(3) Every non zero consistent direct practical on X achieves its most extreme on atmost one point of the unit sphere

(4) If ‖y+z‖< ‖y‖ and [z,y] = O, at that point z = O

(5) Let T be a limited straight administrator on X with ‖I+T‖ < 1 On the off chance that (Tx, x] = 0 for same x in x, Tx=0

(6) If ‖y+z‖ = ‖y‖ and [z,y] = 0, at that point z=0

(7) ‖y+x‖ = ‖y‖+‖x‖, where y, x ≠0 infers x = □y for some □>0.

(8) If [y, x] = ‖y‖ ‖x‖p (1-p< ∞) for any non zero vectors x and y, at that point ys mx for some □>0

(9) [x, z] = [y,z], for some Z in X If and just if y = x.

Confirmation. We initially demonstrate (1) □ (2) □ (3) □ (8) □ (1), and after that we finish the confirmation by appearing

(3) □ (4) □ (5) □ (6) □ (7) □ (8) □ (9) □ (3).

(2) □ (3), suppose condition (3) don’t hold. That point Э a f in X* with =‖f‖=1 to such an extent s.t. f(y) = 1 = f(x), y≠x, and a‖y‖ =‖x‖=1. Presently for any scalar □, one have

f (x-□x+□y) = f(x) – □f(x) +□f (y) =1,

what's more, subsequently,

1 = □f(x-m(y-x))□< ‖f‖ ‖x-□(y-x) ‖=‖x-□ (y-x)‖.

Presently by taking infimum we get

1 = Inf ‖x-m(x-y)‖

or other hand proportionally

‖x-O‖ = Inf ‖x-□ (x-y)‖,

and furthermore we can compose

‖y‖ = ‖x-(y-x)‖ = Inf ‖–□ (y-x)‖.

Subsequently x has two closest focuses, in particular, 0 and x-y in the subspace M = Span {x-y}, which negates condition (2).

(3) □ (8). Actually on the off chance that (8) isn't valid, Э vectors x and y in X with y≠x to such an extent that

[x+y] = ‖x‖ ‖y‖p-1

Presently characterize fy(z) = (z, y] for all z in X. At that point obviously fy□x* and

□fy(z) □ = □[z,y] □ ⊲‖z‖ ‖y‖ p-1

Demonstrates that ‖fy‖ < ‖y‖p-1. then again

‖fy‖ >fy(y/‖y‖) = [y/‖y‖= ‖y‖p-1.

In this manner ‖fy‖ = ‖y‖p-1. Presently we have

fy(y)= ‖y‖p = ‖y‖p-1 ‖y‖= ‖fy‖ ‖y‖

what's more,

fy(x)= [y, x] = ‖y‖ ‖x‖p-1 = ‖y‖ ‖fx‖

Since we can accept without loss of all inclusive statement that ‖x‖=1=‖y‖,the above conditions demonstrate that f accomplishes its most extreme at two unmistakable focuses x and y, which negates condition (3). We presently demonstrate that (8) □ (1). We demonstrate each point u of unit circle is outrageous purpose of unit ball. Give v and w a chance to be with the end goal that ‖v‖,<1 and ‖w‖,<1 and u = television + (1-t)w, 0<t<1. We have to demonstrate that u=v=w. We have.

1 = ‖u‖ = ‖tv+(1-l)w‖ < t‖v‖ + (1-t) ‖w‖ <1,

This demonstrates ‖w‖=1=‖v‖. Presently putting x = television and y = (1-t)w in

3. Rieses representation theorem

Milicie [39] has demonstrated the portrayal hypothesis that on the off chance that X is smooth entirely curved Banach space, for each limited straight practical f on X there is reliable g.s.i.p [.,.] on X and a one of a kind vector y in X with the end goal that f(y) = (y, x] for all x in X. In this area we demonstrate this outcome under a lot more fragile conditions. one review that a vector x in normed straight space X is called to be ordinary (individually, arthogonal as for a predictable g.s.i.p. [.,.] on X) to y E X if ‖y+□x‖>‖y‖ for all scalars m (separately, [x, y] =0). M is called to be typical (separately, symmetrical) to N if every x EM is ordinary (individually, symmetrical) to all y E N, i.e. M & N are subsets of X.

Lemma 1.2 Is demonstrated by Faulkner for p =2 ([11], Th. 1). Here, we give a proof for the Lemma by Suitably altering the verification of Faulkner.

Evidence. Assume M is symmetrical to N regarding a g.s.i.p. [.,.] on X predictable with standard of X. At that point plainly M is typical to N. Alternately, assume that M is ordinary to N, at that point M∩N={0}. For every x EM, characterize fx on the range {x,N} = {□x+y:y E N, □ EC}

By, f_x (□x+y)=□ $‖x‖^p$ (1<p<∞).

Clearly f_x is linear and also bounded, for

│f_x(□x+y) │=│□│$‖x‖^p$ = $‖x‖^{p-1}$ ‖□x+y‖

Since fx(x/‖x‖) = ‖x‖p-1, we have ‖fx‖ = ‖x‖p-1. Additionally one see that fx (x) = ‖x‖p and fx (y) = 0 if y E N. Presently for z M, characterize fz (□z) =□ ‖z‖p on the range {z} and for these z E X, we have

$‖f_z‖ = ‖z‖^{p-1}$ and $f_z (z) = ‖z‖^p$.

Thus, for every x EX, we get a limited direct practical fx which can be stretched out to the entire space x by the Hahn Banach hypothesis, thus we may think about that fx to be characterized on the whole space X. Presently let □ be a well requesting on X and let x be the underlying component of □. Characterize the utilitarian □x to be fx, and if z =□x and if z=□x, characterize □z = □□□p-2 □x. Correspondingly for x', the underlying component of □ not in the range {x}, characterize □x' =fx' and □□□p-2 □x. Proceeding with this by transfinite enlistment, we can characterize for □z for every z E X. Since every z E X has an exceptional beginning generator w, as for the request □, the ordering of the functionals □z is unmistakably very much characterized. We presently set □z (x)= [x, z] and demonstrate that [.,.] is a homogenous g.s.i.p. on X regarding which M is symmetrical to N. Unmistakably [x, y] = □y (x) = o for x E M & y E N, thus we have to demonstrate [.,.] is a g.s.i.p. One initially that the linearity in the principal contention pursues from the linearity of □z. Presently for strict inspiration, assume w is the underlying generator of x E X, state x =□w, at that point

[x, x] = $\phi_{x\,(x)} = \phi_{\square\, w} = \overline{\mu}$ │□│$^{p-2}$ ϕ_w (□w)= │□│^{2}□│□│$^{p-2}$ ϕ_w (w) = $‖x‖^{p}$ >0 for x≠0.

Now for the Holder’s inequality, if x = □w, and □ = □η, where w and η are the initial elements of □, then one have

│[y, x]│ =│ϕ_x (y) =│$\phi_{\square\eta}$ (□w)│

=│ $\overline{\alpha}$ ││□│$^{p-2}$ │□│ϕ_η (w) =$[x, x]^{1/p}$ $[y, y]^{(p-1)/p}$

Finally, for the homogenous condition, we have

[x, □y] = [x, □□)n] = $\phi_{(\square\,\square\eta)}$(x)

= $\overline{\beta\alpha}$ │□□│$^{p-2}$ $\phi_{\eta\square}$(x) = $\overline{\beta}$ │□│$^{p-2}$ (x, y)

This completes the proof

4. Direct sum of q.si.i.p. spaces

Hypothesis 1.3 . Let X and Y by g.s.i.p. spaces. At that point the direct sumTheorem. Let X and Y by g.s.i.p. spaces. At that point the immediate total

y⊕x={(y, x): y E y, x∊ X}

is a g.si.p. space part insightful expansion and sclare duplication together with the g.s.i.p. characterized by

[(x2, y2), (x1, y1)] = [y1, y2] + [x1, x2].

The norm on X ⊕ Y is given by

‖(y, x)‖ = $‖y‖^p$ +$‖x‖p)^{1/p}$ (1<p<∞)

Confirmation. Unmistakably X□Y is a direct space. The linearity in the primary contention and the strist energy of [.,.]

are effortlessly checked. Henceforth, we have to demonstrate the Holder's disparity.

$|[x_1, y_1), (x_2, y_2)] \leq |(x_1, y_1), (x_1, y_1)]^{1/p} [(x_2, y_2), (x_2, y_2)]^{(p-1)/p}$.

one have

$|[y_1, x_1), (y_2, x_2)] \leq |(y_1, y_2)+|(x_1, x_2)]^{1/p} \leq [y_1, y_1)^{1/p} [y2, y2]^{p-1/p} + [x_1, x_1]^{1/p} [x_2, x_2]^{p-1/p}$

$=([x_y, x_1], (y_1,x_1])^{1/p} [(y_2,x_2], [y_2, x_2])^{(p-1)/p}$

On the off chance that x and Y are normed straight spaces, coordinate entirety X□Y turns into a normed direct space with segment savvy expansion and scalar increase together with the standard given

$\|(y, x)\| = \|y\|^p + \|x\|^p)^{1/p}$ $(1<p<\infty)$.

Presently by utilizing Riesz portrayal hypothesis 1.3, we demonstrate the accompanying:

Hypothesis 1.4. Give X and Y a chance to be finished g.s.i.p. spaces which are entirely raised and reflexive. In the event that the standard of X□Y are given:

$\|(x, y)\| = \|x\|^p + \|y\|^p)^{1/p}$ $(1<p<\infty)$,

then $(X\oplus Y)^*$ isisometrically isomorphic to $X^*\oplus Y^*$ with the correspondence $\phi(y,x) = f(y) + g(x)$, and $\|\phi\| = f(x)+g(y)$ for $(y, x) \in y\oplus x$. Then ϕ are bounded linear functional on $X\oplus Y$. For if $(x,y) \in X\oplus Y$, then

(12) $|\phi| (y,x) \leq |f(y)| + |g(x)| \leq \|f\| \|(y)\| + \|(g)\| \|(x)\|$

$\leq \|f\|^q + \|g\|^{q)1/q} (\|x\|p+\|y\|p)^{1/p} = (\|f\|^q + \|g\|^q)^{1/q} \|(x, y)\|$

i.e. if $\phi \in (X\oplus Y)^*$, then by linearity one have $\phi(x,y) = \phi(x,O) + \phi(O,y)$, for any (x, y) E $X\oplus$ y). Now if we set $f(x) = f(x) = \phi(x,O)$ for $x \in X$ and $g(y) = \phi(O,y)$for $y \in Y$, we see that $f \in X^*$ and $g \in Y^*$ and $\phi(x,y) = f(x) + g(y)$. Now the desired result is obtained by showing

$\|\phi\| \geq (\|f\|^{)q} (\|g\| q)^{1/q}$

since $f \in X^*$ and $g \in Y^*$, by Theorems 3.5 and 3.6, there exist unique vectors x_0 and y_0 in X and Y respectively s.t. f(x) = [x, x0, for all x in X with $\|f\| = \|x_0\|^{p-1}$, and $g(y) = [y, y_0]$ Э y in Y with $\|g\| = \|y_0\|^{p-1)}$. Now one have

$\phi (x, y_0) = f (x_0) + g(y_0)$

$= \|x_0\|^p + \|y_0\|^p = (\|x_0\|^p + \|y_0\|^p)^{p-1/p} \|(x_0 , y_0)\|$

Therefore

(13) $\phi(x_0y_0) = \{\|f\|^{p/(p-1)} + \|g\|^{p/(p-1)\}(p-1)/p} = \{\|f\|^q + \|g\|^q\}^{1/q} \|(x_0, y_0)\|$

Hence (12) and (13) reveal that

$\|\phi\| = (\|f\|^q + \|g\|^{q\ 1/q}$, This completes the proof.

References

1) G.D.Faulkner, Representation of linear functionals in a Banach space, Rocky Mountain J. Math. 7 (1977), 789—792.
2) J.R.Giles, Classes of semi—inner—product spaces, Trans. Amer. Math. Soc. 129 (1967), 436—446.
3) S.Gudder and D.Strawther, Strictly convex normed linear spaces, Proc. Amer. Math. Soc. 59 (1976), 263—267.
4) T.Husain and B.D.Malviya, On semi—inner—product spaces I, Colloquim Mathematicum XXIV (1972), 235—240.
5) T.Husain and B.D.Malviya, On semi—inner—product Algebras, Proc. Japan Acad. 46 (1970), 273—276.
6) T.Husain and B.D.Malviya, On semi—inner—product spaces 11,1 Vol. XXVII (1973), 95—105.
7) P..M.Milicie, Surle Semi—produit scalaire generalise, Mathematicki Vesnik Vol (i) (25) (1973), 325—329.