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Some Contribution to the Duality Theory of Barrelledly Nuclear Locally Convex Spaces

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ABSTRACT

In this paper we have established that a given dual nuclear locally convex space can be a dual barrelledly nuclear locally convex space. Further we have proved that a dual barrelledly nuclear convex space is a F-space. In other words, a dually nuclear locally convex space is a F-space.

1. Introduction

According to Akbert[1] a fundamental system B_rf(E) of barreles is like a fundamental system u_f(E) zero neighbourhoods. With the help of a fundamental system us (E) of zero neighbourhoods nuclear locally convex space can be defined. With the help of a fundamental system B_rf(E) of barrels these can be defined a barrelledly nuclear locally convex space. On the basis of the concepts given by Pietsch[2], Tornke[3], Trepark[4] and Volme[5]. We have selected two problems have solved those problems as theorems I and II proved below.

The Theorem – I:

Let E^1 be a dual nuclear locally convex space. Then E^1 can be a dual barrelledly nuclear locally convex space.

Proof:

It is obvious that E is a nuclear locally space convexspace ……………………….(1)

On the basis of the concept of (1) is follows that there can exist a fundamental system u_f(E) of zero neighbourhoods in U_n in E such that

$U_1 \supset U_2 \supset U_3 \supset \cdots\cdots\cdots\cdots\cdots\cdots \supset U_n$ ……………………………………(2)

We consider two zero neighbourhood $U, W \varepsilon\, u_f(E)$ with $W < U$ such that

$pU(x) \leq \sum_N |< x, a_n >|, x\varepsilon\, U\varepsilon\, u_f(E),\ a_n \varepsilon\, W^\circ \subset E'$……................................(3)

Moreover, $\sum_N p' W^\circ(a_n) < +\infty$ for $a_n \varepsilon\, W^\circ \varepsilon E'$ ……..…………………………. (4)

We consider a set $M\varepsilon\, u_f(E)$ such that $x, y\varepsilon M \Longrightarrow \alpha x + \beta y\ \ \varepsilon M$ for $\alpha, \beta\ \varepsilon\ K$ and $|\alpha + \beta| \leq 1$ ………...(5)

On the basis of the concept of (5) it is obvious that M can be absolutely convex. ……………………………………………………….…… (6)

It is known that absolutely convex sets can be balanced, closed and convex sets …………………………………………………. (7)

On the basis of the concept of (6) and (7) it is clear that M can be convex closed and balanced. ……………………………………………………… (8)

We can select M such that $x\ \varepsilon\ M \Longrightarrow hx\ \varepsilon\ M$ where h is a number with $|h| \leq 1$ ………………………….................................. (9)

From the concept of (9) it follows that M can be an absorbing set...............(10)

On the basis of the concept of (8) and (10) it is obvious that each M can be a barrel in E. …………………………..……… (11)

From the concept of (5) and (11) it is clear that the all zero neighbourhoods
U_n in E can be barrels in E. ………,………………………….(12)
On the basis of the concepts of (2) and (12) it is obvious that there can
exist a fundamental system B_rf(E) of barrels in E. ……………………………..….. (13)
Moreover U, W can be barrel ε $B_r f(E)$ with $W < U$ such that the relations (3)and (4)
can be valid for barrels. U, W, ε $B_r f(E)$……………………………………………... (14)

Then from above concepts it is clear that E can be barrelledly nuclear locally convex space on the basis of the convex of (15) it follows that E^1 can be a dual barrelledly nuclear locally convex space (16). Thus, the theorem completely proved.

The Theorem – II:

Let E^1 be a dual barrelledly nuclear locally convex space. Then E^1 can be an (F) space.

Proof:-

It is obvious that E can be a barrelledly nuclear locally convex space. ……….. (1)
On the basis of the concepts of (1) it follows that there can exist a fundamental
system $B_r f(E)$ of barrels W_n with $W_1 \supset W_2 \supset W_3 \supset \cdots\cdots\cdots\cdots \supset W_n$……………(2)
From the concepts of (2) it is obvious that
$W_1{}^\circ C \supset W_2{}^\circ C \supset W_3{}^\circ C \supset \cdots\cdots\cdots\cdots\cdots\cdots \supset W_n{}^\circ C$…………………… (3)

On the basis of the concept of (3) it is clear that there can exist a fundamental countable system $Bf(E^1)$ of bounded subsets $W_n{}^\circ C$ for n=1,2,3,………
……………. (4)
We consider two zero neighbourhoods A°, B° in E^1 such that $A^\circ \supset B^\circ \Longrightarrow A^{\circ\circ} \subset B^{\circ\circ}$
for each zero neighbourhood B° in E^1 and for each countable subset $A^{\circ\circ}$.
The topological dual E^{11} of the locally convex space E^1…………………… (5)

On the basis of the concept of (5) it is clear that each countable subsets of the topological dual E^{11} of the locally convex space E^1 is equicontinuous ……………………………..…… (6)

From the concept of (6) if follows that E^1 can be a σ-quasi-barrelled locally
convex space. …………………... (7)

On the basis of the concept of (4) and (7) it is obvious that E^1 can be a complete dual metric locally convex space. ………..…………………………………….…….. (8)

From the concept of (8) it is clear that E^1 can be an (F^1) space. ………………. (9)
Thus the theorem is completely proved.

References

1. **Albert, P.** A Study of dual barrelledly nuclear locally convex spaces for the application.Springer, Verlag-Berlin Heidelberg
2. New York(1989)
3. **Pietsch, A** Nuclear locally convex spaces. Springer, Verlag-Berlin Heidelberg, New York (1972).
4. **Tomke, E** The application of barrelledly metric locally convex spaces for the application. New York, Dover Publication in (1992)
5. **Trepark, G.** On dual nuclear locally convex spaces for the application. Proc. Amer MATH Soc. 13, 175-195(1995).
6. **Volume, E.** The application of dual barreled spaces and (F) – spaces. London Macmillion and Co. (1998).