

Some Solutions for Combined Scalar and Gravitational Fields

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ABSTRACT

The present paper provides two new exact solutions for the combined scalar and gravitational fields using the technique by De [8] for the static plane symmetric solution of Taub [19] and conformastat solution of Das [7].

1. Introduction

Various relativists have paid their interest in the study of scalar meson field [1, 2]. Janis et. al. [12] have analysed the problem further from the point of view of singularity. Gautreau [9] has extended the study to the case of non spherical Weyl fields. Stephenson [18] has considered the problem of the scalar meson field of non rest mass coupled with electromagnetic fields for static spherically symmetric gravitational fields. Mazumar [14] has obtained solutions of LRS (Locally rationally symmetric) Bianchi I space time filled with perfect fluid. Haji Boutras and Sfeila [11] and Sri Ram [20] have also obtained some solutions for the same field equations by using their solution generating techniques. Pradhan et. al. [21] have studied LRS Bianchi I space time with zero mass scalar field. In fact the need for exact solutions in general relativity is well known. In view of the highly non linear character of the field equations only a limited number of solutions are available in this theory. In this chapter we have tried to generate solutions for the coupled gravitational and scalar fields which study is of considerable interest in preventing singularity.

The possibility of construction of new solutions from existing one in some special cases have been explored by many authors. Majumdar [13] has shown that for static Einstein – Maxwell source free field equations the solutions can be generated from those of corresponding vacuum solutions. This investigation has been further extended by Misra and Radhakrishna [15] and later by Harrison [10] to the case of non-static Weyl fields. Formulating the idea of reciprocal solutions Buchdahl [3-6] has developed methods for generating new solutions from those for empty space. Some other workers in this line are Purushottam et. al [22] Singh [17] Santill [16] and Verma [23].

Solution for the coupled gravitational and zero rest mass scalar fields from those of vacuum static fields has been produced by De [8]. In this chapter we have obtained two new exact solutions for the combined scalar and gravitational fields using the above technique by De for the static plane symmetric solution of Taub [19] and conformastat solution of Das [7].

2. Technique and the application

The result established by De [1964] may be given as follows : Suppose that the metric.

$$(2.1) \quad ds^2 = \exp[\mu]dt^2 - \exp[-\mu][\psi_{ij}dx^i dx^j]$$

is a solution of empty space field equations \square and ψ_{ij} are function of the space coordinate x^1, x^2, x^3 . Then a solution of the combined scalar gravitational field in given by

$$(2.2) \quad ds^2 = \exp[\mu]dt^2 - \exp[-K\psi_{ij}dx^i dx^j]$$

with

$$(2.3) \quad \mu = A\theta$$

where θ is scalar potential and

$$(2.4) \quad K = -c\square$$

where c is a constant and $A^2 = C^2 - 16\square$

Case 1 :

Taub's Solution

Taub (1951) found static plane symmetric solution of Einstein's vacuum field equations which may be written in the form

$$(2.5) \quad ds^2 = -\sqrt{G} \left\{ \frac{dx^2}{G} + \sqrt{G}(dy^2 + dz^2) \right\} + \frac{dt^2}{\sqrt{G}}$$

where x_1, x_2, x_3, x_4 corresponds to x, y, z, t respectively
 Comparing (2.5) and (2.1) we get

$$(2.6) \quad e^\mu = \frac{1}{\sqrt{G}}$$

and

$$(2.7) \quad \psi_{ij} dx^i dx^j = \left\{ \frac{dx^2}{G} + \sqrt{G}(dy^2 + dz^2) \right\}$$

From (2.3) we get the scalar potential \square as

$$(2.8) \quad e^\theta = (G)^{-\frac{1}{2A}}$$

where $G = Bx + D$

where B and D are constants

The metric characterising the coupled scalar gravitational field equation is

$$(2.9) \quad ds^2 = -G^{\frac{1}{2A}} \left\{ \frac{dx^2}{G} + \sqrt{G}(dy^2 + dz^2) \right\} + G^{-\frac{C}{2A}} dt^2$$

Case II :

Das Conformastat Gravitational Universe : In 1971 Das found a solution for a conformastat gravitational model of universe which may be written as

$$(2.10) \quad ds^2 = H^{-2} dt^2 - H^4(dx^2 + dy^2 + dz^2)$$

where $H = (1 - \square x)$ and \square is a constant.

Applying the same procedure as in the previous solutions, we get,

$$(2.11) \quad e^\mu = H^{-2}$$

and

$$(2.12) \quad \psi_{ij} dx^i dx^j = H^2(dx^2 + dy^2 + dz^2)$$

Thus in this case the scalar potential is

$$(2.13) \quad e^\theta = (H)^{-\left(\frac{2}{A}\right)}$$

and the metric for the coupled field equation is

$$(2.14) \quad ds^2 = H^{-\frac{2c}{A}} dt^2 - H^{2+\frac{2c}{A}} (dx^2 + dy^2 + dz^2)$$

or
$$ds^2 = (1 - \alpha x)^{-\frac{2c}{A}} dt^2 - (1 - \alpha x)^{2+\frac{2c}{A}} (dx^2 + dy^2 + dz^2)$$

3. Remarks :

The Kretschmann curvature invariant for the metric (5.2.5) has the form

$$(3.1) \quad S = R_{ijkl} R^{ijkl} = \frac{3B^{418}}{(Bx + D)^3}$$

and for the metric (5.2.9), we have

$$(3.2) \quad S = \left(\frac{B^4}{8} \right) \left(\frac{1}{2} + \frac{C}{2A} \right)^2 \left(2 + \frac{C^2}{A^2} \right) \frac{1}{(Bx + 2)^2} + \frac{C}{R}$$

From the examination of the scalar (S) we find that as $X \rightarrow -\frac{D}{B}$, $S \rightarrow \infty$ in both (3.1) and (3.2). This indicate that both the

metrics (2.5) and (2.9) have intrinsic singularity as $X = -\frac{D}{B}$. Hence it may be concluded that there are no qualitative changes in

the singular structure when a zero-rest – mass scalar field is added to Taub's static plane symmetric gravitational field.

On calculation one finds that the scalar for the metric (2.10) and (2.14) are respectively give by

$$(3.3) \quad S = \{96\alpha^4 H^{-12}\}$$

$$(3.4) \quad S = 4\alpha^4 \left(2 + \frac{2C}{A}\right) \left(\frac{1}{2} + \frac{C^2}{A^2}\right) \frac{1}{H^8 + \frac{4C}{A}}$$

As in the case of previous solution, a study of S in (3.3) and (3.4) reveals that the metric (2.10) and (2.14) both have an intrinsic singularity at $X = \frac{1}{\alpha}$. Hence one may again conclude that the singular structure of the combined gravitational and zero-

rest-mass scalar fields have the same feature as the conformastat gravitational universe of Das. [7]

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