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Development of New Weighted Measures of Fuzzy Divergence

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ABSTRACT

Distance measures are quiet well known and play an important role not only in the literature of information theory but in every discipline of science and engineering. In the present communication, we have developed some new generalized measures of weighted fuzzy divergence (distance), studied their important properties for their authenticity and have presented them graphically.

1. INTRODUCTION

It is well known today that in different disciplines of Science and Engineering, the concept of distance has been proved to be very important because of its applications towards the development of various mathematical models. Keeping in mind the application areas of the distance measures, attempts would be made to extend the concept for applications to problems in Economics, Sociology, Psychology, Genetics, Biology, and other emerging disciplines of Technology and Engineering etc.

In case of a fuzzy distribution, distance measure is a term that describes the difference between fuzzy sets and can be considered as a dual concept of similarity measure. Using the axiom definition of distance measure, Fan, Ma and Xie [5] developed some new formulas of fuzzy entropy induced by distance measure and studied some new properties of distance measure. Dubois and Prade [4] defined the distance between two fuzzy subsets on a fuzzy subset of R^+. Their definition does not generalize the shortest distance between two crisp sets; rather it generalizes the set of distances between two sets. Rosenfeld [12] defined the shortest distance between two fuzzy sets as a density function on the non - negative reals, which generalizes the definition of shortest distance for crisp sets in a natural way. Thus, corresponding to the probabilistic measure of divergence due to Kullback and Leibler [9], Bhandari and Pal [2] introduced the following measure of fuzzy directed divergence:

$$I(A:B)=\sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+(1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_A(x_i)}\right] \quad (1.1)$$

Corresponding to Renyi's [11] and Havrada and Charvat's [7] probabilistic divergence measures, Kapur [8] took the following expressions of fuzzy divergence measures:

$$D_\alpha(A:B)=\frac{1}{\alpha-1}\sum_{i=1}^{n}\log\left[\mu_A(x_i)\mu_B^{1-\alpha}(x_i)+(1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha}\right] \quad (1.2)$$

and

$$D^\alpha(A:B)=\frac{1}{\alpha-1}\left[\sum_{i=1}^{n}\log\ \mu_A^\alpha(x_i)\mu_B^{1-\alpha}+(1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha}-1\right] \quad (1.3)$$

Tran and Duckstein [14] have developed a new approach for ranking fuzzy numbers based on a distance measure. In this communication, the authors have first developed a new class of distance measures for interval numbers that takes into account all the points in both intervals, and then it is used to formulate the distance measure for fuzzy numbers. The approach has been illustrated by numerical examples, showing that it overcomes several shortcomings such as the indiscriminative and counterintuitive behavior of several existing fuzzy ranking approaches. Motivated by the standard measures of divergence, Parkash [10] introduced a new generalized measure of fuzzy directed divergence involving two real parameters, given by

$$ {}_1D^{\beta}_{\alpha}(A,B)=[(\alpha-1)\beta]^{-1}\sum_{i=1}^{n}\left[\left\{\mu_A^{\alpha}(x_i)\,\mu_B^{1-\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}(1-\mu_B(x_i))^{1-\alpha}-1\right\}^{\beta}-1\right] \quad (1.4)$$

In fact, Kapur [8] has developed many expressions for the measures of fuzzy directed divergence corresponding to probabilistic measures of divergence due to Harvada and Charvat [7], Renyi [11], Ferreri [6] etc. Many other measures of fuzzy divergence have been developed by Bhandari, Pal and Majumder [3], Parkash [10], Dubois and Prade [4], Xue, Zhang and Lin [15] etc. Since, we have also to deal with fuzzy measures of information; we take into consideration the notion of fuzzy sets introduced by Zadeh [16] and using the concept of useful information provided by Belis and Guiasu [1], we have discussed and developed some new weighted fuzzy measures of divergence in the following section.

2. DEVELOPMENT OF WEIGHTED FUZZY DIVERGENCE MEASURES

In this section, we introduce some generalized weighted measures of fuzzy directed divergence corresponding to some well known measures of fuzzy entropy.

I. Generalized weighted measure of fuzzy divergence of order α :

$$D_{\alpha}(A,B;W)=\sum_{i=1}^{n}w_i\left[\mu_A^{\alpha}(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+(1-\mu_A(x_i))^{\alpha}\log\frac{(1-\mu_A(x_i))}{(1-\mu_B(x_i))}\right] \quad \textbf{(2.1)}$$

To check the validity of the proposed measure (2.1), we first of all check its convexity.

Convexity: To prove convexity of the proposed measure, we have

$$\frac{dD_{\alpha}(A,B;W)}{d\mu_B(x_i)}=-W_i\frac{\mu_A^{\alpha}(x_i)}{\mu_B(x_i)}+W_i\frac{(1-\mu_A(x_i))^{\alpha}}{(1-\mu_B(x_i))}$$

$$\text{Also }\frac{d^2(D_{\alpha}(A,B;W))}{d\mu_B^2(x_i)}=W_i\frac{\mu_A^{\alpha}(x_i)}{\mu_B^2(x_i)}+W_i\frac{(1-\mu_A(x_i))^{\alpha}}{(1-\mu_B(x_i))^2}>0$$

Hence $D_{\alpha}(A,B;W)$ is a convex function of $\mu_B(x_i)$

Similarly, we can prove that $D_{\alpha}(A,B;W)$ is a convex function of $\mu_A(x_i)$. Thus, we have:

1. $D_{\alpha}(A,B;W)\geq 0$
2. $D_{\alpha}(A,B;W)=0$ iff A=B.
3. $D_{\alpha}(A,B;W)$ does not change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ is replaced by $1-\mu_B(x_i)$.
4. $D_{\alpha}(A,B;W)$ is a convex function of both $\mu_A(x_i)$ and $\mu_B(x_i)$.

Under these conditions, the fuzzy divergence (2.1) is a correct measure of directed divergence.
Moreover, with the help of the data, we have presented the measure (2.1) graphically. For this purpose, we have fixed $\mu_B(x_i)=\frac{1}{2}\ \forall i$ and then computed different values of $D_{\alpha}(A,B;W)$ for $\alpha=2$ corresponding to different fuzzy Values under the weighted distribution as shown in the Table- 2.1:

Table-2.1

$\mu_A(x_i)$	w_i	$D_{\alpha}(A,B;W)$
0.000	5.00	5.0000
0.125	5.25	3.0811
0.250	5.50	1.4660
0.375	5.75	0.3875
0.500	6.00	0.0000

0.625	6.25	0.4212
0.750	6.50	1.7325
0.875	6.75	3.9614
1.000	7.00	7.0000

Next, with the help of the above data, we have obtained the following Fig.-2.1 which shows that the divergence measure introduced in equation (2.1) is a convex function.

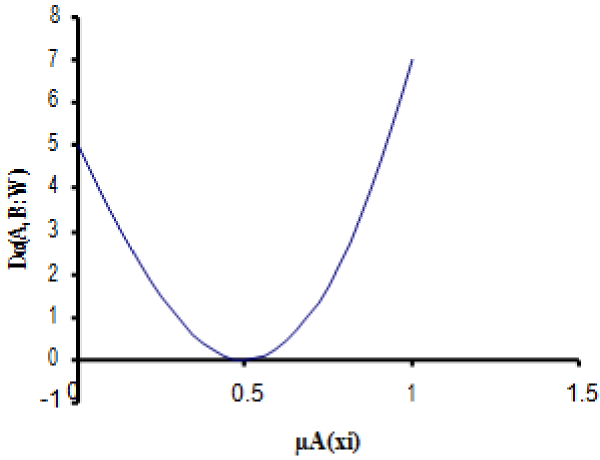

Fig-2.1

II. New non-parametric weighted divergence measure

Next, we propose a new non-parametric weighted measure of fuzzy divergence as given by the following mathematical expression:

$$D(A,B;W)=\sum_{i=1}^{n} w_i \left[\frac{(\mu_A(x_i)-\mu_B(x_i))^2}{(\mu_A(x_i)-\mu_B(x_i))} + \frac{(\mu_B(x_i)-\mu_A(x_i))^2}{(2-\mu_A(x_i)-\mu_B(x_i))} \right] \quad \textbf{(2.2)}$$

To check the validity of the proposed measure (2.2), we first of all check its convexity.

Convexity: The proposed measure is convex Proof. We have

$$\frac{d^2D(A,B;W)}{d\mu_A^2(x_i)} == w_i \frac{8\mu_B^2(x_i)}{(\mu_A(x_i)+\mu_B(x_i))^3} + w_i \frac{2(4+4\mu_B^2(x_i)-8\mu_B(x_i))}{(2-\mu_A(x_i)-\mu_B(x_i))^3} > 0$$

which shows that the weighted measure $D(A,B;W)$ is a convex function of fuzzy values $\mu_A(x_i)$.

Also, we have

$$\frac{d^2D(A,B;W)}{d\mu_B^2(x_i)} = w_i \frac{8\mu_A^2(x_i)}{(\mu_A(x_i)+\mu_B(x_i))^3} + w_i \frac{2(4+4\mu_A^2(x)-8\mu(x))}{(2-\mu_A(x_i)-\mu_B(x_i))^2} > 0$$

which shows that the divergence measure $D(A,B;W)$ is a convex function of $\mu_B(x_i)$

Hence, $D(A,B;W)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$. Thus, we have the following essential properties of $D(A,B;W)$:

1. $D(A,B;W) \geq 0$
2. $D(A,B;W) = 0$ iff A=B.
3. $D(A,B;W)$ does not change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ is replaced by $1-\mu_B(x_i)$.

4. $D(A,B;W)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$.

Under these conditions, the weighted measure of fuzzy divergence (2.2) is a correct measure of directed divergence. Moreover, we have presented (2.2) graphically and obtained the above Fig.-2.2 which shows that the weighted divergence measure introduced in equation (2.2) is a convex function.

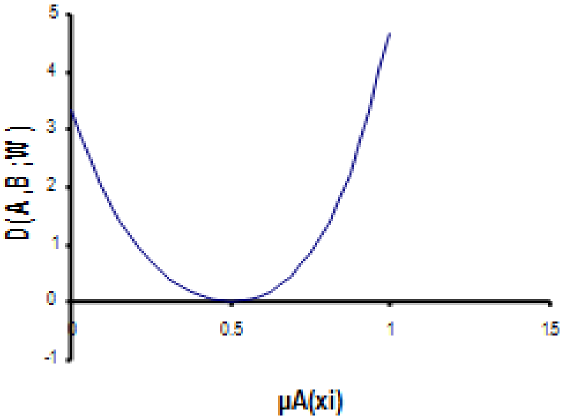

Fig-2.2

III. New non-parametric weighted fuzzy symmetric divergence measure

Here, we propose a new non-parametric weighted measure of fuzzy symmetric divergence, given by

$$\psi(A,B;W)=\frac{1}{2}\sum_{i=1} w_i\left[\left(\sqrt{\mu_A(x_i)}-\sqrt{\mu_B(x_i)}\right)^2+\left(\sqrt{1-\mu_A(x_i)}-\sqrt{1-\mu_B(x_i)}\right)^2\right] \quad (2.3)$$

To check the validity of the proposed measure (2.3), we have verified that $\psi(A,B;W)$ satisfies:

1. $\psi(A,B;W)\geq 0$
2. $\psi(A,B;W)=0$ iff A=B.
3. $\psi(A,B;W)$ does not change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ is replaced by $1-\mu_B(x_i)$.
4. $\psi(A,B;W)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$.

Under these conditions, the weighted measure of fuzzy symmetric divergence proposed in (2.3) is a correct measure of directed divergence. Again, we have presented the above measure graphically and obtained Fig.-2.3 which shows that the weighted divergence measure introduced in equation (2.3) is a convex function.

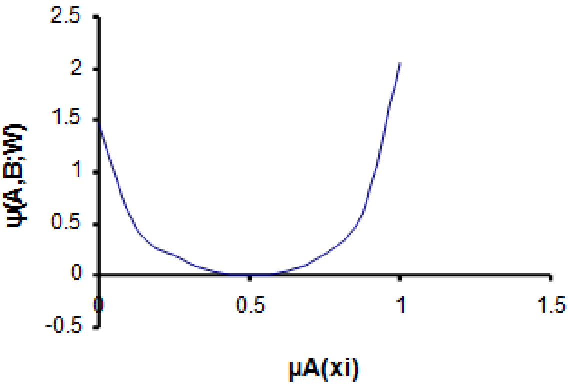

Fig-2.3

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