

A Study of Quadratic-Cubic Functional Equation (FES) In ℓ -Fuzzy Spaces

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ABSTRACT

The functional equations of mathematics are a relatively unpopular area. This is not because of insignificance: the expanding linear algebra which covers linear functions covers a much wider domain. Dynamic systems are generally described in a continuous time domain by differential equations. The dynamics are described by difference equation or by a map for discrete time systems, on the other hand. To create a trajectory or to determine certain device properties, functional equations must be dealt with. Functional equations are not often used to solve practical problems unlike differential calculus or linear algebra. This could be because of practical calculus technical difficulties. In the face of an engineering problem that needs these solutions. The author Lars Kindermann developed successfully methods to solve the equation with the help of neural networks at least numerically. In many fields of mathematics, functional equations exist, such as geometry, arithmetic, theory of probability, measurement theory, algebraic geometry and group theory. The apps are also used in information theory, code theory, fuzzy set theory, decision theory, game theory, artificial intelligence, cluster analysis, multi-value logic and many other fields. It also applies to other applications. Many new issues and hypotheses have inspired the development of new approaches and methods through functional equations. In the second half of the 20th century, the theory of functional equations was rapidly and productively developed. At the same time it was very fruitful to establish this theory. There are more and more mathematical texts and mathematicians working with functional equations every day.

Introduction

One of the problems in ℓ -fuzzy topology is to obtain an appropriate concept of ℓ -fuzzy metric spaces and ℓ -fuzzy normed spaces. R.Saadati and J. Park [158], respectively, introduced and studied a notion of intuitionistic fuzzy metric (normed) spaces and then Deschrijver et al [39] and Saadati generalized the concept of intuitionistic fuzzy metric (normed) spaces and introduced and studied a notion of ℓ -fuzzy metric spaces and ℓ -fuzzy normed spaces [39, 159].

ℓ -FUZZY SPACES

Definition 4.1.1 Let K be a field. A non-Archimedean absolute value on K is a function $\|\cdot\| : K \rightarrow [0, +\infty)$ such that, for any $a, b \in K$,

- (i) $|a| \geq 0$ and the equality holds if and only if $a = 0$,
- (ii) $|ab| = |a||b|$,
- (iii) $|a + b| \leq \max \{|a|, |b|\}$ (the strict triangle inequality).

Note that $|n| \leq 1$ for each integer n . We always assume, in addition, that $k \cdot k$ is non-trivial, i.e., there exists an $a_0 \in K$ such that $|a_0| \neq 0, 1$.

Definition 1 :A non-Archimedean ℓ -fuzzy normed space is a triple (V, τ, τ) , where V is a vector space, τ is a continuous t -norm on V and τ is an ℓ -fuzzy set on $V \times (0, +\infty)$ satisfying the following conditions: for all $x, y \in V$ and $t, s \in (0, +\infty)$,

(a) $0_\ell \leq_\ell p(x, t)$;

(b) $p(x, t) = 1_\ell$ if and only if $x = 0$;

(c) $p(\alpha x, t) = p\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;

(d) $\tau(p(x, t), p(y, s)) \leq_\ell p(x + y, \max(t, s))$

(e) $p(x, \cdot) :]0, \infty[\rightarrow \ell$ is continuous;

(f) $\lim_{t \rightarrow 0} p(x, t) = 0_\ell$ and $\lim_{t \rightarrow 0} p(x, t) = 1_\ell$.

Example 2 Let $(X, \|\cdot\|)$ be a non-Archimedean normed linear space. Then the triple (X, p, \min) , where p is a non-Archimedean ℓ -fuzzy normed space in which $\ell = [0, 1]$.

$$p(x, t) = \begin{cases} 0 & , \text{if } t \leq \|x\| ; \\ 1 & , \text{if } t > \|x\| , \end{cases}$$

Example 3 Let $(X, \|\cdot\|)$ be a non-Archimedean normed linear space.

Denote $\tau_m(a, b) = (\min\{a_1, b_1\}, \max\{a_2, b_2\})$ for all $a = (a_1, a_2), b = (b_1, b_2) \in \ell^*$ and

let $p_{\mu, \nu}$ be the intuitionistic fuzzy set on $X \times]0, +\infty[$ defined as follows:

$$p_{\mu, \nu}(x, t) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|} \right)$$

for all $t \in R^+$. Then $(X, p_{\mu, \nu}, \tau_M)$ is a non-Archimedean intuitionistic fuzzy normed space.

In this study, author investigates the general solution of following new generalized quadratic functional equations

$$f(ax - y) \pm a f(ax \pm y) = (a \pm 1)[af(x) \pm f(y)] \tag{4.1}$$

for any fixed integer a with $a \neq -1, 0, 1$. We also study the Hyers-Ulam-Rassias stability of the functional equation (4.1) in non-Archimedean ℓ -fuzzy normed spaces.

The General Solution of the Functional Equation

Let X and Y be a linear spaces. In this section author will find out the general solution of (4.1).

Theorem 1 If an function $f : X \rightarrow Y$ satisfies

$$f(ax - y) \pm af(ax \pm y) = (a \pm 1)[af(x) \pm f(y)] \tag{4.2}$$

then f is quadratic.

Proof. Setting $x = y = 0$ in (4.2), we obtain $f(0) = 0$ and setting $(x, y) = (x, 0)$ and $(x, y) = (0, x)$ in (4.1), we obtain respectively, for all $x \in X$. Therefore, f is even. Replacing y by $-y$ in (4.2), we obtain

$$f(ax) = a^2 f(x) \text{ and } f(-x) = f(x) \tag{4.3}$$

$$f(ax + y) \pm af(ax \mp y) = (a \pm 1)[af(x) \pm f(y)] \tag{4.4}$$

for all $x, y \in X$. Adding (4.2) and (4.4) and in resultant again using (4.2), we arrive

$$f(ax + y) + f(ax - y) \pm a f(x + y) \pm a f(x - y) = 2f(ax - y) \pm 2af(x \pm y) \quad (4.5)$$

for all $x, y \in X$. If we choose either first order sign or the second order sign in the equation (4.5), in both cases, we obtain

$$f(ax + y) + af(x - y) = f(ax - y) + a f(x + y) \quad (4.6)$$

for all $x, y \in X$. Replacing y by $ax + y$ in (4.6), we obtain

$$f(2ax + y) + af(ax - (x - y)) = f(y) + af(ax + (x + y)) \quad (4.7)$$

for all $x, y \in X$. Replacing y by $-y$ in (4.7), we obtain

$$f(2ax - y) + af(ax - (x + y)) = f(y) + af(ax + (x - y)) \quad (4.8)$$

for all $x, y \in X$. Adding equations (4.7) and (4.8), and using (4.6) in the resultant, we arrive

$$f(2ax + y) + f(2ax - y) + 2a^2f(y) = a^2f(2x + y) + a^2f(2x - y) + 2f(y) \quad (4.9)$$

for all $x, y \in X$. Replacing y by $x + ay$ in (4.6), we obtain

$$f(a(x + y) + x) + af(ay) = f(a(x - y) - x) + af(2x + ay) \quad (4.10)$$

for all $x, y \in X$. Replacing y by $-y$ in (4.10), we obtain

$$f(a(x - y) + x) + af(ay) = f(a(x + y) - x) + af(2x - ay) \quad (4.11)$$

for all $x, y \in X$. Adding equations (4.10) and (4.11), and using (4.6) in the resultant, we arrive

$$f(2x + y) + f(2x - y) + 2f(y) [a^2 - 1] = f(2x + ay) + f(2x - ay) \quad (4.12)$$

for all $x, y \in X$. Setting $(x, y) = (\frac{y}{2}, 2x)$ in (4.12), we get

$$f(2x + y) + f(2x - y) + 2(a^2 - 1)f(2x) = f(2ax + y) + f(2ax - y) \quad (4.13)$$

for all $x, y \in X$. Using (4.13) in (4.9), we arrive

$$f(x + y) + f(x - y) = 2f(x) + 2f(y)$$

for all $x, y \in X$. This shows that f is quadratic, which completes the proof of Theorem.

ℓ -Fuzzy Stability of the Functional

Let K be a non-Archimedean field, X a vector space over K and (Y, p, τ) a non-Archimedean ℓ -fuzzy Banach space over K . In this section, author proves the Hyers-Ulam-Rassias stability of the quadratic functional equation (4.1). We define an ℓ -fuzzy approximately quadratic mapping. Let Ψ be an ℓ -fuzzy set on $X \times X \rightarrow [0, \infty)$ such that $\Psi(x, y, \cdot)$ is non-decreasing,

$$\psi(cx, cx, t) \geq_{\ell} \psi\left(x, x, \frac{t}{|c|}\right), \quad \forall x \in X, c \neq 0$$

and

$$\lim_{t \rightarrow \infty} \psi(x, y, t) = 1_{\ell}, \quad \forall x, y \in X, t > 0.$$

Definition: A mapping $f : X \rightarrow Y$ is said to be ψ -approximately quadratic if *for all $x, y \in X$ and $t > 0$.*

$$\rho(f(ax - y) \pm a f(ax \pm y) - (a \pm 1)[af(x) \pm f(y)], t) \geq_{\ell} \psi(x, y, t) \quad (4.14)$$

Author now investigates the generalized Hyers-Ulam stability problem for functional equation (4.1). The following is one of our main results in this section.

Theorem 4.3.2 Let K be a non-Archimedean field, X a vector space over K and (Y, p, τ) a non-Archimedean τ -fuzzy Banach space over K . Let $f : X \rightarrow Y$ be a ψ -approximately quadratic mapping and $f(0) = 0$. If there exist an $\alpha \in \mathbb{R} (\alpha > 0)$ and an integer $k, k \geq 2$ with $|a^k| < \alpha, |a| \neq 1$ and $a \neq 0$ such that

$$\psi(a^{-k}x, a^{-k}y, t) \geq_{\ell} \psi(x, y, \alpha t) \quad \forall x, y \in X, t > 0 \quad (4.15)$$

and

$$\lim_{n \rightarrow \infty} \tau_{j=n}^{\infty} M\left(x, \frac{\alpha^j t}{|a|^{kj}}\right) = 1_{\ell} \quad \forall x \in X, t > 0$$

then there exists a unique quadratic mapping $Q : X \rightarrow Y$ such that

$$p(f(x) - Q(x), t) \geq_{\ell} \tau_{j=1}^{\infty} M\left(x, \frac{\alpha^{j+1} t}{|a|^{kj}}\right) \quad \forall x \in X, t > 0 \quad (4.16)$$

where

$$M(x, t) = \tau(\psi(x, 0, t), \psi(ax, 0, t) \cdots \psi(a^{k-1}x, 0, t)) \quad \forall x \in X, t > 0.$$

Proof. First, we show, by induction on j , that, for all $x \in X, t > 0$ and $j \geq 1$.

$$p(f(a^j x) - a^{2j} f(x), t) \geq_{\ell} M_j(x, t) = T(\psi(x, 0, t), \dots, \psi(a^{j-1}x, 0, t)). \quad (4.17)$$

Putting $y = 0$ in (4.14), we obtain

$$p(f(ax) - a^2 f(x), t) \geq_{\ell} \psi(x, 0, t)$$

for all $x \in X, t > 0$. This proves (4.17) for $j = 1$. Let (4.17) hold for some $j > 1$.

Replacing y by 0 and x by ax in (4.14), we get

$$p(f(a^{j+1}x) - a^2 f(a^j x), t) \geq_{\ell} \psi(a^j x, 0, t)$$

for all $x \in X, t > 0$. Since $\ell \leq 1$, it follows that

$$\begin{aligned}
 & p\left(f(a^{j+1}x) - a^{2(j+1)}f(x), t\right) \\
 & \geq_l T\left(p\left(f(a^{j+1}x) - a^2f(a^jx), t\right), p\left(a^2f(a^jx) - a^{2(j+1)}f(x), t\right)\right) \\
 & = T\left(p\left(f(a^{j+1}x) - a^2f(a^jx), t\right), p\left(f(a^jx) - a^{2j}f(x), \frac{t}{|l|^2}\right)\right) \\
 & \geq_l T\left(p\left(f(a^{j+1}x) - a^2f(a^jx), t\right), p\left(f(a^jx) - a^{2j}f(x), t\right)\right) \\
 & \geq_l T\left(\psi(a^jx, 0, t), M_j(x, t)\right) \\
 & = M_{j+1}(x, t)
 \end{aligned}$$

for all $x \in X, t > 0, n \geq 0$, and so

$$\begin{aligned}
 & p\left((a^{2k})^n f\left(\frac{x}{(a^k)^n}\right) - (a^{2k})^{n+1} f\left(\frac{x}{(a^k)^{n+1}}\right), t\right) \\
 & \geq_l M\left(x, \frac{\alpha^{n+1}}{|(a^{2k})^n|}t\right) \\
 & \geq_l M\left(x, \frac{\alpha^{n+1}}{|(a^k)^n|}t\right)
 \end{aligned}$$

for all $x \in X, t > 0, n \geq 0$. Hence it follows that

$$\begin{aligned}
 & p\left((a^{2k})^n f\left(\frac{x}{(a^k)^n}\right) - (a^{2k})^{n+p} f\left(\frac{x}{(a^k)^{n+p}}\right), t\right) \\
 & \geq_l T_{j=n}^{n+p}\left(p\left(a^{2k}f\left(\frac{x}{(a^k)^j}\right) - (a^{2k})^{j+p} f\left(\frac{x}{(a^k)^{j+p}}\right), t\right)\right) \\
 & \geq_l T_{j=n}^{n+p} M\left(x, \frac{\alpha^{j+1}}{|(a^k)^j|}t\right)
 \end{aligned}$$

for all $x \in X, t > 0, n \geq 0$. Since $\lim_{n \rightarrow \infty} T_{j=n}^{\infty} M\left(x, \frac{\alpha^{j+1}}{|(a^k)^j|}t\right) = 1_l \forall x \in X, t > 0$,

$\left\{ (a^{2k})^n f\left(\frac{x}{(a^k)^n}\right) \right\}_{n \in \mathbb{N}}$ is a Cauchy sequence in the Non-Archimedean ℓ -fuzzy Banach

space (Y, P, T) . Hence we can define a mapping $Q : X \rightarrow Y$ such that

$$\lim_{n \rightarrow \infty} p\left((a^{2k})^n f\left(\frac{x}{(a^k)^n}\right) - Q(x), t\right) = 1_l, \quad \forall x \in X, t > 0. \tag{4.19}$$

Next, for all $n \geq 1, x \in X$ and $t > 0$, we have

$$\begin{aligned}
 & p\left(f(x) - (a^{2k})^n f\left(\frac{x}{(a^k)^n}\right), t\right) \\
 & = p\left(\sum_{i=0}^{n-1} \left(a^{2k}\right)^i f\left(\frac{x}{(a^k)^i}\right) - (a^{2k})^{i+1} f\left(\frac{x}{(a^k)^{i+1}}\right), t\right) \\
 & \geq_l T_{i=0}^{n-1}\left(p\left((a^{2k})^i f\left(\frac{x}{(a^k)^i}\right) - (a^{2k})^{i+1} f\left(\frac{x}{(a^k)^{i+1}}\right), t\right)\right) \\
 & \geq_l T_{i=0}^{n-1} M\left(x, \frac{\alpha^{i+1}t}{|a^k|^i}\right)
 \end{aligned}$$

and so

$$\begin{aligned}
 & p(f(x) - Q(x), t) \\
 & \geq_l T \left(p \left(f(x) - (a^{2k})^n f \left(\frac{x}{(a^k)^n} \right), t \right), p \left((a^{2k})^n f \left(\frac{x}{(a^k)^n} \right) - Q(x), t \right) \right) \\
 & = p \left(T_{i=0}^{n-1} M \left(x, \frac{\alpha^{i+1}t}{|a^k|^i} \right), p \left((a^{2k})^n f \left(\frac{x}{(a^k)^n} \right) - Q(x), t \right) \right). \tag{4.20}
 \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ in (4.20), we obtain

$$P(f(x) - Q(x), t) \geq T_{i=1}^{\infty} M \left(x, \frac{\alpha^{i+1}t}{|a^k|^i} \right)$$

which proves (4.16). As T is continuous, from a well-known result in ℓ -fuzzy (probabilistic) normed space, it follows that

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} p \left(\begin{aligned} & (a^{2k})^n f(a^{-kn}(ax - y)) \pm a(a^{2k})^n f(a^{-kn}(x \pm y)) \\ & - (a \pm 1)[a(a^{2k})^n f(a^{-kn}x) \pm (a^{2k})^n f(a^{-kn}y)], t \end{aligned} \right) \\
 & = p(Q(ax - y) \pm aQ(x \pm y) - (a \pm 1)(aQ(x) \pm Q(y)), t)
 \end{aligned}$$

for almost all $t > 0$. On the other hand, replacing x, y by $a^{-kn}x, a^{-kn}y$ in equations (4.14)

and (4.15), we get.

$$\begin{aligned}
 & p \left(\begin{aligned} & (a^{2k})^n f(a^{-kn}(ax - y)) \pm a(a^{2k})^n f(a^{-kn}(x \pm y)) \\ & - (a \pm 1)[a(a^{2k})^n f(a^{-kn}x) \pm (a^{2k})^n f(a^{-kn}y)], t \end{aligned} \right) \\
 & \geq_l \psi \left(a^{-kn}x, a^{-kn}x, \frac{t}{|a^{2k}|^n} \right), \forall x \in X, t > 0 \\
 & \geq_l \psi \left(x, y, \frac{\alpha^n t}{|a^k|^n} \right),
 \end{aligned}$$

since $\lim_{n \rightarrow \infty} \psi(x, y, \frac{\alpha^n t}{|a^k|^n}) = 1_l$, we infer that Q is a quadratic mapping.

For the uniqueness of Q , let $Q' : X \rightarrow Y$ be another quadratic mapping such that

$$p(Q'(x) - f(x), t) \geq_l M(x, t),$$

for all $x \in X, t > 0$. Then we have, for all $x, y \in X, t > 0$,

$$\begin{aligned}
 & p(Q(x) - Q'(x), t) \\
 & \geq_l T \left(\begin{aligned} & p(Q(x) - (a^{2k})^n f(\frac{x}{(a^k)^n}), t), \\ & p((a^{2k})^n f(\frac{x}{(a^k)^n}) - Q'(x), t) \end{aligned} \right).
 \end{aligned}$$

Therefore, from (4.19), we conclude that $Q = Q'$. This completes the proof.

Corollary Let K be a non-Archimedean field, X a vector space over K and (Y, p, τ) a non-Archimedean ℓ -fuzzy Banach space over K under a t -norm $\tau \in H$. Let $f : X \rightarrow Y$ be a ψ -approximately quadratic mapping and $f(0) = 0$. If there exist an $\alpha \in R(\alpha > 0)$ and an integer $k, k \geq 2$ with $|a^k| < \alpha, |a| \neq 1$ and $a \neq 0$ such that

$$\psi(a^{-k}x, a^{-k}y, t) \geq_{\ell} \psi(x, y, \alpha t) \quad \forall x, y \in X, t > 0$$

then there exists a unique quadratic mapping $Q : X \rightarrow Y$ such that

$$p(f(x) - Q(x), t) \geq_{\ell} \tau_{j=1}^{\infty} M \left(x, \frac{\alpha^{i+1}t}{|a|^{ki}} \right) \quad \forall x \in X, t > 0$$

Where,

$$M(x, t) = \tau(\psi(x, 0, t), \psi(ax, 0, t) \cdots \psi(a^{k-1}x, 0, t)) \quad \forall x \in X, t > 0.$$

Proof. Since

$$\lim_{n \rightarrow \infty} M \left(x, \frac{\alpha^j t}{|a|^{kj}} \right) = 1_{\ell}, \quad \forall x \in X, t > 0$$

it follows that

$$\lim_{n \rightarrow \infty} T_{j=n}^{\infty} M \left(x, \frac{\alpha^j t}{|a|^{kj}} \right) = 1_{\ell}, \quad \forall x \in X, t > 0.$$

Now, if we apply Theorem 4.3.2, we get the conclusion.

Now, author gives an example to validate the main result as follows:

Example 4.3.4 Let (X, P, T) be a non-Archimedean Banachspace, $(X, \rho_{\mu, \nu}, \tau_m)$ a non-Archimedean ℓ -fuzzy normed space (intuitionistic fuzzy normed space) in which

$$p_{\mu, \nu}(x, t) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|} \right)$$

For all $x \in X, t > 0$ and let $(Y, \rho_{\mu, \nu}, \tau_m)$ be a complete non-Archimedean ℓ -fuzzy normed space (intuitionistic fuzzy normed space) (see Example 4.1.4). Define

$$\psi(x, y, t) = \left(\frac{t}{1+t}, \frac{1}{1+t} \right)$$

It is easy to show that (4.15) holds for $\alpha = 1$ (note that $|a| \neq 1, a \neq 0$). Also, since

$$M(x, t) = \left(\frac{t}{1+t}, \frac{1}{1+t} \right)$$

we have

$$\begin{aligned} \lim_{n \rightarrow \infty} T_{j=n}^{\infty} M \left(x, \frac{\alpha^j t}{|a|^{kj}} \right) &= \lim_{n \rightarrow \infty} \left(\lim_{m \rightarrow \infty} T_{Mj=n}^m M \left(x, \frac{t}{|a|^{kj}} \right) \right) \\ &= \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} \left(\frac{t}{t + |a^k|^n}, \frac{|a^k|^n}{t + |a^k|^n} \right) \\ &= (1, 0) = 1_{\ell}, \quad \forall x \in X, t > 0. \end{aligned}$$

Let $f : X \rightarrow Y$ be a ψ -approximately quadratic mapping. Therefore, all the conditions of Theorem 4.3.2 hold and so there exists a unique quadratic mapping $Q : X \rightarrow Y$ such that

$$p_{\mu, \nu}(f(x) - Q(x), t) \geq t \cdot \left(\frac{t}{t + |a^k|}, \frac{|a^k|}{t + |a^k|} \right)$$

QUADRATIC-CUBIC FES IN PARANORMED SPACE

The FEs

$$f(2x + y) + f(2x - y) = 2 f(x + y) + 2 f(x - y) + 12 f(x)$$

and

$$f(2x + y) + f(2x - y) = 4 f(x + y) + 4 f(x - y) + 24 f(x) - 6 f(y)$$

are called cubic and quartic FEs because the functions $f(x) = cx^3$ and $f(x) = cx^4$ satisfy the equations (4.1) and (4.2), respectively. John Michael Rassias and Kim (78) achieved the general solution and proved the comprehensive HUR permanence for comprehensive additive FEs in the setting of quasi- β -normed spaces. Eshaghi Gordji and Savadkouhi (27) solved for comprehensive key and studied the Ulampermanance outcomes of an assorted type additive-quadratic-cubic FE of the type

$$f(x + 3y) + f(x - 3y) = 9 f(x + y) + f(x - y) - 16 f(x)$$

in random normed spaces. Park (71) confirmed HU permanence of an additivequadratic-cubic-quartic FE

$$g(x + 2y) + g(x - 2y) = 4g(x + y) + 4g(x - y) - 6g(x) + g(2y) + g(-2y) - 4g(y) - 4g(-y)$$

In this study, we introduce the ensuing FEs in paranormed spaces:

$$2\alpha^2 \left[f\left(\frac{u+v}{\alpha}\right) + f\left(\frac{u-v}{\alpha}\right) \right] = (1 + \alpha)[f(u+v) + f(u-v)] \\ + (1 - \alpha)[f(-u+v) + f(-u-v)] \quad (4.5)$$

with $\alpha \neq 0, \pm 1$, and

$$2\alpha^2 \left[f\left(\frac{u+v}{\alpha}\right) + f\left(\frac{u-v}{\alpha}\right) \right] = \left(1 + \frac{1}{\alpha}\right) [f(u+v) + f(u-v)] \\ + \left(1 - \frac{1}{\alpha}\right) [f(-u+v) + f(-u-v)] \quad (4.6)$$

where $\alpha \neq 0, \pm 1, \pm i$. The FEs (4.5) and (4.6) are assorted additive-quadratic and assorted quadratic-cubic FEs, respectively. We solve the FEs (4.5) and (4.6) for their comprehensive keys and investigate their HU permanence in the setting of paranormed space.

CONCLUSION

The possibility of Menger was to utilize circulation function rather than nonnegative genuine numbers as estimations of the measurement. It relates to the circumstance when we don't know precisely the separation between two focuses, we know just probabilities of potential estimations of this separation. The probabilistic speculation of metric spaces seems, by all accounts, to be all around adjusted for the examination of quantum molecule material science particularly in associations with both string and ∞ theory which were given and concentrated contemplated the stability of the cubic functional equation in Mengerprobabilistic normed spaces. Right now, have settled different types of blended kind functional equations for their general arrangement. We have likewise researched summed up Hyers-Ulam-Rassias stability (or Gavruta stability including a general control function as upper bound) of blended sort functional equations, for example, added substance quadratic functional equation in semi β -normed spaces, quadratic-cubic functional equation in paranormed spaces, additive-quartic functional equation in semi Banach spaces and non-Archimedean spaces and added substance quadraticcubic-quartic functional equations in Banach spaces. Functional equations speak to an elective method for displaying issues in Physics. The enthusiasm of displaying physical issues by functional equations is that we don't need to accept the differentiability of the function f . Thus, the functional equations lead regularly to different arrangements than those given by fractional differential equations, and these different arrangements can hold any importance with physicists. The most engaging attribute of functional equation is its ability to plan numerical models. Functional equations are a generally disagreeable zone of mathematics. This isn't because of an absence of significance: Extending linear polynomial math which manages linear functions, functional variable based math covers a substantially more broad area. Typically dynamical frameworks are depicted by differential equations in a constant time space. For discrete time frameworks then again, the elements is characterized by a distinction equation or an iterated map. Building a direction or deciding different properties of the framework requires managing functional equations. As opposed to differential math or linear variable based math, functional equations are seldom utilized to take care of reasonable issues. This might be because of the specialized challenges of the functional analytics.

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