RESEARCH REVIEW JOURNALS
Volume-03
Issue-12
December -2018
ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

Fractional Order Heat Equation in Higher Space-Time Dimensions

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ARTICLE DETAILS

Article History
Published Online: 10 December 2018

Keywords
Fractional Calculus, Fractional Solutions, Fractional Heat Equation

ABSTRACT
In this paper, we study fragmentary request heat condition in higher space-time measurements and offer explicit job of warmth streams in different partial measurements. We offer partial arrangements of the warmth conditions therefore acquired, and look at the related ramifications in different constraining cases. We foresee point of view uses of fragmentary warmth stream arrangements in physical frameworks.

1. Introduction

The incomplete differential conditions whose fragmentary arrangements we consider in this paper are the warmth conditions. This condition assumes a significant job in creating laws of material science while concentrating this present reality wonders happening at a given scale, including both the naturally visible and minute portrayals. So as to concentrate on the discerning request augmentation of the answers for conventional and halfway differential conditions, we first focus on the infinitesimal versus plainly visible inspirations undermining the De Broglie thought of the wave molecule duality.

1.1 Heat Equation:

The warmth condition is an allegorical incomplete differential condition that portrays the dissemination of warmth, to be specific, the varieties in the temperature in a given spatial locale after some time. Given a temperature profile T (x, t), the warmth condition in one spatial measurement [1] is given by

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial x^2},$$

whereK is a positive constant defining the thermal diffusivity.

The warmth condition could be gotten from Fourier law and preservation of the vitality [1]. In reality, it merits referencing that the warmth condition is of essential significance in various logical fields running from material science, science and arithmetic. For example, in the hypothesis of differential conditions, it is a model of explanatory incomplete differential conditions [2]. Essentially, in the likelihood hypothesis, the warmth condition is helpful in the investigation of Brownian movement through the Fokker-Planck condition [3]. The significance of the warmth condition doesn't stop here. In any case, it proceeds in different spaces, too. For instance, in the money related science, the warmth condition could be utilized to comprehend the Black-Scholes halfway differential condition [4]. Physically, the second law of thermodynamics [5] guarantees that the warmth streams from a more sizzling locale to colder areas.

In this paper, the above condition is broke down with regards to the partial analytics, empowering one to think about the warmth stream with better subtleties; in particular, we can get the separation of the temperature profile between given two numbers. The above examination empowers us in high-helping the significance of partial request heat conditions, where the progression sizes of the factors undermining the temperature profile is considered over sound/genuine numbers.

In the spin-off, we audit basics of the partial analytics in segment 2. In area 3, as one of the principle topic of the present paper, we centeraround the definition of fragmentary request halfway differential conditions, to be specific, the partial warmth condition. In area 4, we offer individual fragmentary answers for the previously mentioned partial differential conditions hence created and their separate summed up arrangements utilizing partial request subordinates. At last, in area 5, we present ends and open issues for future research.

2. Review on fractional calculus

In this area, we wish reviewing nuts and bolts of fragmentary analytics. Verifiably, the partial analytics is a term utilized for the hypothesis of subsidiaries and integrals of self-assertive requests over a sound/genuine number, which sums up the thought of customary whole number request separations and - crease combinations, see Ref. [6] for a survey on partial math. The focal motivation behind the partial analytics is to sum up the standard meanings of separation and mix with an essential request $n \in \mathbb{N}$ to a genuine request $s \in \mathbb{R}$. The main talk on the fragmentary math started as before as 1695 of every a letter of Leibniz to L'Hopital tending to about the analytics of a discretionary request. Up to this point, the fragmentary math is of age three centuries. Among the others, Abel, Liouville, Riemann, Euler, and Caputo set out the establishments of the partial math [7, 8]. In this manner, it merits referencing that the partial analytics finds different research applications in unadulterated sciences, money related arithmetic, applied science, and designing and innovation, e.g., see Ref. [9] for deterministic partial models in bioengineering and nanotechnology.

Further, it merits referencing that the subordinates of non-number requests are significantly seen in depicting physical and concoction properties of different genuine materials including polymer, rocks and different conditions of issue [10]. With respect to as the progression size is concerned, the fragmentary request models were discovered increasingly legitimate in uncovering the undermining properties lying between the picked two whole numbers than the relating exchanges rendering from the separate whole number request models. In this paper, we centeraround fragmentary request halfway differential conditions and their partial arrangements. There are various meanings of fragmentary subsidiary and partial integrals, see Refs. [11, 12] and references in that. Some of meanings of the fragmentary subsidiary utilized in the later areas are enrolled as beneath.

2.1 Grunwald-Letnikov Fractional Derivative:

For a given genuine esteemed capacity f (t) and a discerning number, the Grunwald-Letnikov meaning of the - th request partial subsidiary of f (t) as for is given by

$$ {}_{a}D_{t}^{p}f(t) = \lim_{\substack{h\to 0\\ nh=t-a}} h^{-p}\sum_{r=0}^{n}(-1)^{r}\binom{p}{r}f(t-rh), $$

where$h$ is the step size and$a$ is a fixed real number.

2.2 Riemann-Liouville Derivative:

Considering the previously mentioned genuine esteemed capacity f (t), the Riemann-Liouville meaning of the - th request fragmentary subordinate concerning is given by

$$ {}_{a}D_{t}^{p}f(t) = \left(\frac{d}{dt}\right)^{m+1}\int_{a}^{t}(t-\tau)^{m-p}f(\tau)d\tau,\ \ m \le p < m.+1 $$

2.3 Caputo's Fractional Derivative:

In the above line of the thought, the Caputo's definition of ∝-th order fractional derivative of$f$: ℝ → ℝ reads as

$$ {}^{c}D^{\alpha}f(x) = \frac{1}{\Gamma(\alpha-n)}\int_{a}^{x}\frac{f^{(n)}(u)}{(x-u)^{(\propto-n+1)}}\ ,\ n-1 < \propto < n\ ,\propto \in \mathbb{R}\ , n \in \mathbb{N}. $$

2.4 Euler's Fractional Derivative:

Specifically, keeping the number request standard subordinate of a monomial, the Euler's ∝-th request fragmentary subsidiary of a monomial, viz. f (t) = tβ takes the accompanying structure.

$$ \frac{d^{\propto}}{dt^{\propto}}\left[t^{\beta}\right] = D_{t}^{\alpha}\left[t^{\beta}\right] = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}t^{\beta-\alpha}, \alpha \in \mathbb{R} $$

whereΓ(r) is the standard Gamma function for a given r∈ ℝ.

In this paper, we will concentrate on the Euler's meaning of partial subordinates while understanding the previously mentioned fragmentary differential conditions. It is basic to see that partial subordinates fulfill practically every one of the properties that hold for conventional subsidiaries with necessary requests. Following the general properties of essential request subordinate administrator: ,∈ℕ, we outline beneath some clearly irrefutable properties [10] concerning the fragmentary request subsidiaries for a given pair of genuine esteemed capacities f , g : ℝ → ℝ as under

- $D_t^{\propto}[f(t)g(t)] = \sum_{k=0}^{\infty}\binom{\propto}{k} D_t^{\propto-k}[f(t)]D_t^{k}[g(t)]$, where $\binom{\propto}{k} = \frac{\Gamma(\propto+1)}{\Gamma(k+1)\Gamma(\propto+1-k)}$.
- $D_t^{\propto}[f(t)C] = \sum_{k=0}^{\infty}\binom{\propto}{k} D_t^{\propto-k}[f(t)]D_t^{k}[C] = D_t^{\propto}[f(t)]C$ where C is an arbitrary constant.
- $D_t^{\propto}[h(t)+g(t)] = \sum_{k=0}^{\infty}\binom{\propto}{k} D_t^{\propto-k}[t^0]D_t^{k}[h(t)+g(t)] = D_t^{\propto}[h(t)]+D_t^{\propto}[g(t)]$.
- $D_t^{\propto}[h(at)] = a^{\propto}D_x^{\propto}[h(x)]$ under the scaling $x = at$.
- $D_t^{\propto}[t^{-m}] = (-1)^{\propto}\frac{\Gamma(m+\propto)}{\Gamma(m)}t^{-(m+\propto)}$ for a given $m \in \mathbb{R}$.
- $D_t^{\mu+\nu}[f(t)] = D_t^{\mu}[D_t^{\nu}(f(t))] = D_t^{\nu}[D_t^{\mu}(f(t))]$ under the composition of D_t^{ν} and D_t^{μ} on $f(t)$.
- $D_t^{-1}[t^{\beta}] = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1+1)}t^{\beta+1} = \frac{t^{\beta+1}}{\beta+1}$, where $\beta \in \mathbb{R}$ is corresponding to a negative order derivative.

3. Formulation Of Fractional Order Partial Differential Equation In Higher Dimension

In this segment, we figure the partial request fragmentary warmth condition in higher measurements expanding the particular necessary request plans, see Ref. [1] for an essential survey on the above class of second request incomplete differential condition.

3.1 Fractional Heat Equation in Higher Dimensions:

To define a partial variant of the warmth condition, we should consider two proclamations as in the standard case, see Ref. [1] for example, to be specific, the way that the warmth streams toward diminishing temperature and the rate at which the vitality as warmth is moved through a region is relative to the zone and the fragmentary request temperature slope ordinary to the picked territory. In this way, it pursues that the warmth transition through the zone, which is ordinary to the - pivot is given by

$$Q = -kA\frac{\partial^r T}{\partial x^r},$$

where r is a fragmentary number lying somewhere in the range of 0 and 1, viz. we have 0 ≤ r ≤ 1. Therefore, the vitality picked up or lost by a group of mass that experiences in uniform partial temperature change Δ might be communicated as

$$\Delta^r E = cm\Delta^r T = c\rho\Delta^r x\Delta^r y\Delta^r z\Delta^r T$$

where the total mass mcontained in the fractionalregion $(\Delta^r x, \Delta^r y, \Delta^r z)$ is given by m=$\rho\Delta^r x\Delta^r y\Delta^r z$ for a given volume density ρ.Thus, the energy flowing into the element through the face in XZ-plane in $\Delta^r t$ is

$$\Delta^r E_{XZ} = Q_{XZ}\Delta^r t = -k\Delta^r x\Delta^r z\Delta^r t\,\frac{\partial^r T}{\partial y^r}\Big|_{x+\frac{\Delta^r x}{2},\ y,\ z+\frac{\Delta^r z}{2}}$$

Similar expressions could indeed be obtained for the fractional order changes in energy from other planes, and thus the total fractional change of order in the total energy $\Delta^r E$ can readily be evaluated. With this consideration, it hereby follows that we have

$$c\rho\Delta^r x\Delta^r y\Delta^r z\Delta^r T =$$
$$k\Delta^r x\Delta^r z\Delta^r t\left(\frac{\partial^r T}{\partial y^r}\Big|_{x+\frac{\Delta^r x}{2},\ y+\Delta^r y,\ z+\frac{\Delta^r z}{2}} -\right.$$
$$\left.\frac{\partial^r T}{\partial y^r}\Big|_{x+\frac{\Delta^r x}{2},\ y,\ z+\frac{\Delta^r z}{2}}\right) + \dots\dots$$

In the limit of $\Delta^r x \to 0,\ \Delta^r y \to 0,\ \Delta^r t \to 0,$ the above equation reduces as

$$\frac{\partial^r T}{\partial t^r} = \frac{k}{c\rho}\lim_{\Delta^r y\to o}\left[\frac{\frac{\partial^r T}{\partial y^r} - \frac{\partial^r T}{\partial y^r}}{\Delta^r y}\right] + \cdots$$

Herewith, the factional contributions resulting from various faces of the element in (3 + 1) dimensions along (x, y,z) spatial directions give the following fractional heat equation

$$\frac{\partial^r T}{\partial t^r} = \frac{k}{c\rho}\nabla^{2r} T,$$

where∇^{2r} is the fraction Laplacian operator defined as

$$\nabla^{2r} = \frac{\partial^{2r}}{\partial x^{2r}} + \frac{\partial^{2r}}{\partial y^{2r}} + \frac{\partial^{2r}}{\partial z^{2r}}.$$

4. Fractional Solutions

In this area, we offer answers for fragmentary request differential conditions in this manner defined in the past segment, specifically, the partial warmth condition in higher measurements by following the arrangement of the fragmentary math.

4.1 Solution of Fractional Heat Equation In Higher Dimensions:

For the fractional heat equation as brought out into the consideration in section 3.1, namely, for a given thermal diffusivity, it follows that we have

$$\frac{\partial^r T}{\partial t^r} = K\left[\frac{\partial^{2r} T}{\partial x^{2r}} + \frac{\partial^{2r} T}{\partial y^{2r}}\right] \text{ and}$$

$$\frac{\partial^r T}{\partial t^r} = K\left[\frac{\partial^{2r} T}{\partial x^{2r}} + \frac{\partial^{2r} T}{\partial y^{2r}} + \frac{\partial^{2r} T}{\partial z^{2r}}\right]$$

in two and three spatial measurements separately. Herewith, by taking various estimations of , we find that the summed up answer for the above fragmentary warmth condition can be communicated as:

$$T_r(x,y,t) = \frac{1}{\left(\frac{Px^{2r}}{2r}+c_1\right)^{\frac{1}{n_1 r}}}\cdot\frac{1}{\left(\frac{Py^{2r}}{2r}+c_2\right)^{\frac{1}{n_1 r}}}\cdot r\,\frac{1}{\left(\frac{\lambda t^r}{r}+c_3\right)^{\frac{1}{n_2 r}}}$$

and

$$T_r(x,y,z,t) = \frac{1}{\left(\frac{Px^{2r}}{2r}+c_1\right)^{\frac{1}{n_1 r}}}\cdot\frac{1}{\left(\frac{Py^{2r}}{2r}+c_2\right)^{\frac{1}{n_1 r}}}\cdot\frac{1}{\left(\frac{Pz^{2r}}{2r}+c_3\right)^{\frac{1}{n_1 r}}}\cdot\frac{1}{\left(\frac{Pt^r}{r}+c_4\right)^{\frac{1}{n_2 r}}},$$

Where $n_1 r = (1-2r),\ n_2 r = (1-r),\ p = \frac{\lambda}{K}$ { C_1, C_2, C_3, C_4} are arbitrary constants. It turns out thatthe parameter λ could further be expressed in termsof K, wherefore rendering {C_1, C_2, C_3, C_4} as solely independent integration constants.

5. Conclusion

In this paper, we consider plan of fragmentary request heat conditions in higher measurements and offer the comparing summed up partial arrangements. The arrangements concerning fragmentary warmth conditions as brought out in this paper are foreseen to be of a model thought towards point of view improvements of the partial thermodynamics, temperature profiles with fragmentary request inclinations, partial liquid elements, and Schwartz disseminations [13] by considering partial expansions of the standard elliptic, illustrative and hyperbolic incomplete differential conditions. We leave such contemplations open for a future research.

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