A Study of Mixed Convection MHD and Oscillatory MHD Convective Flow of a Viscoelastic Fluid

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ABSTRACT

The magneto hydrodynamic (MHD) blended convection stream of an electrically directing, viscoelastic and incompressible liquid through a permeable medium filled in a vertical permeable channel is investigated. The liquid is infused into the channel with a steady speed through one of the plates and at the same time evacuated through the different permeable plate of the channel. The stream is produced by an occasional weight angle fluctuating with time. An attractive field of uniform quality is additionally connected opposite to the plates. Shut structure arrangement of the issue is acquired for the speed, temperature, skin rubbing and the rate of warmth move as far as their sufficiency and stage. The magneto hydro dynamic (MHD) convection courses through a permeable medium are of essential enthusiasm attributable to their applications in geophysics, astronomy, and building especially in the plan of underground water vitality stockpiling frameworks, oil extraction, geothermal vitality recuperation, soil sciences, and so on. The learning of move through permeable media is valuable in the recuperation of raw petroleum proficiently from the pores of repository shakes by uprooting with immiscible water.

1. Introduction

The wide scope of mechanical and innovation utilizations of these streams has pulled in the consideration of a substantial number of researchers. El-Hakiem (2000) has examined the magneto hydro dynamic (MHD) oscillatory free convection course through a permeable medium with consistent suction speed within the sight of warmth radiation. Gholizadeh (1990) examined the MHD oscillatory stream past a vertical permeable plate through a permeable medium within the sight of warm and mass dispersion with the consistent warmth source. Aldoss et al. (1995) considered magneto hydro dynamic blended convection from a vertical plate inserted in a permeable medium. Makinde and Mhone (2005) have investigated the MHD oscillatory stream of a thick liquid in a planer channel loaded up with permeable medium. Alagoa et al. (1999) considered the issue of radiative and free convection impacts of move through the permeable medium between two unbounded parallel plates with time-subordinate suction. Mebine (2007) examined warm radiation impact on MHD Couette stream with warmth exchange between two parallel plates. Thinking about the occasional divider temperature, Israel-Cookey et al. (2010) explored MHD oscillatory Couette stream of a transmitting thick liquid in a permeable medium. Singh (2011) broke down an oscillatory convective course through a permeable medium limited by two vertical permeable plates. Attia and Ewis (2010) broke down the flimsy magneto hydro dynamic Couette stream of an electrically directing incompressible non-Newtonian viscoelastic liquid between two parallel flat non-leading permeable plates with warmth exchange The stream of visco versatile liquids through permeable media is one more marvel which has pulled in the consideration of a substantial number of researchers and designers in view of its significance in the stream of oil through permeable rocks, the extraction of vitality from geothermal areas, the filtration of solids from fluids and medication pervasion through human skin. Ariel (1994) acquired careful arrangements of stream issues of a second-grade liquid through two parallel dividers. Rajagopal et al. (2006) considered the oscillatory movement of an electrically directing viscoelastic liquid over an extending sheet in a soaked permeable medium. Gupta and Sridhar (1985) examined the viscoelastic impacts of non-Newtonian course through a permeable medium. Petrov (2000) logically analyzed the temperamental stream of Bingham liquid brought about by the suddenly connected weight inclination. Considering the consistent weight inclination Sarpkaya (1961) talked about the relentless stream of a consistently leading non-Newtonian incompressible liquid between two parallel plates. Prasuna et al. (2010) analyzed a shaky stream of a viscoelastic liquid through a permeable medium between two impermeable parallel plates. The issue is examined for two phases (I) when the weight inclination is connected to achieve the relentless state and (ii) at that point the weight slope is all of a sudden pulled back. As of late Choudhury and Das (2012) considered warmth exchange to MHD oscillatory viscoelastic stream in a channel with impermeable dividers loaded up with permeable medium. Singh (2013) dissected MHD blended convection visco-versatile slip-move through a permeable medium in a vertical permeable channel with warm radiation. Singh (2013) has additionally examined visco-flexible MHD convective occasional course through a permeable medium in a pivoting vertical channel with warm radiation. The point of the present investigation is to consider the impact of infusion/suction on a flimsy blended convection stream of a viscoelastic, incompressible and electrically leading liquid through a permeable medium in a vertical permeable channel. The stream stays oscillatory because of the intermittent weight angle considered. The attractive field of uniform quality is connected toward the path ordinary to the plates. Likewise, the non-uniform temperature distinction of the plates is sufficiently high to initiate heat because of radiation.

2. Review of literature

Charles et. al. [2010] analyzed the viscoelastic fluid stream and warmth trade traits over a broadening sheet with frictional warming and inward warmth age/ingestion. The force condition is decoupled from the vitality condition for the present incompressible limit layer stream issue with consistent physical parameters. Positive answers for the speed field and the skin disintegration are procured. Moreover the answers for temperature and warmth trade characteristics are procured similar to Kummer's capacity.

Kelly et. al. [2009] investigated the direct of the warmth and mass trade characteristics of an incompressible and electrically driving viscoelastic fluid past a dimension versatile sheet. Expository course of action of the resulting straight non-homogeneous limit esteem issues, imparted the extent that Kummer's capacities, were presented for the occurrence of PST similarly as PHF, the two of which are believed to be elements of separation. They in like manner considered the asymptotic farthest reaches of the response for little and sweeping Prandtl numbers.

Hayat et.al. [2008], inquired about the MHD slip stream of a second grade fluid with warmth trade examination. The stream in porous space is a direct result of an expanding sheet which in like manner shows slip condition. The non-direct limit condition developing utilizing slip condition is considered. Rehash conditions in the course of action answers for speed and temperature are shown. The effects of various stream controlling parameters on the dimensionless speed and temperature are addressed graphically. It is found that the even piece of speed augments as the slip parameter increases.

Ghosh and Beg (2008). Hypothetical examination of radiative effects on transient free convection heat trade past a hot vertical surface in the penetrable media was discussed.

Srinivasacharya et al. (2008) inspected the effect of divider properties on peristaltic transport of a dusty fluid under the long wavelength gauge. Bother arrangements were gotten for the stream elements of both fluid particles and solid particles, to the extent the divider incline parameter. The verbalization for ordinary speed of the fluid particles and the solid particles, and the typical fluid stream rate were derived. The effect of various versatile parameters and mass centralization of the buildup particles on the streamline structure and the ordinary stream rate were examined. The wonder of getting was viewed and the zone of the got bolus extended close by the weight parameter, yet lessened with damping and mass union of buildup particles.

Chiam (2007) showed an answer of the vitality condition for the limit layer stream of an electrically driving fluid influenced by a consistent transverse alluring field (suction/blowing) over a straightly expanding non-isothermal dimension sheet. Effects in light of dispersing, stress work and warmth age were considered. Investigative course of action of the resulting straight nonhomogeneous limit esteem issue, imparted the extent that Kummer's capacities, were shown for the cae of PST similarly as PHF, the two of which were believed to be quadratic elements of separation.

Kumari and Nath(=2009) considered the effect of the appealing field on the stagnation point stream and warmth trade of a thick electrically conduction fluid on a straightly broadening sheet, when the speed of the sheet and the free stream speed are not comparable. The issue may be seen as a mix of two issues, to be explicit, two-dimensional stagnation-point stream and stream over an expanding sheet in a surrounding fluid. Exact arrangements of the Navier-Stokes condition were gotten.

Layek et al[2011] considered the examination of two-dimensional stagnation point stream of an incompressible thick fluid towards a penetrable broadening surface embedded in a porous medium subject to suction/blowing with interior warmth age or maintenance. The development of this examination is to explore the effect of suction/blowing on the control of stream parcel similarly as warmth trade and moreover to inquire about the effects of warmth source or sink parameter on hear trade. The energy and warm limit layer conditions are settled numerically using shooting strategy.

AnuarIshak et .al[2009] considered the predictable two-dimensional MHD stagnation point stream towards a broadening sheet with variable surface temperature. In this paper the overseeing game plan of mostly differential conditions are moved into standard differential conditions, which are comprehended numerically using a restricted qualification contrive known as the Keller-box strategy. The effects of the administering parameters on the stream field and warmth trade qualities are gotten.

Mahapatraet.al[2009] pondered the Analytical course of action of magneto hydrodynamic stagnation point stream of a power-law fluid towards an expanding sheet. Here the regulating conditions of the stream warmth and mass trade are clarified by Homotopy Analysis Method and pondered effects of each and every directing parameter on stream, warmth and mass trade.

Kuznetsov and Nield [2010] have considered the standard convection limit layer stream, warmth and mass trade of Nano fluid due to past a vertical plate. Here the regulating conditions of the stream warmth and mass trade are handled by expository technique and considered effects of each directing parameter on regular convection limit layer stream, warmth and mass trade.

3. Mathematical Analysis

An insecure occasional stream of a viscoelastic, incompressible and electrically leading liquid through a permeable medium in a vertical channel is considered. The two unbounded plates of the vertical channel remove'd' separated are permeable, and the liquid is infused through one of the plates and at the same time sucked through the other with a similar speed. A Cartesian organize framework is expected with the end goal that the X*-hub lies vertically upwards toward the lightness constrain along the centerline of the channel and Y*-hub is opposite to the parallel plates. A transverse attractive field of uniform quality B is connected along Y*-pivot. The attractive Reynolds number is accepted exceptionally little with the goal that the instigated attractive field is immaterial. The temperature of one of the plates is non-uniform and wavers occasionally. The physical issue is schematically exhibited in Fig. 1. All the physical amounts are free of x* for this issue of completely created laminar stream. Every single liquid property are expected consistent aside from that the impact of thickness variety with temperature is viewed as just in the body drive term. In this manner, under the typical Boussinesq guess the stream of radiative liquid is represented by the accompanying conditions:

$$\frac{\partial v^*}{\partial y^*} = 0,$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial y^{*2}} + \vartheta_2 \frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta_1}{K^*} u^* + g\beta T^*$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q^*}{\partial y^*},$$

where t* is the time, p* is the pressure, is the fluid density, is the kinematic viscosity, is the viscoelasticity of the fluid, K* is the permeability of the porous medium, g is the acceleration due to gravity, is the coefficient of volume expantion, k is the thermal conductivity, cp is the specific heat at constant pressure.

Following Cogley et al. (1968) it is assumed that the fluid is optically thin with relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2 T^*,$$

where is the mean radiation absorption coefficient.

The boundary conditions of the problem are

$$u^* = 0, \quad v^* = V, \quad T^* = T_0 \cos\omega^* t^* \quad at \quad y^* = \frac{d}{2},$$

$$u^* = 0, \quad v^* = V, \quad T^* = 0 \quad at \quad y^* = -\frac{d}{2},$$

where is the frequency of oscillations. For the oscillatory internal flow in the channel, the periodic pressure gradient variations are assumed to be of the form

$$y^* = -\frac{d}{2}, u^* = 0, v^* = V, \; T^* = 0.$$

$$y^* = \frac{d}{2}, u^* = 0, v^* = V, T^* = T_0 \cos\omega^* t^*$$

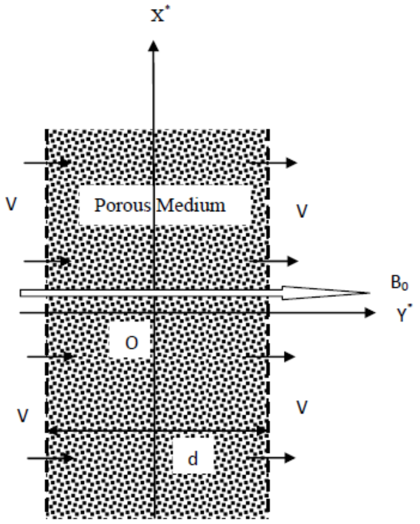

Figure 1 Physical model of the problem with the coordinate system. Because of the assumption of constant injection and suction velocity V at the left and the right plates respectively, continuity equation (3A.1) integrates to Substituting equation (3A.8) and introducing the following non-dimensional quantities

$$x = \frac{x^*}{d}, \; y = \frac{y^*}{d}, \; u = \frac{u^*}{V}, \; T = \frac{T^*}{T_0}, \; t = \frac{t^* V}{d}, \; \omega = \frac{\omega^* d}{V}, \; p = \frac{p^*}{\rho V^2},$$

Into equations (3A.2) and (3A.3), we get

$$\lambda\left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y}\right) = -\lambda\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \gamma\frac{\partial^3 u}{\partial y^2 \partial t} - M^2 u - K^{-1}u + GrT$$

$$\lambda Pr\left(\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} - N^2 T,$$

whereGrashof number $Gr = \frac{g\beta d^2 T_0}{\vartheta_1 V}$

Hartmann number M = $B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$

Injection/suction parameter λ = $\frac{Vd}{\vartheta_1}$

Viscoelastic parameter $\gamma = \frac{\vartheta_2 \lambda}{d^2}$

Permeability of the porous medium K = $\frac{K^*}{d^2}$

Prandtl number Pr = $\frac{\mu c_p}{k}$

Radiation parameter N = $2\alpha \frac{d}{\sqrt{k}}$

The boundary conditions in dimensionless form become

$$u = 0, \quad T = \cos \omega t, \quad at \quad y = \tfrac{1}{2},$$

$$u = 0, \quad T = 0, \quad at \quad y = -\tfrac{1}{2}\ .$$

For the mathematical solution of this unsteady MHD periodic flow in the porous channel when the fluid is also acted upon by a periodic drop in pressure, we assume the solution in complex variable notations as

$$u(y,t) = u_0(y)e^{i\omega t}, \quad T(y,t) = \theta_0(y)e^{i\omega t}, \quad -\frac{\partial p}{\partial x}$$

The real part of the solution will have physical significance. The boundary conditions (3A.12) and (3A.13) can also be written in complex notations as

$$u = 0, \quad T = e^{i\omega t} \quad at \quad y = \tfrac{1}{2},$$

$$u = 0, \quad T = 0 \qquad at \quad y = -\tfrac{1}{2}.$$

Substituting expressions (3A.14) into equations (3A.10) and (3A.11), we obtain following equations:

$$(1 + i\omega\gamma)u_0'' - \lambda u_0' - (M^2 + K^{-1} + i\omega\lambda)u_0 = -\lambda A - Gr\theta_0$$

$$\theta_0'' - \lambda Pr\theta_0' - \left(N^2 + i\omega\lambda Pr\right)\theta_0 = 0$$

where the primes in these ordinary differential equations denote differentiation with respect to y. The boundary conditions (3A.15) and (3A.16) reduce
to

$$u_0 = 0, \quad \theta_0 = 1, \quad at \quad y = \tfrac{1}{2}$$

$$u_0 = 0, \quad \theta_0 = 0, \quad at \quad y = -\tfrac{1}{2}.$$

The solution of equation (3A.17) for the velocity field under the boundary conditions
(3A.19) and (3A.20) is obtained as

$$u(y,t) = \left[\begin{array}{c} \frac{\lambda A}{l}\left\{1 + \frac{e^{my}\sinh\frac{n}{2} - e^{ny}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right\} + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l} \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{my-\frac{n}{2}} - e^{ny-\frac{m}{2}}\right) \\ + \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(e^{my+\frac{n}{2}} - e^{ny+\frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2}} \end{array}\right\} \\ -\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{ry-\frac{s}{2}}}{C_1} - \frac{e^{sy-\frac{r}{2}}}{C_2}\right) \end{array}\right] e^{i\omega t},$$

where $C_1 = (1 + i\omega\gamma)r^2 - \lambda r - l$, $C_2 = (1 + i\omega\gamma)s^2 - \lambda s - l$, $l = M^2 + K^{-1} + i\omega\lambda$,

$$m = \frac{\lambda + \sqrt{\lambda^2 + 4l(1+i\omega\gamma)}}{2(1+i\omega\gamma)}, \quad n = \frac{\lambda - \sqrt{\lambda^2 + 4l(1+i\omega\gamma)}}{2(1+i\omega\gamma)},$$

$$r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}, \quad s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}.$$

Similarly, the solution of equation (3A.18) for the temperature field under the boundary conditions (3A.19) and (3A.20) is obtained as

$$T(y,t) = \left(\frac{e^{ry-\frac{s}{2}} - e^{sy-\frac{r}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}\right)e^{i\omega t}\ .$$

From the velocity field obtained in equation (3A.21) we can get the skin-friction τ at the left plate (y = -0.5) in terms of its amplitude $|F|$ and phase angle φ as $\tau = |F| \cos(t+\varphi)$, with

$$F = F_r + i\,F_i = \left[\begin{array}{c} \frac{\lambda A}{l}\left(\frac{me^{-\frac{m}{2}}\sinh\frac{n}{2} - ne^{-\frac{n}{2}}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right) + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{ \begin{array}{l} \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m-n)e^{-\frac{\lambda}{2(1+i\omega\gamma)}} + \\ \left(\frac{C_1-C_2}{C_1C_2}\right)\left(me^{-\frac{m-n}{2}} - ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2}} \end{array} \right\} \\ -\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1} - \frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2}} \end{array} \right],$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$; and the phase angle $\varphi = \tan^{-1}\frac{F_i}{F_r}$.

Similarity, we can get the Nusselt number Nu, in terms of its amplitude $|H|$ and the phase angle ψ from equation for the temperature field as

$$q = |H| \cos(t+\psi) ,$$

With $H = Hr + i\,Hi = \frac{(r-s)e^{-\frac{\lambda Pr}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}$,

where the amplitude $|H|$and the phase angle β of the rate of heat transfer are given as

$$|H| = \sqrt{Hr^2 + Hi^2},\quad \psi = \tan^{-1}\frac{Hi}{Hr} .$$

The temperature field, amplitude, and phase of the Nusselt number need no further discussion because these have already been discussed in detail by Singh

MHD CONVECTIVE FLOW OF VISCOELASTIC FLUID THROUGH POROUS MEDIUM FILLED IN A ROTATING VERTICAL POROUS

An oscillatory MHD convective stream of incompressible, visco flexible and electrically directing liquid in a vertical permeable channel is examined. An attractive field of uniform quality is connected opposite to the plates of the channel. The attractive Reynolds number is thought to be exceptionally little so the incited attractive field is immaterial. The whole framework pivots around a hub opposite to planes of the plates. The temperature distinction between the plates is sufficiently high to initiate the warmth because of radiation. A shut structure arrangement of the simply oscillatory stream is acquired. The speed, temperature, and the skin-erosion regarding its sufficiency and stage point have been appeared at watch the impacts of turn parameter Ω, suction parameter , Grashof number Gr, Hartmann number M, the weight A, Prandtl number Pr, Radiation parameter N and the recurrence of wavering ω.

Magneto hydro dynamic (MHD) convection streams of electrically directing gooey incompressible liquids in the turning framework have increased extensive consideration in view of its various applications in material science and building. In geophysics, it is connected to quantify and think about the positions and speeds as for a fixed casing of reference on the outside of the earth which pivots concerning an inertial casing within the sight of its attractive field. The subject of geophysical elements these days has turned into a critical part of liquid elements because of the expanding enthusiasm to think about condition. In astronomy, it is connected to consider the excellent and sun oriented structure, interplanetary and interstellar issue, sun powered tempests and flares, and so on. Amid the most recent couple of decades, it likewise discovers its application in building. Among the utilizations of turning stream in permeable media to designing controls, one can discover the nourishment preparing industry, concoction process industry, centrifugation and filtration forms and pivoting apparatus.

The goal of the present examination is to consider an oscillatory convection stream of an incompressible, electrically leading and visco-versatile liquid in a vertical permeable channel. Steady infusion and suction is connected at the left and the correct unbounded permeable plates individually. The whole framework pivots around a hub opposite to the planes of the plates, and a uniform attractive field is likewise connected along this hub of revolution. A general accurate arrangement of the halfway differential conditions overseeing the stream issue is acquired, and the impacts of different stream parameters on the speed field and the skin grinding are talked about in the last segment of the paper with the assistance of Figures.

Consider the stream of a visco versatile, incompressible and electrically directing liquid in a pivoting vertical channel. So as to determine the essential conditions for the issue under thought following suppositions are made:

(i) The two infinite vertical parallel plates of the channel are permeable and electrically non-conducting.
(ii) The vertical channel is filled with a porous medium.
(iii) The flow considered is fully developed, laminar and oscillatory.
(iv) The fluid is visco elastic, incompressible and finitely conducting.
(v) All fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term.
(vi) The pressure gradient in the channel oscillates periodically with time.
(vii) A magnetic field of uniform strength B0 is applied perpendicular to the plates of the channel.
(viii) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(ix) Hall effect, electrical and polarization effects are also neglected.
(x) The temperature of a plate is non-uniform and oscillates periodically with time.

(xi) The temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation.

(xii) The fluid is assumed to be optically thin with relatively low density.

(xiii) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.

Under these assumptions we write hydro magnetic governing equations of motion and continuity in a rotating frame of reference as:

$$\nabla.\vec{V} = 0$$

$$\frac{\partial V}{\partial t} + (\vec{V}.\nabla\vec{V}) + 2\vec{\Omega} \times \vec{V} = -\frac{1}{\rho}\nabla p^* + \vartheta_1\nabla^2\vec{V} + \frac{\vartheta_1}{K^*}\vec{V} + \nabla.\exists + \frac{1}{\rho}(\vec{J} \times \vec{B}) + \vec{F}$$

$$\rho c_p\left[\frac{\partial T}{\partial t} + (\vec{V}.\nabla)T\right] = k\nabla^2 T - \nabla q$$

In equation (3B.2) the last term on the left hand side is the Coriolis force. On the right hand side of (3B.2) the last term $\vec{F}(= g\beta T^*)$ accounts for the force due to buoyancy. The second last term is the Lorentz Force due to magnetic field $\vec{B}$ and is given by

$$\vec{J} \times \vec{B} = \sigma(\vec{V} \times \vec{B}) \times \vec{B},$$

and the modified pressure $p^* = p' - \frac{\rho}{2}|\vec{\Omega} \times \vec{R}|^2$, where $\vec{R}$ denotes the position vector from the axis of rotation, p' denotes the fluid pressure, $\vec{J}$ is the current density and all other quantities have their usual meaning and have been defined from time to time in the text.

In the term third from last of equation (3B.2), $\exists$is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll (1960) for an incompressible homogeneous fluid of second order is

$$\exists = -p_1 I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2.$$

Here $-p_1 I$ is the inter determinate part of the stress due to constraint of incompressibility, μ_1, μ_2, μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematics A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla\vec{V}) + (\nabla\vec{V})^T,$$

$$A_2 = \frac{dA_1}{dt} + (\nabla\vec{V})^T A_1 + A_1(\nabla\vec{V}),$$

Where ∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman (1964), the material constants $\mu_1, \mu_3$are taken as positive and μ_2 as negative. The modified pressure $p^* = p' - \frac{\rho}{2}|\vec{\Omega} \times \vec{R}|^2$, where $\vec{R}$ denotes the position vector from the axis of rotation, p' denotes the fluid pressure.

4. Formulation of the problem

In the present analysis, we consider an unsteady flow of a visco elastic incompressible and electrically conducting fluid bounded by two infinite vertical porous plates distance 'd' apart. A coordinate system is chosen such that the X* -axis is oriented upward along the centerline of the channel and Z* -axis taken perpendicular to the planes of the plates lying in $z^* = \pm\frac{d}{2}$ planes. The fluid is injected through the porous plate at $z^* = -\frac{d}{2}$ with constant velocity w0 and simultaneous sucked through the other porous plate at $z^* = +\frac{d}{2}$ with the same velocity w_0. The non-uniform temperature of the plate at $z^* = +\frac{d}{2}$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation. The Z* - axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity Ω* . A transverse magnetic field of uniform strength $\vec{B}(0, 0, B_0)$ is also applied along the axis of rotation.

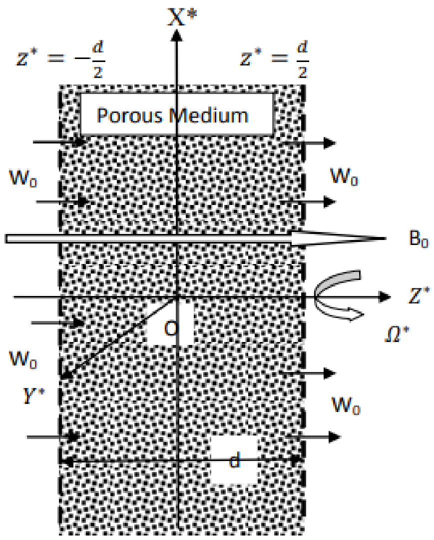

Figure 2 Physical configuration of the flow.

All physical quantities depend on z* and t* only for this problem of fully developed laminar flow. The equation of continuity $\nabla.\vec{V} = 0$ gives on integration w* = w_0. when the velocity may reasonably be assumed with its components along x* , y * , z * directions as $\vec{V}$ (u* , v * , w_0). A schematic diagram of the flow problem is shown in Fig. 2

Following Attia (2005) and under the usual Boussinesq approximation and by using the velocity and the magnetic field distribution as stated above the magneto hydro dynamic (MHD) flow in the rotating channel is governed by the following Cartesian equations:

$$\frac{\partial u^*}{\partial t^*} + w_0\frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1\frac{\partial^2 u^*}{\partial z^{*2}} + \vartheta_2\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho}u^* - \frac{\vartheta_1 u^*}{K^*} + g\beta(T^* - T_1)$$

$$\frac{\partial v^*}{\partial t^*} + w_0\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta_1\frac{\partial^2 v^*}{\partial z^{*2}} + \vartheta_2\frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho}v^* - \frac{\vartheta_1 u^*}{K^*}$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}$$

$$\frac{\partial T^*}{\partial t^*} + w_0\frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho c_p}\frac{\partial q}{\partial z^*}$$

Where p is the density, ϑ_1 is the kinematic viscosity, ϑ_2 is visco elasticity, p* s the modified pressure, t* is the time, σ is the electric conductivity, B_0 is the component of the applied magnetic field along the z* -axis, g is the acceleration due to gravity, k is the thermal conductivity, cp is the specific heat at constant pressure, and the last term in equation (3B.8) is the radiative heat flux.

Following Cogley et al. (1968) it is assumed that the fluid is optically thin with a relatively low density and the heat flux due to radiation in equation (3B.8) is given by

$$\frac{\partial q}{\partial z^*} = 4\alpha^2(T^* - T_1).$$

where∝ is the mean radiation absorption coefficient. After the substitution of equation (3B.9) equation (3B.8) becomes

$$\frac{\partial T^*}{\partial t^*} + w_0\frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{4\alpha^2}{\rho c_p}(T^* - T_1)\ .$$

Equation (3B.7) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the X* -axis is of the form

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = A\cos\omega^* t^* \quad \text{and} \quad -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} = 0, \text{ where A is a constant.}$$

The boundary conditions for the problem are

$$z^* = \frac{d}{2}:\ u^* = v^* = 0,\ T^* = T_1 + (T_2 - T_1)\cos\omega^* t^*$$

$$z^* = -\frac{d}{2}:\ u^* = v^* = 0,\ T^* = T_1$$

where T_0 is the mean temperature and w* is the frequency of oscillations. Introducing the following non-dimensional quantities:

$$\eta = \frac{z^*}{d}, x = \frac{x^*}{d}, y = \frac{y^*}{d},\ u = \frac{u^*}{w_0}, v = \frac{v^*}{w_0},\ T = \frac{T^* - T_1}{T_2 - T_1}, t = \frac{t^* w_0}{d}, \omega = \frac{\omega^* d}{w_0}, p = \frac{p^*}{\rho w_0^2}$$

into equations (3B.5), (3B.6) and (3B.10), we get

$$\lambda\left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \eta}\right) = -\lambda\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + \gamma\frac{\partial^3 u}{\partial \eta^2\partial t} + 2\Omega v - M^2 u - K^{-1}u + Gr\,T$$

$$\lambda\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \eta}\right) = -\lambda\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} + \gamma\frac{\partial^3 v}{\partial \eta^2\partial t} - 2\Omega u - M^2 v - K^{-1}v$$

$$\lambda Pr\left(\frac{\partial T}{\partial t} + \frac{\partial T}{\partial \eta}\right) = \frac{\partial^2 T}{\partial \eta^2} - N^2 T$$

where '*' represents the dimensional physical quantities,

$\lambda = \frac{w_0 d}{\vartheta_1}$ is the injection/suction parameter,

$\gamma = \frac{v_2\lambda}{d^2}$ is the visco-elastic parameter,

$\Omega = \frac{\Omega^* d^2}{\vartheta_1}$ is the rotation parameter,

$M = B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,

$K = \frac{K^*}{d^2}$ is the permeability of the porous medium,

$Gr = \frac{g\beta d^2(T_2 - T_1)}{\vartheta_1 w_0}$ is the Grashof number,

$Pr = \frac{\rho\vartheta_1 c_p}{k}$ is the Prandtl number,

$N = \frac{2\alpha d}{\sqrt{k}}$ is the radiation parameter,

$\omega = \frac{\omega^* d}{w_0}$ is the frequency of oscillations.

The boundary conditions in the dimensionless form become

$$\eta = \tfrac{1}{2}: \quad u = v = 0, \ T = \cos \omega t,$$

$$\eta = -\tfrac{1}{2}: \quad u = v = 0, \ T = 0.$$

For the oscillatory internal flow, we shall assume that the fluid flows under the influence of a non-dimension pressure gradient varying periodically with time in the direction of X-axis only which implies that

$$-\frac{\partial p}{\partial x} = A \cos \omega t \ \text{ and } \ -\frac{\partial p}{\partial y} = 0.$$

SOLUTION OF THE PROBLEM Now combining equations into single equation by introducing a complex function of form F = u + iv and with the help of equation (3B.20), we get

$$\lambda\left(\frac{\partial F}{\partial t} + \frac{\partial F}{\partial \eta}\right) = -\lambda \frac{\partial p}{\partial x} + \frac{\partial^2 F}{\partial \eta^2} + \gamma \frac{\partial^3 F}{\partial \eta^2 \partial t} - (M^2 + K^{-1} + 2i\Omega)F + Gr\,T, \quad (3B.21)$$

with corresponding boundary conditions as

$$\eta = \tfrac{1}{2}: \quad F = 0, \ T = \cos \omega t,$$

$$\eta = -\tfrac{1}{2}: \quad F = 0, \ T = 0.$$

In order to solve equation under boundary conditions it is convenient to adopt complex notations for the velocity, temperature, and the pressure as under:

$$F(\eta, t) = F_0(\eta) e^{i\omega t}, \quad T = \theta_0(\eta) e^{i\omega t}, \quad -\frac{\partial y}{\partial x} = A e^{i\omega t}.$$

The solutions will be obtained in terms of complex notations, the real part of which will have physical significance.
The boundary conditions in complex notations can also be written as

$$\eta = \tfrac{1}{2}: \quad F = 0, \ T = e^{i\omega t},$$

$$\eta = -\tfrac{1}{2}: \quad F = 0, \ T = 0.$$

Substituting expressions in equations we get

$$(1 + i\omega\gamma)\frac{d^2 F_0}{d\eta^2} - \lambda \frac{dF_0}{d\eta} - (M^2 + K^{-1} + 2i\Omega + i\omega\lambda)F_0 = -\lambda A - Gr\,\theta_0$$

$$\frac{d^2\theta_0}{d\eta^2} - \lambda Pr \frac{d\theta_0}{d\eta} - (N^2 + i\omega\lambda Pr)\theta_0 = 0$$

The transformed boundary conditions reduce to

$$\eta = \tfrac{1}{2}: \quad F_0 = 0, \ \theta_0 = 1,$$

$$\eta = -\tfrac{1}{2}: \quad F_0 = 0, \ \theta_0 = 0.$$

The solution of the ordinary differential equation under the boundary conditions gives the velocity field as

$$F(\eta, t) = \left[\begin{array}{c} \frac{\lambda A}{l}\left\{1 + \frac{e^{m\eta}\sinh\frac{n}{2} - e^{n\eta}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right\} + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l} \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{m\eta - \frac{n}{2}} - e^{n\eta - \frac{m}{2}}\right) \\ + \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(e^{m\eta + \frac{n}{2}} - e^{n\eta + \frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2}} \end{array}\right\} \\ -\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{r\eta - \frac{s}{2}}}{C_1} - \frac{e^{s\eta - \frac{r}{2}}}{C_2}\right) \end{array}\right] e^{i\omega t},$$

Where

: $l = (M^2 + K^{-1} + 2i\Omega + i\omega\lambda)$, $C_1 = (1 + i\omega\gamma)r^2 - \lambda r - l$,

$C_2 = (1 + i\omega\gamma)s^2 - \lambda s - l$,

$$m = \frac{\lambda + \sqrt{\lambda^2 + 4l(1 + i\omega\gamma)}}{2}, \qquad n = \frac{\lambda - \sqrt{\lambda^2 + 4l(1 + i\omega\gamma)}}{2},$$

$$r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}, \qquad s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}.$$

Similarly, the solution of equation for the temperature field can be obtained under the boundary conditions as

$$T(\eta, t) = \left(\frac{e^{r\eta - \frac{s}{2}} - e^{s\eta - \frac{r}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}\right) e^{i\omega t}.$$

From the velocity field obtained in equation (3B.31) we can get the skin-friction τ_l at the left plate (η = -0.5) in terms of its amplitude $|F|$ and phase angle φ as

$\tau = |F| \cos(t + \varphi)$, with

$$F = F_r + i\,F_i = \begin{bmatrix} \frac{\lambda A}{l}\left(\frac{me^{-\frac{m}{2}}\sinh\frac{n}{2} - ne^{-\frac{n}{2}}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right) + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{matrix}\left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m-n)e^{-\frac{\lambda}{2(1+i\omega\gamma)}} + \\ \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(me^{-\frac{m-n}{2}} - ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{matrix}\right\} \\ -\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1} - \frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2}} \end{bmatrix}.$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle is $\varphi = \tan^{-1}\frac{F_i}{F_r}$.

Similarly the Nusselt number Nu in terms of its amplitude $|H|$ and the phase angle ψ can be obtained from equation (3B.32) for the temperature field as

$$q = |H|\cos(t + \psi),$$

$$Hr + i\,Hi = \frac{(r-s)e^{-\frac{r+s}{2}}}{2\,\sinh\left(\frac{r-s}{2}\right)},$$

where the amplitude $|H|$nd the phase angle ψ of the rate of heat transfer are given as $|H| = \sqrt{Hr^2 + Hi^2}, \quad \psi = \tan^{-1}\frac{Hi}{Hr}$. The temperature field, amplitude and phase, of the Nusselt number need no further discussion because these have already been discussed in detail by Singh.

5. Conclusion

The examination of the streams of visco versatile liquids is basic in the fields of oil development and in the decontamination of foul oils. Of late, streams of viscoelastic liquids pulled in the thought of a couple of specialists in context on their reasonable and fundamental centrality related with various present day applications. Composing is stacked with the distinctive stream issues considering a variety of geometries analyzed discontinuous insecure streams of a non-Newtonian liquid. It analyzed the oscillatory visco flexible stream in a channel stacked up with the porous medium inside seeing radiative warmth trade. The streams issues of electrically driving liquids are at present getting amazing thought. The magneto hydrodynamic (MHD) stream has various feasible applications, for instance, electromagnetic stream meters, electromagnetic siphons and hydro magnetic generators, etc. The excitement for MHD convective streams with warmth trade is reestablished as a result of its essentialness in the structure of MHD generators and stimulating specialists in geophysics, in underground water and vitality amassing systems. A couple of specialists have exhibited their excitement for considering MHD, and warmth move streams in penetrable and non-porous media. Exactly when the nature of the appealing field is adequate, by then one can't ignore the effects of Hall streams. In spite of the way that it is of great hugeness to look at how the aftereffects of the hydro dynamical issues get balanced by the effects of Hall streams. The Hall and Ion-slip impacts in MHD Couette stream with warmth trademoreover inspected Hall impacts in MHD Couette stream with warmth trade. Inquire about Hall sway on hydromagnetic free convection stream along a penetrable dimension plate with mass trade. Effects of Hall streams on free convective stream past a stimulated vertical penetrable plate in a turning structure with warmth source/sink is bankrupt somewhere around Singh and Garg (2010). The liquid move through a penetrable medium is another essential edge which has pulled in the thought of analysts and masters because of its handiness in the fields of rustic designing to analyze the underground water resources, spillage of water in stream beds, in compound building for filtration and cleansing structures. The radioactive and free convective effects of a MHD course through a porous medium between tremendous parallel plates with time-subordinate suction. Thinking about the warmth radiation and the Hall streams Singh et al. (2012) considered warmth and mass move in a touchy MHD free convective course through a penetrable medium constrained by vertical porous channel.

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