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Some Non-Static Spherically Symmetric Models In General Relativity

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ABSTRACT

This research paper provides some solution's of Einstein's field equation for non-static spherically symmetric metric. Pressure and density have been calculated. As these solutions afford suitable models of a universe which is assumed to consist of isotropic and homogeneous matters, so these are of special interest in general relativity.

1. Introduction

Mehra, Vaidya and Kushwaha [5] have obtained a general solution of the field equations for a complete sphere having a number of shells, one above the other, or different densities. Durgapal and Gehlot [1] have obtained exact internal solutions for dense massive stars in which control pressure and density are infinitely large. Durgapal and Gehlot [2,3] have obtained exact solutions for a massive sphere with two different density distributions. The density being minimum at the surface varies inversely as the square of the distance from the centre. The distribution has a core of constant density and radius. Hargreaves [4] has discussed the stability of a static spherically symmetric fluid spheres, consisting of a core of ideal gas and radiation, in which the ratio of the gas pressure to the total pressure is a small constant, and an envelope consisting of an adiabatic gas. J. P. Deleon [6] has presented two new exact analytical solutions to Einstein's field equations representing static fluid spheres with anisotropic pressures while Yadav and Saini [8] have obtained an exact, static spherically symmetric solution of Einstein's field equation for the perfect fluid with $p = \varepsilon$.

In this paper we have considered some solution of Einstein's field equation for nan- static spherically symmetric metric. Pressure and density have been calculated. As these solutions afford suitable models of a universe which is assumed to consist of isotropic and homogeneous matter so these are of special interest in general relativity.

2. Solutions of the Field Equations

Here we consider the non-static metric in the form given by

$$ds^2 = e^{\beta} dt^2 \; e^{\alpha} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \qquad \textbf{(2.1)}$$

where, α, β are functions of r and t (i.e. non-static case). The two similar solutions are given by

$$e^{\alpha} = \frac{16 e^{\Psi(t)}}{\left(4 + \frac{r^2}{R^2}\right)^2} \qquad \textbf{(2.2)}$$

$$e^{\alpha} = \frac{16R^2}{A^2(4 + Kr^2)^2}, R = R(t) \qquad \textbf{(2.3)}$$

Clearly is e^{β} functions of t only. By using a simple transformation $e^{\beta}dt^2$ can be transformed as dt^2 and thus e^{α} has to be expressed in the form

$$e^{\alpha} = \mu_1(t)\, \mu_2(r). \qquad \textbf{(2.4)}$$

The usual condition of isotropy is obtained by using (2.4) in

$$e^{\frac{\alpha}{2}}\left(\alpha'' - \frac{1}{2}\alpha'^2 - \frac{\alpha'}{r}\right) = g(r) \qquad \textbf{(2.5)}$$

We get solutions (2.1) and (2.4) by suitable adjustment of constant if we choose $g(r) = 0$. When $g(r) = 0$, we get

$$\alpha'' - \frac{1}{2}\alpha'^2 - \frac{\alpha'}{r} = 0 \qquad \textbf{(2.6)}$$

With the condition (2.6), the most general possible solutions are given above. The first solution gives the Friedman Lemaitre model of the expanding universe and the second one gives the solutions due to Tolman [7]. We can also obtain some other solution of (2.5). Let us consider one such solution given by

$$e^{-\frac{\alpha}{2}} = \nu(c^2t^2 - r^2) \quad (2.7)$$

Use of (2.7) in (2.5) yields

$$\frac{d^2\nu}{dz^2} + \frac{g(r)}{8r^2}.\nu^2 = 0 \quad (2.8)$$

where $Z = t^2 - \dfrac{r^2}{A^2}$

Case (a)

If $g(r) = 0$ them v is given by

$$y'(A^2t^2 - r^2) = cz + d \quad (2.9)$$

where c and d are integrating constants and $A^2z = A^2t^2 - r^2$.

The metric in this case represents Milne's model and is given by

$$ds^2 = dt^2 - (cz + d)^{-2}(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \quad (2.10)$$

Case (b)

By choosing $g(r) = 12\, r^2$ and putting in (2.8) we get

$$\frac{d^2\nu}{dz^2} = \frac{-3}{2}\nu^2 \quad (2.11)$$

whose solutions after some readjustment is given in terms of elliptic functions as

$$\nu = 1 + \sqrt{3}\,\frac{an(cz+d) - 1}{an(cz+d) + 1} \quad (2.12)$$

which in terms of a series can be written as

$$\nu = \frac{1 + \sqrt{3}\,\dfrac{\pi}{K}\sum \dfrac{\cos(2n^{-1})(cz+d)^{\frac{\pi}{2k}}}{\cosh(2n^{-1})^{\frac{\pi k'}{2k}}} - k}{\pi K \sum \dfrac{\cos(2n-1)(cz+d)^{\frac{\pi}{2k}}}{\cosh(2n-1)^{\frac{\pi k'}{2k}}} + k} \quad (2.13)$$

where K is the modulus, K is the quarter period and K' is complementary to K. The surviving components of the energy-momentum tensor usually expressed as the isotropic pressure P and density e are given by

$$8\pi P = -4r^2\left\{2\nu.\frac{d^2\nu}{dx^2} - 3\left[\frac{d\nu}{dx}\right]^2\right\} - \frac{3}{4} + \eta \quad (2.14)$$

and

$$8\pi\varepsilon = 4r^2\left\{2\nu.\frac{d^2\nu}{dx^2} - 3\left[\frac{d\nu}{dx}\right]^2\right\} - \frac{12}{dx} + \frac{3}{4} - \eta \quad (2.15)$$

These can be written for case (a) and case (b) as follows :

Case (a)

$$8\pi P = 12c^2r^2 - \frac{3}{4} + \eta \quad (2.16)$$

$$8\pi\varepsilon = -12c(cd^2t^2 + d) + \frac{3}{4} - \eta \quad (2.17)$$

These expressions satisfy the relation

$$\frac{\delta^{\varepsilon}}{r} + \frac{1}{2}(P + \varepsilon)\frac{\delta^{\beta}}{r} = 0 \quad (2.18)$$

which is the relativistic analogue of dependence of pressure on gravitational potential in Newtonian theory [7].

Case (b)

Here the pressure and density are expressed in terms of elliptic functions, namely.

$$8\pi P = 16\sqrt{3}c^2 r^2 \frac{\{2gn^2(cz+d-an)(cz+d)\}}{[an(cz+d)+1]^2}$$ **(2.19)**

$$+3\frac{sn^2(cz+d)\ [gn^2(cz+d)+an(cz+d)]}{[an(cz+d)+1]^4}+\frac{3}{4}-\eta$$

$$8\pi\varepsilon = -16\sqrt{3}c^2 r^2 \frac{\{2gn^2(cz+d)-an(cz+d)\}}{[an(cz+d)+1]^2}$$ **(2.20)**

$$+\frac{\sqrt{3}sn^2(cz+d)[gn^2(cz+d)+an(cz+d)]}{[an(cz+d)+1]^2}$$

$$24\sqrt{3}\ c\left\{1+\sqrt{3}\frac{an(cz+d)-1}{an(cz+d)+1}\right\}\frac{sn(cz+d)an(cz+d)}{[an(cz+d)+1]^2}-\frac{3}{4}+\eta$$

The relation (2.18) though not satisfied in this case renders the left hand side of it into a perfect differential of the form

$$\frac{d}{dx}\left\{\log\left(\frac{1}{v^2}\frac{d^2v}{d^2v}\right)\right\}-\frac{d}{dr}\left(\frac{1}{2r}\right)$$

Besides (a) and (b) the differential equation (2.8) has apparently no other solution possible, that is, $g(r)$ = 0 and $g(r)$ = $12r^2$ afford the only solutions given above. For let us take another metric of a model having spherical symmetry, namely.

$$ds^2 = -e^{\eta}dr^2 - r^2\,d\theta^2 - r^2\sin^2\theta d\phi^2 + e^{\beta}dt^2$$ **(2.21)**

where η and β are functions of r and t only.

The relation (2.18) is satisfied and it gives the following differential equation.

$$\frac{\delta}{\delta r}\left[\frac{1}{r}e^{\frac{\beta}{2}-\frac{\eta}{2}}\frac{\beta}{r}\right]=\frac{2}{r^3}(1-e^{\eta})\exp\left(\frac{\beta}{2}-\frac{\eta}{2}\right)$$ **(2.22)**

Further in addition to this, if we suppose that the condition of isometry is also satisfied, then we get the following differential equation.

$$\frac{\delta}{\delta r}\left[\frac{1}{r}\frac{\delta^{\beta}}{\delta^{r}}+\frac{1}{r^3}\right]\exp\left(\frac{\beta}{2}-\frac{\eta}{2}\right)=\frac{-2(1+e^{\eta})\exp.\left(\frac{\beta}{2}-\frac{\eta}{2}\right)}{r^4}$$ **(2.23)**

Solving (2.22) and (2.23) for η and β we get

$$e^{\eta}=\frac{1}{1-m^2r^4}$$ **(2.24)**

and

$$e^{\beta}=A^2\cos^2\left\{\frac{1}{2}\sin^{-1}(mr^2)+\alpha\right\}$$ **(2.25)**

Here the isotropic pressure and density are found to be

$$8\pi P=\left[\frac{1}{r^2}-\frac{m^2r^4}{r^2m^2r}(1-m^2r^4)^{\frac{1}{2}}\right]\tan\left\{\frac{1}{2}\sin^{-1}(mr^2)+\alpha\right\}-\frac{1}{r^2}+\eta$$ **(2.26)**

and

$$8\pi\varepsilon=-\frac{(1-6m^2r^4+m^4r^8)}{r^2(1-m^2-r^4)}+\frac{1}{r^2}-\eta$$ **(2.27)**

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