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# Static Charged Fluid Spheres In Einstein-Cartan Theory

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**ABSTRACT**

The present paper provides solution of Einstein-Maxwell-Cartan field equations for static charged fluid sphere in Einstein-Cartan theory using judicious choice of metric potentials g11 and g44. We have assumed that the spins of the individual particles composing the fluid are all aligned along radial direction.

## 1. Introduction

Various relativists have shown their interest for solutions of perfect fluid spheres in Einstein-Maxwell Cartan theory. Attempts have been made to investigate the problem of charged fluid in E-C theory by Nduka [7], Singh and Yadav [ 10 ], Prasanna [ 8 ], Kopezynki [ 5,6 ] and Raychaudhuri [ 9 ]. They have considered the generalization of Maxwell's equations in space having torsion but this idea leads to a breakdown in the gauge invariance and charge conservation principle. However, Raychaudhuri [ 9 ], Nduka [ 7 ] and Singh and Yadav [10] have taken the equation in a form so as to preserve the charge conservation principle. With this formulation Raychaudhuri has investigated the possibility of bounce in the presence of a magnetic field for Bianchi type-1 universes with p =0 and p = $\rho$. Further Nduka has discussed the static charged fluid spheres in E-C theory and has found that the pressure is discontinuous at the boundary of the fluid sphere. Recently Katkar [3] and Katkar and Phadatare [4] have also considered Einstein-Cartan theory of gravitation.

In the paper we have studied the interior field of a static spherically symmetric charged fluid distribution with non-zero spin density. Assuming that the spins of the individual particles composing the fluid are all aligned in radial direction, we have obtained solutions by choosing metric potential $\lambda$(r) and $\nu$(r) in different suitable forms. Pressure and density have been also found for the distribution and the physical constants appearing in the solution have been evaluated by matching the solution to the Reissnar-Nordstrom metric at the boundary. Unlike general relativity, p as discontinuous at the boundary of the fluid sphere.

## 2. The field equations :

The Einstein-Cartan-Maxwell equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\, t_{ij} \qquad \textbf{(2.1)}$$

$$Q^k_{ij} - \delta^k_i Q^l_{jl} - \delta^k_i Q^l_{jl} = -8\pi\, S^k_{ij} \qquad \textbf{(2.2)}$$

$$\left[(-g)^{1/2} F^{ij}\right]_{;j} = (-g)^{1/2} J^i = (-g)^{1/2} \sigma\, u^i \qquad \textbf{(2.3)}$$

$$\left[F_{ij\,k}\right] = 0 \qquad \textbf{(2.4)}$$

Where $R_{ij}$ is the Ricci tensor of asymmetric connection and also the energy momentum tensor $t_{ij}$ is not symmetric, $F_{\mu\nu}$ is the electromagnetic field tensor and $J^{\mu}$ is current four vector (we have set c and the gravitational constant also to be equal to unity).

We use here the static spherically symmetric metric

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2) \qquad \textbf{(2.5)}$$

where $\nu$ and $\lambda$ are functions of *r* only.

For the system under study the symmetric energy momentum tensor $\overline{T}^i_j$ splits into two parts viz. $T^i_j$ and $E^i_j$ for matter and electromagnetic field respectively as

$$\overline{T}^i_j = T^i_j + E^i_j \qquad \textbf{(2.6)}$$

where

 

$$T_j^i = (\rho + p)u_j u^i - p\delta_j^i$$

$$E_j^i = -F_{j\alpha}F^{i\alpha} + \frac{1}{4}\delta_j^i F_{lm}F^{lm}$$

with $u_i u^i = 1$. Here p is internal pressure. ρ, σ are the densities of matter charged respectively and $u^i$ is the velocity vector of matter. We use co-moving co-ordinates so that

$$u^i = \delta_4^i$$

The non-vanishing components of $T_i^j$ are

$$T_4^4 = \rho, T_1^1 - T_2^2 = T^3 = -p$$

Because of spherical symmetry, the only non-vanishing components of $F^{ij}$ is $F^{41} = -F^{14}$. Then from (2.1) and (2.6) the field equations may be written as

$$8\pi\ \bar{p} - E = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \quad \textbf{(2.7)}$$

$$8\pi\ \bar{p} + E = e^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right), \quad \textbf{(2.8)}$$

$$8\pi\ \bar{\rho} + E = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \quad \textbf{(2.9)}$$

where following Hehl [1,2] we have defined effective density $\bar{\rho}$ and effective pressure $\bar{p}$ as

$$\bar{\rho} = \rho - 2\pi K^2 \text{ and } \bar{p} = p - 2\pi K^2 \quad \textbf{(2.10)}$$

with

$$K = He^{-\nu/2}, \quad \textbf{(2.11)}$$

Here H is constant dashes denote differentiation with respect to r.
Also

$$E = -F^{41}F_{41} \quad \textbf{(2.12)}$$

and

$$8\pi\ \sigma = \left(\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\lambda' + \nu'}{2}F^{41}\right)e^{\nu/2} \quad \textbf{(2.13)}$$

It is clear from these quantities it is $\bar{p} = p - 2\pi K^2$ and not the p which is continuous across the boundary $r = r_o$ of the fluid sphere. The continuity of $\bar{p}$ across the boundary ensures that of $\nu'$ exp.(ν). Further with $\bar{p}$ and $\bar{\rho} = \rho - 2\pi K^2$ replacing p and ρ respectively, we are assured that the metric coefficients are continuous across the boundary. Hence we shall apply the usual boundary conditions to the solutions of equations (2.7) and (2.8) and (2.9).

The exterior metric is taken as usual Reissner-Nordstrom line element given by

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q_0^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{Q_0^2}{r^2}\right)^{-1}dr^2 - r^r(d\theta^2 + \sin^2\theta d\phi^2) \quad \textbf{(2.14)}$$

Where $Q_o = Q(r_o)$ and M is the total mass of the fluid sphere.

## 3. Solution of the field equations

Equations (2.7)-(2.9) provides us

$$8\pi\ \bar{p} = \frac{e^{-\lambda}}{2}\left(\frac{3\nu'}{2r} + \frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{\lambda'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2} \quad \textbf{(3.1)}$$

$$E = \frac{e^{-\lambda}}{2}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{\nu'}{2r} - \frac{\lambda'}{2r} - \frac{1}{r^2}\right) + \frac{1}{2r^2} \quad \textbf{(3.2)}$$

$$8\pi\ \bar{\rho} = e^{-\lambda}\left(\frac{5\lambda'}{4r} - \frac{\nu''}{4} + \frac{\lambda'\nu'}{8} - \frac{\nu'^2}{8} + \frac{\nu'}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2} \quad \textbf{(3.3)}$$

Use of equations (2.10) in (3.1) and (3.3) yield pressure and density as

$$8\pi\ \rho = \frac{e^{-\lambda}}{2}\left(\frac{3\nu'}{2r}+\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}-\frac{\lambda'}{2r}+\frac{1}{r^2}\right)-\frac{1}{2r^2}+16\pi^2K^2 \quad \textbf{(3.4)}$$

$$8\pi\ \rho = e^{-\lambda}\left(\frac{5\lambda'}{4r}-\frac{\nu''}{4}+\frac{\lambda'\nu'}{8}-\frac{\nu'^2}{8}+\frac{\nu'}{4r}-\frac{1}{2r^2}\right)+\frac{1}{2r^2}+16\pi^2K^2 \quad \textbf{(3.5)}$$

We have three equations (2.7)-(2.9) in five variables (ρ, E, p, λ, ν). Hence we have two variables free. We take λ and ν as two free variables. We choose these fluid variables in the following forms.

$$e^{\lambda} = \frac{k_1 r^{2n}+k}{r^{2n}+k} \quad \textbf{(3.6)}$$

$$e^{\nu} = \frac{k_2 r^2+k_4 r+k}{4k_3} \quad \textbf{(3.7)}$$

where k, $k_1$,$k_2$,$k_3$,$k_4$ are arbitrary constants and n is positive integer. Using equations (2.11) the spin density K is given by

$$K^2 = \frac{4k_3 H^2}{k_2 r^2+k_4 r+k} \quad \textbf{(3.8)}$$

Using equations (3.6), (3.7) and (3.8) in (2.13), (3.2), (3.4), (3.5) we get

$$16\pi\ p = \frac{128\pi^2 k_3 H^2}{k_2 r^2+k_4 r+k}+\frac{r^{2n}+k}{k_1 r^{2n}+k}\left[\frac{12k_2^2 r^3+18k_2k_4r^2+16kk_2r+5k_4^2r+6kk_4}{4r(k_2r^2+k_4r+k)^2}-\frac{nr^{2n-2}(k)(k_1-1)(4k_2r^2+3k_4r+2k)}{2(r^{2n}+k)(k_1r^{2n}+k)(k_2r^2+k_4r+k)}+\frac{1}{r^2}\right]-\frac{1}{2r^2}, \quad \textbf{(3.9)}$$

$$16\pi\ \rho = \frac{128\pi^2 k_3 H^2}{k_2 r^2+k_4 r+k}+\frac{r^{2n}+k}{k_1 r^{2n}+k}\left[\frac{nr^{2n-1}(k)(k_1-1)(12k_2r^2+11k_4r+10k)}{2r(r^{2n}+k)(k_1r^{2n}+k)(k_2r^2+k_4r+k)}+\frac{4k_2^2r^3+6k_2k_4r^2+3k_4^2r+2kk_4}{4r(k_2r^2+k_4r+k)^2}-\frac{1}{r^2}\right]+\frac{1}{r^2} \quad \textbf{(3.10)}$$

$$2E = \frac{r^{2n}+k}{k_1 r^{2n}+k}\left[\frac{-4k_2^2r^3-6k_2k_4r^2-3k_4^2r-2kk_4}{4r(k_2r^2+k_4r+k)^2}-\frac{nr^{2n-2}(k)(k_1-1)(k_2r^2+k_4r+2k_2r+k_4+k)}{(r^{2n}+k)(k_1r^{2n}+k)(k_2r^2+k_4r+k)}-\frac{1}{r^2}\right]+\frac{1}{r^2} \quad \textbf{(3.11)}$$

$$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r}+\frac{2}{r}F^{41}+\frac{\left(\frac{nr^{2n-1}.k(k_1-1)}{(r^{2n}+k)(k_1r^{2n}+k)}+\frac{2(k_2r+k_4)}{(k_2r^2+k_4r+k)}\right)}{2}F^{41}\right]\times\left(\frac{k_2r^2+3k_4r+k}{4k_3}\right)^{1/2}. \quad \textbf{(3.12)}$$

Now using the boundary conditions as discussed in section 2. The constants are fixed by

$$\frac{r_o^{2n}+k}{k_1 r_o^{2n}+k}=\left(1-\frac{2M}{r_o}+\frac{Q_0^2}{r_o^2}\right), \tag{3.13}$$

$$\frac{k_2 r_o^2+k_4 r_o+k}{4k_3}=\left(1-\frac{2M}{r_o}+\frac{Q_0^2}{r_o^2}\right), \tag{3.14}$$

$$\frac{2k_2 r_o+k_4}{8k_3}=\left(\frac{M}{r_o^2}-\frac{Q_0^2}{r_o^3}\right). \tag{3.15}$$

Also the constant H is given by

$$H^2=\frac{k_2 r_o^2+k_4 r_o+k}{128\pi^2 k_3}\left[16\pi\rho(\mathrm{r_o})-\frac{1}{r_o^2}-\frac{r_o^{2n}+k}{k_1 r_o^{2n}+k}\right.$$
$$\left\{\frac{n r_o^{2n-1}k(k_1-1)(12k_2 r_o^2+11k_4 r_o+10k)}{2r_o(r_o^{2n}+k)(k_1 r_o{}^{2n}+k)(k_2 r_o^2+k_4 r_o+k)}\right.$$
$$\left.\left.+\frac{4k_2^2 r_o^3+6k_2 k_4 r_o^2+3k_4^2 r_o+2kk_4}{4r(k_2 r_o^2+k_4 r_o+k)^2}\right\}\right]+\frac{1}{r_o^2} \tag{3.16}$$

**4. Discussion and Conclusion**

For a realistic model p > 0, ρ > 0 which will impose further restrictions on these solutions. We, therefore, restrict our solutions to only those values of constants for which pressure and density are positive.

Further solutions obtained in this paper are singularity free.

**Reference**


1. Hehl, F.W. (1973): G.R.G., 4,333.
2. Hehl, F.W. (1974): G.R.G., 5,491.
3. Katkar, L.N. (2015) : Int. Jour. Of Theor. Phys. 54(3), pp.971.
4. Katkar, L.N. and Phadatare, D.R. (2019) : Indian Jour. of Phys. 93(4), pp. 543-552.
5. Kopczynki, W. (1973a); Phys. Lett., 43A, 63.
6. Kopczynki, W. (1975); Scripts Fac Sci. Nat. Ujep Brunensis Physica, 3-4, 5, 255.
7. Nduka, A. (1977) : Gen. Rel. Grav., 8,371.
8. Prasanna, A.R. (1975c) : Phys. Lett., A54, 17.
9. Raychaudhuri, A.K. (1975): Phys. Rev. D12, 952.
10. Singh, T. and Yadav, R.B.S. (1978) : Acta. Phys. Pol., B9, 837.