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A study on Thermal Stresses in A Viscoelastic Tube

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ABSTRACT

Viscoelastic materials contain the properties of elasticity and viscous both. Temperature affects in viscoelastic bodies. The present paper deals with the study on thermal stresses in viscoelastic tube assuming that tube wall is thin.

1. Introduction

Many research scholars paid considerable attention in finding solution of thermoviscoelastic problems.Valanis (5) represented his paper thermal stresses in a viscoelastic cylinder with temperature dependent properties.Muki & Sternberg(3), Morland &Lee(4),Christensen(1) did sufficient work in this field. The problem of thermal stresses in an isotropic viscoelastic thin –walled circular cylinder with temperature dependent has been considered by Lockett & Morland (2).On the basis of this theory,a perturbation scheme using the thin walled approximation has been used which leads to a set of simpler

Equation .The response in dilation has been taken as elastic. In this paper we have studied on thermal stresses in a viscoelastic tube assuming wall is thin.

2. Constitutive equations

The consecutive equations are taken as (if disturbance strain at t=0)

2.1 $$\frac{1}{2}\hat{S}_{ij}\left(\underline{x,\zeta}\right)=R(0)\xi_{ij}\left(\underline{x,\zeta}\right)+\int_0^{\xi} R'(\zeta-\zeta')\hat{\xi}_{ij}\left(\underline{x,\zeta'}\right)d\zeta'$$

2.2 $$\frac{1}{3}\hat{\phi}_{\kappa\kappa}(\underline{x},t)=\kappa\{\hat{\ell}_{\kappa\kappa}\underline{(x,\zeta)-3\alpha_0}\hat{\theta}(\underline{x,\zeta})\},$$

where R(t) is the shear relaxation modulus at temperature $T_0\{R(t)=0,\forall t\langle 0\}, R'(t)$ is its derivative and κ is the bulk modulus .

We take the equation of heat conduction in the form

2.3 $k\nabla^2 T\left(\underline{x,t}\right)=\frac{\partial T}{\partial t}\left(\underline{x,t}\right)$, where k is assumed to be temperature –independent .

We take the bounding surfaces of the cylinder to be r=a and r=b (a<b), as cylindrical polar co-ordinate system (r,θ,z),with origin on the axis of the cylinder and the x-axis coinciding with the axis, taking the geometric ratio

2.4 $$h=\frac{b-a}{a}$$

And dimensionless radial co-ordinate x,

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2.5 $$x=\frac{r-a}{b-a}=\frac{r-a}{ah}$$

then the cylinder covers the region $0\le x\le 1$ .Suppose the cylinder is thin –walled,h<<1.Taking plane strain conditions(in the r,θ-plane) and radial symmetry , so that all field variable are functions of(x,t) only.The non-vanishing quantities are the radial displacement k, radial and circumferential strain ℓ_r and ℓ_θ ;radial ,circumferential and axial stresses $\phi_r,\phi_\theta and\phi_z$, and the prescribed pseudo-temperature θ. The strain-displacement relations are

2.6 $\ell_r = \frac{1}{h}\frac{\partial U}{\partial x}, \ell_\theta = \frac{U}{1+hx}$, where U is the dimensionless displacement defined by

2.7 $U = \frac{u}{a}$.

The single compatibility equation is

2.8 $\frac{\partial \ell_\theta}{\partial x} = h\left[\ell_r - \frac{\partial}{\partial x}(x\ell_\theta)\right]$

The equation of equilibrium (in the absence of body forces and with quasi-static conditions),

2.9 $\frac{\partial \phi_r}{\partial x} = h\left[\phi_\theta - \frac{\partial}{\partial x}(x\phi_r)\right]$

Then equation (2.1)and (2.2) becomes 3

2.10 $\frac{1}{2}\{\hat{\phi}_r - \hat{\phi}_\theta\}(x,\varsigma) = R(0)\{\hat{\ell}_r - \hat{\ell}_\theta\}(x,\varsigma) + \int_0^\varsigma R'(\varsigma-\varsigma')(\hat{\ell}_r - \hat{\ell}_\theta)(x,\varsigma')d\varsigma'$,

2.11 $\frac{1}{2}\{\hat{\phi}_r + \hat{\phi}_\theta - 2\hat{\phi}_x\}(x,\varsigma) = R(0)\{\hat{\ell}_r + \hat{\ell}_\theta\}(x,\varsigma) + \int_0^\varsigma R'(\varsigma-\varsigma')\{\hat{\ell}_r + \hat{\ell}_\theta\}(x,\varsigma')d\varsigma'$,

2.12 $\frac{1}{3}\{\hat{\phi}_r + \hat{\phi}_\theta + \hat{\phi}_z\}(x,\varsigma) = k\{\hat{\ell}_r + \hat{\ell}_\theta - 3\alpha_0\hat{\theta}\}(x,\varsigma)$.

In (2.10), (2.11) & (2.12) using the Laplace transform and calculating for the individual stress components ,we get

2.13 $3\bar{\hat{\phi}}_r(x,s) = [3k + s\bar{R}(s)]\bar{\hat{\ell}}_r(x,s) + [3k - s\bar{R}(s)]\bar{\hat{\ell}}_\theta(x,s) - 9k\alpha_0\bar{\hat{\theta}}(x,\theta)$

2.14 $3\bar{\hat{\phi}}_\theta(x,s) = [3k - s\bar{R}(s)]\bar{\hat{\ell}}_r(x,s) + [3k + s\bar{R}(s)]\bar{\hat{\ell}}_r(x,s) - 9k\alpha_0\bar{\hat{\theta}}(x,s)$

2.15 $\bar{\hat{\phi}}_z(x,s) = \frac{3k - 2s\bar{R}(s)}{2[3k + s\bar{R}(s)]}\{\bar{\hat{\phi}}_r + \bar{\hat{\phi}}_\theta\}(x,s) - \frac{9\alpha_0 ks\bar{R}(s)}{3k + s\bar{R}(s)}\bar{\hat{\theta}}(x,s)$

from (2.13) & (2.14),

2.16 $\bar{\hat{\phi}}_\theta(x,s) = s\bar{A}(s)\bar{\hat{\phi}}_r(x,s) + s\bar{B}(s)\bar{\hat{\ell}}_\theta(x,s) - 3s\bar{C}(s)\alpha_0\bar{\hat{\theta}}(x,s)$

where

2.17 $s\bar{A}(s) = \frac{3k - 2s\bar{R}(s)}{3k + 4s\underline{R}(s)}$, $s\bar{B}(s) = \frac{4s\bar{R}(s)\{3k + s\bar{R}(s)\}}{3k + 4s\bar{R}(s)}$ & $s\bar{C}(s) = \frac{6ks\bar{R}(s)}{3k + 4s\bar{R}(s)}$

From this we get $A(\varsigma), B(\varsigma) \& C(\varsigma)$ and, the convolution theorem defines $A(\varsigma), B(\varsigma) \& C(\varsigma)$ to be solutions of a Volterra integral equation 4

2.18 $[3k + 4R(0)]y(\varsigma) + 4\int_0^\varsigma R'(\varsigma-\varsigma')y(\varsigma')d\varsigma' = 3\gamma(\varsigma)$

where $\gamma(\varsigma)$ is

2.19

$$\gamma(\varsigma) = \begin{cases} k-\frac{2}{3}R(\varsigma),\ for, y=A \\ 4kR(\varsigma)+\frac{4}{3}\left\{R(0)R(t)+\int_0^{\xi}R'(\varsigma,\varsigma')R(\varsigma')d\varsigma'\right\},\ for, y=B \\ 2kR(\varsigma),\ for, y=C \end{cases}$$

Also applying the convolution theorem to (2.16), we get

2.20
$$\hat{\phi}_\theta(x,\varsigma)=A(0)\hat{\phi}_r(x,\varsigma)+\int_0^{\varsigma}A'(\varsigma-\varsigma')\hat{\phi}_r(x,\varsigma')d\varsigma'+B(0)\hat{\ell}_\theta(x,\varsigma)+\int_0^{\varsigma}B'(\varsigma-\varsigma)\hat{\ell}_\theta(x,\varsigma')d\varsigma'-3\alpha_0\left\{C(0)\hat{\theta}(x,\varsigma)+\int_0^{\xi}C'(\varsigma-\varsigma')\hat{\ell}(x,\varsigma')d\varsigma'\right\}$$

3. Solution

The equations obtained do not exhibit the constitutive laws to be evaluated in reduced time while treating the remaining equations in real time. This uncoupling is achieved in the iteration scheme. We take the maximum strain and maximum stress divided by an instantaneous modulus, as a first order quantity ,is to matching powers of h in an expansion solution. Expanding f(x ,t) in ascending powers of h, we have

3.1 $f(x,t)=f^{(0)}(x,t)+hf(x,t)+h^2f^{(2)}(x,t)+h^3f^{(3)}(x,t)+..............+h^nf^{(n)}(x,t)+......$

Where f(x,t) may be a strain, stress, displacement or temperature.
Substituting the corresponding series in (2.8) and (2.9) and equating

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equal powers of h from both sides ,we get

3.2 $\frac{\partial \ell_\theta^{(0)}}{\partial x}=0$

3.3 $\frac{\partial \ell_\theta^{(n+1)}}{\partial x}=\ell_r^{(n)}-\frac{\partial}{\partial x}\left(x\ell_\theta^{(n)}\right), n=0,1,2,...$

3.4 $\frac{\partial \phi_r^{(0)}}{\partial x}=0$

3.5 $\frac{\partial \phi_\theta^{(n+1)}}{\partial x}=\phi_r^{(n)}-\frac{\partial}{\partial x}\left(x\phi_\theta^{(n)}\right), n=0,1,2,...$

From (3.2) and (3.4), it is clear that $\ell_\theta^{(0)}$ and $\phi_r^{(0)}$ are functions of t only , therefore

3.6 $\ell_\theta^{(0)}=\ell_\theta^{(0)}(t)$, $\phi_r^{(0)}=\phi_r^{(0)}(t)$

If the stress boundary conditions are in the form

3.7 $\phi_r(x,t)=-P_0(t), x=0$

$-P_1(t), x=1$

Then from (3.6) and (3.1) $P_1(t)-P_0(t)$ must be of order h or similar .Thus if we write $P_1(t)=P_0(t)-h\Delta(t)$,then

3.8 $\phi_r^{(0)}=-P_0(t), \frac{\partial \phi_r^{(1)}}{\partial x}=\phi_\theta^{(0)}+P_0(t)$,

3.9 $\phi_r^{(1)}=0, when x=0; \phi_r^{(1)}=\Delta(t), when x=1$

3.10 $\phi_r^{(n)} = 0, x = 0 \,\&\, x = 1, (n = 2,3,4,....)$

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provided $P_0(t)$ is the 1st order magnitude of the pressures applied to both the boundaries .

We get from (2.20) written in (x,t) co-ordinate

$$\phi_\theta^{(0)}(x,t) = A(0)\phi_t^{(0)}(t) - \int_0^t \phi_t^{(0)}(t')\frac{\partial}{\partial t'}[A\{\varsigma(x,t) - \varsigma(x,t')\}]dt'$$

3.11 $$+ B(0)\ell_\theta^{(0)}(t) - \int_0^t \ell_\theta^{(0)}(t')\frac{\partial}{\partial t'}[B\{\varsigma(x,t) - \varsigma(x,t')\}]dt' - 3\alpha_0 C(0)\theta^{(0)}(x,t)$$

$$+ 3\alpha_0 \int_0^t \theta^{(0)}(x,t')\frac{\partial}{\partial t'}[C\{\varsigma(x,t) - \varsigma(x,t')\}]dt'$$

Integrating through the wall thickness, using (3.8) and (3.9) , we get

3.12 $g(t) = B(0)\ell_\theta^{(0)}(t) - \int_0^t \ell_\theta^{(0)}(t')\frac{\partial \beta(t,t')}{\partial t'}dt'$,

Where

3.13 $\beta(t,t') = \int_0^t B\{\varsigma(x,t) - \varsigma(x,t')\}dz$,

& $g(t) = \Delta(t) - P_0(t) - \int_0^1 Q^{(0)}(x,t)dx$

3.14
$$Q^{(0)}(x,t) = A(0)\phi_r^{(0)}(t) - \int_0^t \phi_r^{(0)}(t')\frac{\partial}{\partial t'}[A\{\varsigma(x,t) - \varsigma(x,t')\}]dt'$$

$$- 3\phi_0 C(0)\theta^{(0)}(x,t) + 3\alpha_0 \int_0^t \theta^{(0)}(x,t')\frac{\partial}{\partial t'}[C\{\varsigma(x,t) - \varsigma(x,t')\}]dt'$$

Here $Q^{(0)}(x,t)$ is a known function and $\theta^{(0)}(x,t)$ is the first term in the development of $\theta(x,t)$ as a power series in h.Equation (3.12) is a Volterra Integral equation of the second kind for $\ell_\theta^{(0)}(t)$. we find $\ell_\theta^{(0)}(t)$,

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then $\phi_\theta^{(0)}(x,t)$ using (3.11) and $\phi_x^{(0)} \,\&\, \phi_r^{(0)}$ using (2.15) and (2.13).If we take $P_1(t) = 0$, then we have $P_0(t) = h\Delta(t)$. As such $P_0(t)$ is not of the first order and from (3.8).

3.15 $\phi_r^{(0)}(t) = 0$

DISCUSSION

$\phi_r^{(1)}$ can be obtained by (3.8)&(3.9) and $\ell_\theta^{(1)}$ to within an arbitrary function of time. The remaining quantities can be find as in the first iteration and so on. The only first order stresses are $\phi_\theta^{(0)} \,\&\, \phi_z^{(0)}$ and the maximum cross-section shear stresses at fixed (x,t) is given by $\frac{1}{2}\left[\phi_\theta^{(0)}(x,t)\right]$.The required

stress is given by (3.11)and (3.12) with the $\phi_r^{(0)}(t) \,\&\, P_0(t)$ omitted from (3.13) and (3.14) with $\Delta(t)$ replaced by $P_0(t)/h$.

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