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An another approach for obtaining initial feasible solution of Transportation Problem

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ABSTRACT

In the Operations Research, transportation problems have a great importance. To solve these problems, it is very important to start with a good initial basic feasible solution. In this paper, we have proposed a new technique to find the best initial solution, which is based on making allocations in zero cost cells of reduced transportation table. The proposed technique yields near optimal solution as compare to other existing techniques.

1. Introduction

Transportation problem (TP) is an important aspect for logistics as well as for layman. This problem is developed by Hitchcock [10] and Koopmans [15]. To solve this problem, several methods are developed, such as, stepping stone method developed by Charnes et al. [4], modified distribution method by Dentzig [6] and modified stepping stone method by Shih [25] etc. Also to solve TP with well known existing methods, it is necessary that one have to start with initial basic feasible solution (IBFS). For this purpose, some of well known techniques are developed by Reinfeld et al. [23] in 1958, which are north west corner method (NWCM), least cost method (LCM), Vogel's approximation method (VAM). In 1969, Russells approximation method was developed by Russell [24]. Also some improved versions of VAM given by Shimshak et al. [26] in 1981, Goyal [8] in 1984, Ramakrishnan [20] in 1988, Balakrishnan [3] in 1990 and by Das et al. [5] in 2014. In 1990, Kirca et al. [14] developed new approach named total opportunity cost method (TOM). Gass [7] reviewed various existing methods in 1990 and discussed on solving transportation problems. Also Goyal [9] given a note on TOM in 1991. An effective approach to solve TP is developed by Adlakha et al.

[1] in 1999. Mathirajan et al. [18] coupled the total opportunity cost matrix (obtained by Kirca et al.) with varients of VAM in 2004 and named this technique as TOCMVAM. In 2004, Extreme difference method (EDM) developed by Kasana et al. [12]. In 2009, Pargar et al. [19] developed maximum demand heuristic. Then a new heuristic to solve modified unbalanced transportation problems is given by Kulkarni et al. [17] in 2010. Also, highest cost difference method(HCDM) by Khan [13] in 2011, cost sum method(SUM) by Kulkarni [16] in 2012, average cost method(ACM) by Rashid et al. [22] in 2013, A cost minimization approach by Ahmed et al. [2] in 2014, Juman and Horque method (JHM method) by Juman et al. [11] in 2015 and Rashid [21] developed theorems to solve TP in 2016.

In this paper, we have proposed a simple technique to find best initial solution of transportation problem, which is based on making allocations in the zero cost cell corresponding to highest cost cell of reduced transportation table.

This paper is organized as follows: Section 2 contains Model Representation. Section 3 contains basic definitions. In Section 4, An Algorithm to obtain IBFS of TP is proposed. Numerical examples are given in Section 5 to illustrates the algorithm. In Section 6, comparison between existing methods and proposed method is given. The last section contains conclusion.

2. Model Representation

In general terms, the transportation problems are related to transporting commodities from m sources to n destinations with least expenses while satisfying all supply and demand limitations. The sources may be production facilities, warehouses etc and the destinations may be sales, warehouses, outlets etc. These problems are widely known as cost minimizing transportation problem. In these problems, the decision maker is sure about the transportation cost, availability and demand of the product to be transported from factories to retail stores.

A cost minimization transportation problem (ρ) is formulated as:

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r}\, x_{ij}\ ; r = 1, 2, ...,$$

$$Subject\ to \sum_{j=1}^{n} x_{ij} = a_i,\ a_i > 0$$
$$\sum_{i=1}^{m} x_{ij} = b_j,\ b_j > 0$$

$$x_{ij} \geq 0, for\ i = 1, 2, ..., m; j = 1, 2, ..., n.$$

Where
m number of sources (Si)
n number of destinations (Dj)
ai supply amount of the product at Si,
bj demand of the product at Dj
xi j amount of homogeneous product to be transported from Si to Dj
ci j unit transportation cost of the product from Si to Dj

ai and bj are given non-negative numbers.

Remark 1. The problem (ρ) is said to be balanced if the total supply of the goods is same as its demand, otherwise the problem is said to be unbalanced.

The problem can also be represented as table 1.

Table 1: Tabular representation of model (ρ)

Destination → Source ↓	D_1	D_2	...	D_n	Supply(a_i)
S_1	c_{11}	c_{12}	...	c_{1n}	a_1
S_2	c_{21}	c_{22}	...	c_{2n}	a_2
⋮	⋮	⋮	...	⋮	⋮
S_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand (b_j)	b_1	b_2	...	b_n	

3. Basic definitions

In this section, some basic definitions and theorems related to our proposed methods are reviewed.

Definition 1. [27] An n-tuple (y_1, $y_{2, \dots,}$ y_n) of real numbers which satisfies the constraints of a general L.P.P. is called a solution to the general L.P.P.

Definition 2. [27] Any solution to a general L.P.P. which also satisfies the non-negativity condition of general L.P.P. is called feasible solution of general L.P.P.

Definition 3. [27] For m simultaneous linear equation in n variable (m < n) in general L.P.P., a solution obtained by setting n-m variable zero and by solving the resulting system in m variables, is called basic solution. The m variables, which may be all different from zero, are called basic variables.

Definition 4. [27] A feasible solution to general L.P.P., which is also basic solution is called basic feasible solution to general L.P.P.

Definition 5. [27] The cells of transportation table, in which allocations are made called basic cells and remaining are called non basic cells.

4. Proposed Technique

To proceed with proposed heuristic the given steps are followed:

Step 1.

Represent the given TP into the form of cost matrix as table 1

Step 2.

Balance the given transportation problem, if it is not balanced by adding dummy row/column

according to requirement of supply/demand

Step 3.

Reduce the transportation cost matrix by using 3a and then 3b so that there should be at least one zero in each row and column:

3a.

For all i and fixed j choose c_{tj} = min(c_{ij}) and calculate c'_{ij} = c_{ij} - c_{tj} (i.e subtract minimum

cost of each column from all costs of corresponding column).

3b.

For all j and fixed i choose c'_{ip} = min(c'_{ij}) and calculate c''_{ij} = c'_{ij} - c'_{tj} (i.e subtract minimum cost of each row from all costs of corresponding row after applying step 3a).

Step 4.

Choose c''_{rs} = max(c''_{ij}), for i, j (i.e select maximum cost cell from the obtained cost table in step 3b).

Step 5.

Except from dummy row/column, choose δ=($c'_{ip} = 0, 1 \leq i \leq m, and\ c''_{rj} = 0, 1 \leq j \leq n$) (i.e select zero cost cells (δ), which are presented in corresponding row/column of selected maximum cost cell in step 4).

Step 6.

Assign maximum possible x_{ij} to that δ, in which maximum x_{ij} is possible. If possibility of x_{ij} is also equal in all δ, then allocate x_{ij} to that δ, which has corresponding minimum c_{ij}. Cross out the row/column for which whole supply/demand is satisfied and go to step 3.

Step 7.

Repeat above process until each demand and supply is not satisfied

Step 8.

Finally, calculate total transportation cost by $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij}$ where value of all x_{ij} for non

basic cells should be taken zero.

Remark 2. Make allocations in dummy column or row at the end, when whole supply and demand satisfied for all other rows and columns.

As the proposed heuristic is based on giving allocations to zero cost cell in the reduced table, therefore present method can be called as 'Reduced table zero allocation method'.

5. Numerical Examples

Numerical examples are taken from different research papers for the application of proposed method (PM). So that new methodology can be clearly specified. Input data of all considered examples is given in Table 2 with IBFS. Here IBFS is obtained by using proposed heuristic.

Table 2: Input data of numerical examples and IBFS using proposed method

Ex.	Input data	Source	Obtained allocations	Obtained cost
1	$[c_{ij}]_{4\times3}$=[3 4 6; 7 3 8; 6 4 5; 7 5 2]; $[a_i]_{4\times1}$=[100, 80, 90, 120]; $[b_j]_{1\times3}$=[110, 110, 60]	[16]	$x_{11}=100, x_{22}=80, x_{31}=10, x_{32}=30, x_{43}=60$	840
2	$[c_{ij}]_{4\times5}$=[10 2 16 14 10; 6 18 12 13 16; 8 4 14 12 10; 14 22 20 8 18]; $[a_i]_{4\times1}$=[300, 500, 825, 375]; $[b_j]_{1\times5}$=[350,400, 250, 150, 400]	[2]	$x_{12}=300, x_{21}=350, x_{23}=150, x_{32}=100, x_{33}=100, x_{34}=400, x_{44}=150$	11500
3	$[c_{ij}]_{3\times4}$=[13 18 30 8; 55 20 25 40; 30 6 50 10]; $[a_i]_{3\times1}$=[8, 10, 11]; $[b_j]_{1\times4}$=[4, 7, 6, 12]	[11]	$x_{11}=4, x_{14}=4, x_{22}=4, x_{23}=6, x_{32}=3, x_{34}=8$	412
4	$[c_{ij}]_{3\times4}$=[18 27 13 19; 25 21 24 14; 23 15 21 17]; $[a_i]_{3\times1}$=[40, 40, 20]; $[b_j]_{1\times4}$=[30, 15, 30, 20]	[9]	$x_{11}=10, x_{13}=30, x_{21}=15, x_{24}=20, x_{31}=5, x_{32}=15$	1565
5	$[c_{ij}]_{3\times3}$=[6 10 14; 12 19 21; 15 14 17]; $[a_i]_{3\times1}$=[50, 50, 50]; $[b_j]_{1\times3}$=[30, 40, 55]	[8]	$x_{11}=10, x_{12}=40, x_{21}=20, x_{23}=5, x_{33}=50$	1655
6	$[c_{ij}]_{4\times6}$=[5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9]; $[a_i]_{4\times1}$=[4, 3, 3, 7]; $[b_j]_{1\times6}$=[3, 3, 6, 2, 1, 2]	[1]	$x_{12}=2, x_{16}=2, x_{21}=3, x_{33}=3, x_{42}=1, x_{43}=3, x_{44}=2, x_{45}=1$	96
7	$[c_{ij}]_{3\times3}$=[6 8 10; 7 11 11; 4 5 12]; $[a_i]_{3\times1}$=[150, 175, 275]; $[b_j]_{1\times3}$=[200, 100, 300]	[11]	$x_{11}=25, x_{13}=125, x_{23}=175, x_{31}=175, x_{32}=100$	4525
8	$[c_{ij}]_{3\times4}$=[19 30 50 10; 70 30 40 60; 40 8 70 20]; $[a_i]_{3\times1}$=[7, 9, 18]; $[b_j]_{1\times4}$=[5, 8, 7, 14]	[21]	$x_{11}=5, x_{14}=2, x_{22}=2, x_{23}=7, x_{32}=6, x_{34}=12$	743
9	$[c_{ij}]_{5\times7}$=[12 7 3 8 10 6 6; 6 9 7 12 8 12 4; 10 12 8 4 9 9 3; 8 5 11 6 7 9 3; 7 6 8 11 9 5 6]; $[a_i]_{5\times1}$=[60, 80, 70, 100, 90]; $[b_j]_{1\times7}$=[20, 30, 40, 70, 60, 80, 100]	[5]	$x_{12}=20, x_{13}=40, x_{21}=20, x_{25}=60, x_{34}=70, x_{47}=100, x_{52}=10, x_{56}=80$	1900

5.1. Illustration

Here detailed solution of example 1 is given, which is proceeded by steps of proposed method.

Step 1 Represent the data of example 1 into the form of cost matix as shown in table 3.

Table 3: Tabular representation of model (ρ)

Destination → source ↓	D_1	D_2	D_3	supply(a_i)
S_1	3	4	6	100
S_2	7	3	8	80
S_3	6	4	5	90
S_4	7	3	2	120
Demand (b_j)	110	110	60	

Step 2

As total supply is 110 units more than total demand, therefore add dummy column with transportation cost zero and demand 110 units as shown in table 4.

Table 4: Tabular representation of example 1

Destination → Source ↓	D_1	D_2	D_3	Dummy	Supply(a_i)
S_1	3	4	6	0	100
S_2	7	3	8	0	80
S_3	6	4	5	0	90
S_4	7	3	2	0	120
Demand (b_j)	110	110	60	110	390

Step 3

Reduce the transportation cost matrix by applying step 3a and then 3b of proposed method as follows: 3.a On subtracting minimum cost 3, 3, 2 and 0 of column 1, 2, 3 and 4 respectively from each cost of corresponding column, we get table 5.

Table 5: Reduced Cost Matrix with allocation in c''_{22}

Destination → source ↓	D_1	D_2	D_3	Dummy	supply(a_i)
S_1	0	1	4	0	100
S_2	4	(80) 0	6	0	80
S_3	3	1	3	0	90
S_4	4	2	0	0	120
Demand(b_j)	110	110	60	110	390

3.b In table 5, minimum cost in each row is zero, therefore after subtracting minimum cost of each row from corresponding cost of each row, table 5 remains unchanged. So it represents the reduced cost matrix.

Step 4

In table 5, cell c''_{23} contains maximum reduced cost.

Step 5

In corresponding row/column of c''_{23}, zero cost cells are c''_{22} and c''_{43}.

Step 6

Make allocation in the cell c''_{22} in table 5 to which maximum allocation 80 is possible as compare to c''_{43} and cross out the row 2, for which whole supply(a_1) is satisfied as shown in table 6 and go to step 3.

Table 6: Reduced Cost Matrix after removing row 2

Destination → Source ↓	D_1	D_2	D_3	Dummy	Supply(a_i)
S_1	0	1	4	0	100
S_3	3	1	3	0	90
S_4	4	2	0	0	120
Demand(b_j)	110	30	60	110	390

Step 3

In table 6 after crossing out row 2, remaining each row and column contains zero except column 2. So select minimum cost i.e 1 from column 2 and subtract it from all costs of corresponding column 2 and we get table 7.

Table 7: Reduced transportation cost table with allocation in c''_{11}

Destination → source ↓	D_1	D_2	D_3	Dummy	supply(a_i)
S_1	(100) 0	0	4	0	100
S_3	3	0	3	0	90
S_4	4	1	0	0	120
Demand(b_j)	110	30	60	110	390

Step 4

In table 7, maximum cost cells are c''_{13} and c''_{41},

Step 5

c''_{11}, c''_{12} and c''_{43}, contains zero cost cells in corresponding row/column of c''_{13} and c''_{41}.

Step 6

Make maximum possible allocation 100 to cell c''_{11} in table 7 and cross out the row 1 for which supply (a_2) is satisfied. We get table 8 and again go to step 3.

Table 8: Reduced transportation cost table after removing row 1

Destination → source ↓	D_1	D_2	D_3	Dummy	supply(a_i)
S_3	3	0	3	0	90
S_4	4	1	0	0	120
Demand(b_j)	10	30	60	110	390

Step 3

In table 8, all row/columns contains zero cost cells except 1st column. So subtract minimum cost 3 from all costs of column 1 and we get table 9.

Table 9: Reduced transportation cost table with allocation in c_{43}

Destination → source ↓	D_1	D_2	D_3	Dummy	supply(a_i)
S_3	0	0	3	0	90
S_4	1	1	(60) 0	0	120
Demand(b_j)	10	30	60	110	390

Step 4

Maximum cost cell in table 9 is c''_{33}.

Step 5

c''_{21} , c''_{22} and c''_{43} have zero cost in corresponding row/column of c''_{33}.

Step 6

Make maximum possible allocation 60 to cell c''_{43} in table 9 and cross out the column 3 for which demand (s_3) is satisfied. We get table 10 and again go to step 3.

Table 10: Further reduced transportation cost table after removing column 3

Destination → source ↓	D_1	D_2	Dummy	supply(a_i)
S_3	0	0	0	90
S_4	1	1	0	60
Demand(b_j)	10	30	110	390

Similarly proceeding with same way as explained above, we get next allocations 30 and 10 in cells c''_{32} and c''_{31} respectively and each supply/demand satisfied except dummy demand.

According to algorithm, we allocate dummy demand at the end of procedure and we get table 11 with all allocations.

Table 11: Final allocation table

Destination → source ↓	D_1	D_2	D_3	Dummy	supply(a_i)
S_1	(100) 3	4	6	0	100
S_2	7	(80) 3	8	0	80
S_3	(10)6	(30) 4	5	(50)0	90
S_4	7	3	(60) 2	(60) 0	120
Demand (b_j)	110	110	60	110	390

Total TP=100_3+80_3+10_6+30_4+60_2 = 840

6. Comparative Study and Result Analysis

For making comparison between new heuristic and commonly used existing methods, IBFS of Numerical Examples given in table 2 are also solved by using different existing methods. So, IBFS obtained by applying existing methods and Optimal Solution by modified distribution method [6] for all examples are given in table 12.

Table 12: Results (IBFS by using existing methods and Optimal Solution)

Examples → Methods ↓	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7	Ex-8	Ex-9
NWCM	1010	19700	484	1960	1815	103	4725	1015	3180
LCM	990	13700	516	1600	1885	110	4550	814	2080
VAM	880	12250	475	1565	1745	96	5125	749	1930
TOM	990	13750	516	1600	1885	110	4600	814	1900
TOCM-VAM	840	11800	536	1575	1650	96	4525	743	2040
EDM	950	12250	476	1600	1695	99	4550	749	1955
HCDM	990	11500	427	1600	1650	108	4550	814	2320
Optimal Solution	840	11500	412	1565	1650	96	4525	743	1900

After comparison between IBFS table 12 and our proposed heuristic, it is analyzed that our proposed heuristic gives direct optimal solution for 8 problems out of 9 and for remaining one problem, it needs only one iteration to reach optimal solution. TOCMVAM gives optimal solution for 5 problems and even needs more calculation to obtain IBFS as compare to our proposed heuristic. Also VAM and HCDM gives optimal solution for 2 problems and TOM gives for only one problem. Remaining methods not give optimal solution for single problem. So our proposed method performs better than 10 all methods. Also for more comparison, we have obtained average relative deviation of existing methods and proposed method corresponding to optimal solution by using measure given by Mathirajan et al. [18]. Which is as follows:

$$\text{ARPD (M)} = \frac{1}{N}\sum_{k=1}^{N}(RPD(M,k)), \quad where \quad \text{RPD (M,k)} = \frac{(IBFS)_k - (OS)_k}{(OS)_k} \times 100$$

Where ARPD (M)-Average relative percentage deviation of the given method "M", where "M" indicates NWCM, or VAM, or LCM, or HCDM, or EDM, or TOM, or TOCM-VAM, or proposed method; RPD (M, K)-Relative percentage deviation of the kth problem between IBFS using method "M" and optimal solution; N-the number of problems presented; (IBFS)k-IBFS of kth problem using method M; (OS)k-optimal solution of kth problem. By using above measures RPD and ARPD of different method are as given in table 13.

Table 13: Results (RPD and ARPD in %)

Examples → Methods ↓	RPD									ARPD
	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7	Ex-8	Ex-9	
NWCM	20.24	71.30	17.48	25.24	10	7.29	4.42	36.61	67.37	28.88
LCM	17.86	19.13	25.24	2.24	14.24	14.58	.55	9.56	9.47	12.54
VAM	4.76	6.52	15.29	.64	5.76	0	13.26	.81	1.58	5.40
TOM	4.76	6.52	25.24	2.24	14.24	14.58	1.66	9.56	0	8.76
TOCM-VAM	0	2.61	30.10	.64	0	0	0	0	7.37	4.52
EDM	13.10	6.52	15.53	2.24	2.72	3.13	.55	.81	2.89	5.28
HCDM	17.86	0	3.64	2.24	0	12.5	.55	9.56	22.11	7.61
Proposed Method	0	0	0	0	.30	0	0	0	0	.03

APRD of various techniques for finding initial solution of transportation problem, which is obtained in table 13, is interpreted graphically as shown in figure 1.

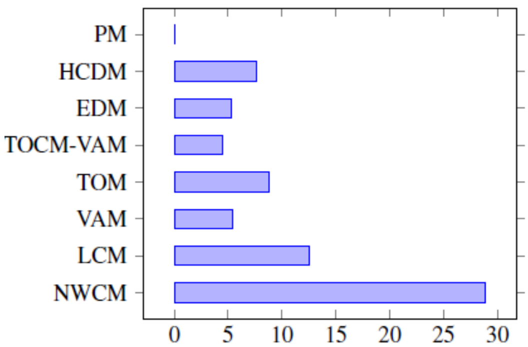

Figure 1: ARPD Graph Representation

In the above graph, ARPD of our proposed method is less than all other existing methods, which shows that there is very less deviation between our proposed heuristic and optimal solution as compare to other existing methods.

7. Conclusions and future scope

In the above result analysis, comparison between different existing methods is given, from which it is analyzed that our proposed method gives more times best initial solution than other existing methods specially for unbalanced transportation problems. Most of times gives optimal solution or near optimal solution, which needs less number of iterations to reach optimal solution. But it always not insure optimality. So it is necessary to check optimality after obtaining solution by our proposed method. Also it is very simple and easy to apply. Further research can also be possible by applying this method on assignment problems and traveling salesman problems. Also the present paper can be extend in future by using linear programming software for reporting obtained results as CPU times and computer memory.

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