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Certain Investigation On Spherically Symmetric Cosmological Model in General Relativity

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ABSTRACT

In this paper, we have presented a spherically symmetric a cosmological model in general relativity with the help of Lyra's geometry with time dependent displacement field. The Lyra's geometry which is a generalization of Weyl's geometry (removing the defect of non-integrability of length transfer). gravitational theory and the cosmology based on this geometry have been reviewed. Exact solutions have been obtained for constant deceleration parameter models of the universe. Further, cosmological models with both constant and time dependent displacement field have been discussed. We discuss a model universe with different situations, by solving the modified Einsteinfield equation within the framework of Lyra geometry, specially for Non-flat models. This model is suitable for an early stage of the universe, that is, before the universe underwent the compactification transition. The explicit solution of the scale factor are found via the assumption of an equation of state p=Y ρ, where Y=0.5. varies cases have been considered. When the space-time dimension N = 4, our solution reduces to that of Singh and Desikan. Some physical properties of the models are also discussed.

1. Introduction

Kaluza and Klein [1] tried to unify gravity with electromagnetic interaction by introducing an extra dimension. Their theory is an extension of Einstein's general relativity to five dimensions. According to Chodos and Detweiler [2], the present four-dimensional stage of the universe could have been preceded by a higher-dimensional stage, which at a later period becomes four-dimensional in the sense that the extra dimensions contract to unobserved planckian length scale due to dynamical contraction. In view of the emergence of superstring theory as the most promising theory developed so far, having the potential to lead us to a step closer towards unification of four forces, studies in higher-dimensional cosmology have obtained renewed importance inspiring a host of workers to enter into this field of study.

Einstein developed his general theory of relativity, where gravitation is described in terms of geometry. Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the Universe, because without the cosmological term his field equations admit only non-static cosmological models for non-zero energy density. Later, Weyl [9] proposed a more general theory in which electromagnetism is also described geometrically. He showed how one can introduce a vector field in the Riemannian space-time with an intrinsic geometrical significance. But this theory was based on non-integrability of length transfer so that it had some unsatisfactory features, and hence this theory which is known as Weyl's geometry still today did not gain general acceptance. After having these concepts, Lyra [10] suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structure-less manifold, which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. Halford [14-14a] pointed out that in the normal general relativistic treatment the constant displacement vector field φi in Lyra's geometry plays the role of cosmological constant and the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits, as far as the classical solar system test are concerned (as in the Einstein's theory of relativity).

Already a number of important solutions of Einstein's equations in higher dimensions have been obtained by many authors [3-8] constructed higher dimensional cosmological models in general theory of relativity. Subsequently Sen [11], Sen and Dunn [12] suggested a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry, which in normal gauge may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \varphi_i \varphi_j - \frac{3}{4} g_{ij} \varphi_k \varphi^k = -\chi T_{ij} \quad (1)$$

Where, φ_i is the displacement vector, c = 1, 8πG = χ and other symbols have their usual meaning in the Reimannian geometry. A brief note on Lyra's geometry is given by Beesham [22], Singh and Singh [13], Halford [14-14a]has pointed out that the constant vector displacement field φ, in Lyra geometry plays the role of cosmological constant in the normal general relativistic treatment. Fivedimensional cosmological model in Lyra's manifold are constructed by Rahman et al. [15-17], Singh et al. [18] and Mohanty et al. [19-21]. Beesham [22] considered four-dimensional FRW cosmological model in Lyra's geometry with time dependent displacement field. He has shown, by assuming the energy density of the universe to be equal to its critical value, the models have the k = -1 geometry. Singh and Desikan [23] obtained the exact solutions for four-dimensional FRW cosmological model in Lyra's geometry with constant deceleration parameter. They examined the behavior of the displacement field α and the energy density ρ for perfect fluid distribution. Mohanty et al. [20-21] showed the non-existence of five-dimensional perfect fluid cosmological model in Lyra's geometry. Mollah and Singh [24-25], Rao and Reddy[26] obtained the exact solution for five-dimensional LRS Biranchi type I cosmological model in Lyra's geometry with constant deceleration parameter. Further, they obtained the exact solutions of the field equations for empty universe.

Moreover, solutions of Einstein field equation in higher dimensional space times are believed to be of physical relevance possibly at extremely early times before the Universe underwent the compactification transitions. As a result, now the higher dimensional theory is receiving great attention in both cosmology and particle physics. Particle physicists and cosmologists predicted the existence of GUT (Grand Unified Theory). Using a suitable scalar field it was shown that the phase transitions on the early universe can give rise to such objects which are nothing but the topological knots in the vacuum expectation value of the scalar field and most of their energy is concentrated in a small region. As the necessity to study higher dimensional space-time in this field aiming to unify gravity with other interactions the concept of extra dimension is relevant in cosmology. In particular, for early stage of the Universe and theoretically the present four dimensional stage of the Universe might have been preceded by a multi-dimensional stage.

In this paper we have constructed a higher dimensional (N-dimensional) spherically symmetric cosmological model in Lyra's geometry. Exact solutions of the field equations are obtained with constant deceleration parameter. Here we discuss a model universe with different situations, by solving the modified Einstein field equation with the framework of Lyra geometry; specially for Non-flat models and flat models. The explicit solutions of the scale factor are found via the assumption of an equation of state

p = γ ρ, where γ =1/2 Varies cases have been considered. When the space-time dimensions N = 4, our solutions reduces to that of Singh and Desikan [23]. Some physical properties of the models are also discussed.

2. Field Equations:

Here we consider the N-dimensional spherically symmetric space-time in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\omega^2\right] \qquad (2)$$

Where $d\omega^2$ is the line element for a (N-2) sphere and k =1,-1,0.

The N-dimensional time like displacement vector φ_i in (1) is defined as

$$\varphi_i = (\alpha(t)0,0,0,0\ldots,0) \qquad (3)$$

The energy-momentum tensor is taken as

$$T_{ij} = (\rho+\rho)u_i u_j - p g_{ij} \qquad (4)$$

Together with the coordinate satisfying

$$g_{ij}u^i u^j = 1$$

Where p,ρ and u^i are pressure, energy density and N-dimensional velocity vector of the field distribution, respectively. That is u^i has a component (1,0,0,0,0….,0).

The field equations (Eqn.(1)) for the metric (Eqn. (2))with eqns. (3) and (4), become

$$[\tfrac{1}{2}(N-1)(N-2)]H^2 + \tfrac{1}{2}(N-1)(N-2)\frac{k}{R^2} - \tfrac{3}{4}\alpha^2 = \chi\rho \qquad (5)$$

$$(N-2)\dot{H} + [\tfrac{1}{2}(N-1)(N-2)]H^2 + \tfrac{1}{2}(N-2)(N-3)\frac{k}{R^2}\frac{3}{4}\alpha^2 = -\chi\rho(6)$$

Where $H=\frac{\dot{R}}{R}$is the Hubble's parameterand dot denotes differentiation with respect to t.

$$\chi\dot{\rho} = [(N-1)(N-2)]H\dot{H} - (N-1)(N-2)\frac{k\dot{R}}{R^3} - \frac{3}{2}\alpha\dot{\alpha} \qquad (7)$$

Adding Eqn. (5) and (6), we get

$$\chi(\rho+p) + \frac{3}{2}\alpha^2 = (N-2)\frac{k}{R^2} - (N-2)\dot{H} \qquad (8)$$

From Eqn. (7) and (8), we get

$$\chi\dot{\rho} + \frac{3}{2}\alpha\dot{\alpha} + (N-1)\left[\chi(\rho+p) + \frac{3}{2}\alpha^2\right]H = 0 \qquad (9)$$

Eqn. (9) is the equation of continuity.

3. Solutions of Field Equations :

There are two field equations with four unknowns viz R,ρ,α,p.

Assuming the equation of state

P = 2p (**Here equation of state p= γρ and γ =** $\frac{1}{2}$)

(10)

The number of unknowns is reduced to three.

Making use of equation of state and eliminating ρ(t) Eqns. (5) and (6) we get,

$$(N-2)\dot{H} + [\tfrac{3}{4}(N-1)(N-2)]H^2 + \tfrac{1}{4}(N-2)[2(N-3)+(N-1)]\frac{k}{R^2} + \frac{3}{8}\alpha^2 = 0 \qquad (11)$$

Here α^2 plays the role of cosmological term ʌ (t) as one can see from field equations with cosmological term. There are three unknowns and two independent equations (eqns. (5) and (6)). To obtain a unique solution, one more equation is needed. So, we have considered the deceleration parameter to be constant.

Let the deceleration parameter

$$Q = -\frac{R\ddot{R}}{\dot{R}^2} = -\left(\frac{\dot{H}+H^2}{H^2}\right) = l \qquad (12)$$

where l is a constant

The above equation may be written as

$$\frac{\ddot{R}}{R} + l\frac{\dot{R}^2}{R^2} = 0(13)$$

Integrating (13) we get the exact solution

$$(t) = (\mu t + v)^{\frac{1}{t+l}}, \quad l > -1 \qquad (14)$$

and R(t) = $R_0 e^{H_0 t}$, l =−1 (15)

where u, v, R_0 and $H_0$are constant of integration.

Using Eqn. (12) in Eqn. (11) we get,

$$\alpha^2 = \frac{2(N-2)}{3}[\{4l - 2(N-3) - (N-1)\}H^2 - \{2(N-3) + (N-1)\}\frac{k}{R^2}] \qquad (16)$$

Using Eqn. (16) in Eqn. (5) we get,

$$\chi\rho = 2(N-2)[(N-2)\frac{k}{R^2} - \{l - (N-2)\}H^2] \qquad (17)$$

Case-1 Non-Flat Models

- **When l> −1**

For singular models R(0)=0, from eqn. (14) with v=0 and $S_0 =$ $u^{\frac{1}{1+l}}$ we get,

$$R(t) = S_0 t^{\frac{1}{1+l}}, \quad l > -1 \qquad (18)$$

We get using Eqn. (18) from Eqns. (16) and (17),

$$\alpha^2 = \frac{2(N-2)}{3}[\{4l - 2(N-3) - (N-1)\}\frac{1}{(1+l)^2 t^2}$$

$-\{2(N-3)+(N-1)\}\frac{k}{S_0^2 t^{\frac{2}{1+l}}}]$ (19)

χρ =2(N−2)[(N − 2)$\frac{k}{S_0^2 t^{\frac{2}{1+l}}}$ - $\{l-(N-2\}\frac{1}{(s+l)^2t^2}]$ (20)

Here ρ>0 can be obtained for k ≥ 0 with l≤ $(N-2)$ for γ < 1.

From Eqn. (10) and (20), we obtain

ρ+3p =$\frac{5(N-2)}{\chi}\left[\{(N-2)-l\}\frac{1}{(1+l)^2t^2}+(N-2)\frac{k}{S_0^2 t^{\frac{2}{1+l}}}\right]$ (21)

When k=1, Eqn. (19) and (20) reduces to

$$\alpha^2=\frac{2(N-2)}{3}[\{4l-2(N-3)-(N-1)\}\frac{1}{(1+l)^2t^2}$$
$$-\{2(N-3)+(N-1)\}\frac{1}{S_0^2 t^{\frac{2}{1+l}}}] \quad (22)$$

χρ = 2(N−2)[$(N-2)\frac{1}{S_0^2 t^{\frac{2}{1+l}}}-\{l-(N-2)\}\frac{1}{(1+l)^2t^2}]$ (23)

Again from Eqn. (22) we see that when

$$\frac{2(N-3)+(N-1)}{4}<l\le(N-2)$$

$\alpha^2>0\ if\ 0<\frac{2}{t^{1+l}}<\frac{[4l-\{2(N-3)+(N-1)\}]}{\{2(N-3)+(N-1)\}(1+l)^2}S_0^2$(24)

$\alpha^2<0\ if\ \frac{2}{t^{1+l}}>\frac{[4l-\{2(N-3)+(N-1)\}]}{\{2(N-3)+(N-1)\}(1+l)^2}S_0^2$ (25)

But if $\alpha^2=0\ if\ \frac{2}{t^{1+l}}=\frac{[4l-\{2(N-3)+(N-1)\}]}{\{2(N-3)+(N-1)\}(1+l)^2}S_0^2$ (26)

When 4l = {2(N−3) + $(N-1)$}, Eqns. (22) and (23) reduce to

$\alpha^2=\frac{2(N-2)}{3}[-\{2(N-3)+(N-1)\}\frac{1}{S_0^2 t^{\frac{8}{3(N-1)}}}]$ (27)

χρ =2(N−2)[$\frac{(N-2)}{S_0^2 t^{\frac{8}{3(N-1)}}}$] (28)

from Eqn.(27), it s evident that α^2< 0 for all times for γ < 1 and from Eqn. (28) it is clear that ρ > 0 for all times as γ < 1.

- When l = −1

Eqn. (12) becomes

$\dot{H}=0 \Rightarrow H=H_0=constant$ (29)

Using Eqn. (29) in Eqn. (16) and (17), we get

$$\alpha^2=\frac{2(N-2)}{3}[-\{3(N-1)\}H_0^2$$
$$-\{2(N-3)+(N-1)\}\frac{k}{S_0^2}e^{-2H_0t}] \quad (30)$$

χρ =2$(N-2)[(N-1)H_0^2+(N-2)\frac{k}{S_0^2 t^{\frac{2}{1+l}}}]$ (31)

From Eqn. (30), it is clear that $\alpha^2<0$ for all times for γ < 1 and from Eqn. (31) it is clear that ρ > 0 for all times as γ < 1 and k≥ 0

Case-II <u>Flat Models</u>

With k=0 Eqns. (16) and (17) reduce to

$\alpha^2=\frac{2(N-2)}{3}[\{4l-2(N-3)(N-1)\}]H^2$(32)

And χρ = 2(N−2)[{(N−2) – $l\}H^2$](33)

From Eqn. (33), we see ρ≥ 0 if l≤ $(N-2)$, for γ < 1, therefore

$\alpha^2>0$ if l >$\frac{2(N-3)+(N-1)}{4}$ (34)

$\alpha^2<0$ if l < $\frac{2(N-3)+(N-1)}{4}$ (35)

Also from Eqn. (32) we get

When l = $\frac{2(N-3)+(N-1)}{4}$, $\alpha^2=0$ (36)

- **When l>−1**

For singular models R(0)=0, from Eqn. (14) with ρ=0 and $S_0=\mu^{\frac{1}{1+l}}$ we get,

R(t) = $S_0 t^{\frac{1}{1+l}}$, l >−1 (37)

Eqns. (32) and (33) with Eqn. (37) become

$\alpha^2=\frac{2(N-2}{3}[\{4l-2(N-3)-(N-1)\}\frac{1}{(1+l)^2t^2}]$(38)

And χρ =2 (N −2)[{(N −2) − $l\}\frac{1}{(1+l)^2t^2}]$ (39)

Now, since ρ=2p, from Eqn. (41) we get,

ρ+3p = $\frac{S(N-2)}{\chi}[\{(N-2)-l\}\frac{1}{(1+l)^2t^2}]$ (40)

From Eqn. (42) it can be seen that ρ+3p$\ge 0, if\ \gamma\ge -\frac{1}{3}$

Here α^2 plays the role of a variable cosmological term. α^2 is large during the early stage of the universe and it decreases with the increase of time. The energy density ρ(t) decreases with the increase of cosmic time.

From Eqns. (39) and (40), we see that the expressions α^2 and ρ will not be valid for empty universe (i.e. p= ρ=0) and the stiff or Zel'dovich matter (i.e., p= ρ, where $\delta=1$).

4. Discussion:

In this paper, we have studied N-dimensional spherically symmetric FRW cosmological models in Lyra geometry. We have found exact solutions for constant deceleration parameter. Singh and desikan [23]. have presented four-dimensional FRW cosmological models in Lyra geometry with both positive and negative α^2(t). In our problem, we have obtained also solutions with positive and negative α^2(t). But negativeα^2(t) will make α(t) an imaginary quantity. Although purely imaginary values of α(t) have already been considered by Sen [11] and by some other authors, imaginary quantitieshave no physical significance in cosmology. This paper provides comparing between flat and non-flat models.

- We have considered for Non-flat models (i.e. k=1) two cases -

(i) l > −1 and (ii) for l = −1.

- It is seen that for case (i) i.e., (l > −1), α^2(t), which plays the role of cosmological term, is positive ($\alpha^2>0$) under condition for l as

$$\frac{2(N-3)+(N-1)}{4}<l\le(N-2)$$

Considering the possible values of δ, it appears that on calculating Eqn.(24) is true for early universe. $\alpha^2(t)$ is negative for other values of l. In second case (l = −1) α^2(t) is also negative.

- We have also considered for flat models (i.e. k=0) two cases – (i) for deceleration parameter l–1 and (ii) for l =–1.
- It is seen that for case (i)i.e., (l> −1), α^2(t), which plays the role of cosmological term, is positive under condition for l as

l> $\frac{2(N-3)+(N-1)}{4}$

α^2(t) is large during the early stage of the universe and it decreases with the increase of time and also the energy density ρ> 0 decreases with the increase of cosmic time. But case (ii) (l = −1) corresponds to negative α^2(t)

- Our N-dimensional model is suitable for early stage of the universe and explains the different types of distributions of matter and for different type of symmetries of space time.
- It is important in a natural way to make a search for exact solutions for constant deceleration parameter with differenttypes of distributions of matter and for different type of symmetries of space time.
- This is also helpful to provide the idea about study of physical situation at the early stages of the formation of the universe.

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