

A Study of Transportation Problems

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1. Introduction

Industries now a days require to transport their products available at several sources or production centres to a number of destinations or markets. In the process of distributing to various destinations, high transportation costs are involved. Minimizing the transportation cost will benefit the organisation by increasing the profit. To analyse and minimize the cost of transportation, transportation model is used. The name "transportation model" is, however, misleading. This model can be used for a wide variety of situations such as scheduling, personnel assignment, product mix problems and many others, so that the model is really not confined to transportation or distribution only.

The origin of transportation models dates back to 1941 when F.L. Hitchcock presented a study entitled 'The Distribution of a Product from Several Sources to Numerous Localities'. The presentation is regarded as the first important contribution to the solution of transportation problems. In 1947, T.C. Koopmans presented a study called 'Optimum Utilization of the Transportation System'. These two contributions are mainly responsible for the development of transportation models which involve a number of production centres / sources and a number of destinations / markets. Each shipping source has a certain capacity and each destination has a certain requirement associated with a certain cost of transportation from the sources to the destinations. The objective is to minimize the cost of transportation while meeting the requirements at the destinations. Transportation problems may also involve movement of a product from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, retailers to customers, etc.

2. Assumptions in the Transportation Model

Total quantity of the items available at different sources/ supply is equal to the total requirement/ demand at different destinations / markets.

Items can be transported conveniently from all sources to destinations.

The unit transportation cost of the item from all sources to destinations is known.

The transportation cost on a given route is directly proportional to the number of units shipped on that route.

The objective is to minimize the total transportation cost for the organization as a whole and not for individual supply and distribution centres.

In the supply chain environment, several problems related to transportation and others apparently unrelated can be formulated and solved by the technique used for the typical transportation problem, frequently simply denoted by the initials transportation problem.

Besides the transportation problem proper, we shall address: the (simple) production scheduling; the transshipment problem; and the assignment problem (AP). These problems can be solved by their own algorithms: the transportation problem, the production scheduling and the transshipment, by the "stepping-stone" method; and the AP by the Hungarian method. As all these problems are particular cases of Linear Programming (LP), the problems will be presented and then formulated as LP problems. Indeed, with the current availability of high quality LP software, it looks unnecessary to go into the details of those other methods.

The general goal is to "transport" (whatever that may be) goods to the customers at minimum global cost of transportation, according to the unit costs of transportation (certainly according to distance, etc.) from the sources to the destinations.

The transportation problem is a distribution-type problem, the main goal of which is to decide how to transfer goods from various sending locations (also known as origins) to various receiving locations (also known as destinations) with minimal costs or maximum profit. As long as the number of origins and destinations is low, this is a relatively easy decision. But as the numbers grow, this becomes a complicated linear programming problem. Think about Walmart. In 2016, it had 5,229 stores and 166 distribution centres in the US! It would be impossible to calculate the optimal shipping routes without a computer algorithm.

It deals with sources where a supply of some commodity is available and destinations where the commodity is demanded. The classic statement of the transportation problem uses a matrix with the rows representing sources and columns representing destinations. The algorithms for solving the problem are based on this matrix representation. The costs of shipping from sources to destinations are indicated by the entries in the matrix. If shipment is impossible between a given source and destination, a large cost of M is entered. This discourages the solution from using such cells. Supplies and demands are shown along the margins of the matrix. As in the example, the classic transportation problem has total supply equal to total demand.

	D1	D2	D3	Supply
S1	3	1	M	5
S2	4	2	4	7
S3	M	3	3	3
Demand	7	3	5	

Figure 9. Matrix model of a transportation problem.

The transportation problem and cycle cancelling methods are classical in optimization. The usual attributions are to the 1940's and later. However, Tolsto(1930) was a pioneer in operations research and hence wrote a book on transportation planning which was published by the National Commissariat of Transportation of the Soviet Union, an article called Methods of ending the minimal total kilometrage in cargo-transportation planning in space, in which he studied the transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost cycle in its residual graph. He might have been the first to observe that the cycle condition is necessary for optimality. Moreover, he assumed, but did not explicitly state or prove, the fact that checking the cycle condition is also sufficient for optimality. The transportation problem is concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from sources to destinations that minimizes the total transportation cost.

There are various types of transportation models and the simplest of them was first presented by Hitchcock (1941). It was further developed by Koopmans(1949) and Dantzig (1951). Several extensions of transportation model and methods have been subsequently developed.

Transportation Problem (TP) is based on supply and demand of commodities transported from several sources to the different destinations. The sources from which we need to transport refer the supply while the destination where commodities arrive referred the demand. It has been seen that on many occasion, the decision problem can also be formatting as transportation problem. In general we try to minimize total transportation cost for the commodities transporting from source to destination. There are two types of Transportation Problem namely (1) Balanced Transportation Problem and (2) Unbalanced Transportation Problem.

Definition of Balanced Transportation Problem: A Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand.

Definition of Unbalanced Transportation Problem: A Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

Transportation problem can also be formulated as linear programming problem that can be solved using either dual simplex or Big M method. Sometimes this can also be solved using interior approach method. However it is difficult to get the solution using all this method. There are many methods for solving transportation problem. Vogel's method gives approximate solution while MODI and Stepping Stone (SS) method are considered as a standard technique for obtaining

to optimal solution. Since decade these two methods are being used for solving transportation problem. Goyal (1984)improving VAM for the Unbalanced Transportation Problem, Ramakrishnan(1988) discussed some improvement to Goyal's Modified Vogel's Approximation method for Unbalanced Transportation Problem. Moreover Sultan (1988) andSultan and Goyal (1988) studied initial basic feasible solution and resolution of degeneracy in Transportation Problem. Few researchers have tried to give their alternate method for overcoming major obstacles over MODI and SS method. Adlakha and Kowalski (1999, 2006) suggested an alternative solution algorithm for solving certain transportation problem based on the theory of absolute point. Ji and Chu (2002)discussed a new approach so called Dual Matrix Approach to solve the Transportation Problem which gives also an optimal solution. Recently Adlakhaand Kowalski (2009) suggested a systematic analysis for allocating loads to obtain an alternate optimal solution. However study on alternate optimal solution is limited in the literature of transportation problem. In this chapter we have tried an attempt to provide two alternate algorithms for solving transportation problem. It seems that the methods discussed by us in this chapter are simple and a state forward. We observed that for certain transportation problem, our method gives the optimal solution. However for another certain transportation problem, it gives the near to optimal solution. In this chapter we have discussed only balanced transportation problem for minimization case however these two methods can also be used for maximization case. Moreover, we may also use these two methods for unbalanced transportation problem for minimization and maximization case.

Transportation Method : The transportation method consists of the following three steps.

1. Obtaining an initial solution, that is to say making an initial assignment in such a way that a basic feasible solution is obtained.
2. Ascertaining whether it is optimal or not, by determining opportunity costs associated with the empty cells, and if the solution is not optimal.
3. Revising the solution until an optimal solution is obtained.

The first step in using the transportation method is to obtain a feasible solution, namely, the one that satisfies the rim requirements (i.e. the requirements of demand and supply). The initial feasible solution can be obtained by several methods. The commonly used are

- (I). North – west Corner Method
- (II). Least Cost Method (LCM)
- (III). Vogel's Approximation Method (VAM)
- (I) North-West corner method (NWCM)The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North –West corner (i.e., top left corner)

Steps

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, i.e., $\min(s_1, d_1)$.

2. Adjust the supply and demand numbers in the respective rows and columns allocation.
3. If the supply for the first row is exhausted then move down to the first cell in the second row.
4. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.
5. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.
6. Continue the procedure until the total available quantity is fully allocated to the cells as required.

This chapter deals two alternate algorithms for transportation problem as very few alternate algorithms for obtaining an optimal solution are available in the textbook and in other literature. These methods are so simple and easy that makes understandable to a wider spectrum of readers. The methods discussed in this gives a near optimal solution for certain transportation problem while it gives optimal solution for other certain transportation problem.

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