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Forced Longitudinal Oscillations of A Semi Infinite Viscoelastic Rod

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ABSTRACT

In this paper we have followed the analysis process of Hunter (4) and studied that the effect of the neglected initial transients longitudinal force on the temperature field is very small.

1. Introduction

Singh and Singh (5) considered wave propagation in a linear non-homogenous thin viscoelastic semiinfinite rod.

Stouffer (6) did sufficient work on a thermal-Hereditary Theory for linear viscoelasticity. Christensen (2) also worked on theory of viscoelasticity.

In this paper we have followed the analysis process of Hunter (4). We have obtain that the effect of the neglected initial transients longitudinal force on the temperature field is very small.

2. Basic Equations

We consider the propagation of harmonic waves in an isotropic infinite medium with temperature independent mechanical properties, and for convenience we take $F_i = 0, Q = 0$

We now consider

(2.1) $(\sigma_{jk}, e_{jk}, u_j, T) = (\hat{\sigma}_{jk}, \hat{e}_{jk}, \hat{u}_j, \hat{T}) e^{i\omega t}$

Next we write

(2.2) $(\hat{\sigma}_{jk}, \hat{e}_{jk}, \hat{u}_j, \hat{T}, \hat{\phi}, \hat{\Psi})$

$$= (\Omega_{jk}, E_{jk}, U_j, \theta, A, \underline{B}) \, exp\, (-il_j x_j)$$

where the constants $\Omega_{jk}, E_{jk}, U_j, \theta, A, B$ are amplitudes not all dependent; and l_j are constants, in general depending on ω which will be related to phase velocity and attenuation and which the direction of wave propagation.

From (2.2) and (2.1) we have

(2.3) $$u_1 = u(x,t) = \hat{u}(x) e^{i\omega t} = \frac{\partial \hat{\theta}}{\partial x} e^{i\omega t}$$

$$= i\eta A \exp[i(\omega t - \eta x)]$$

(2.4) $$u_2 = u_3 = 0, \underline{\hat{\Psi}} = 0$$

Writing A' for $i\eta A$ and taking $\eta = \eta_1 = A + \epsilon A_2$ which corresponds to waves propagating in the positive x-direction.

We have then

(2.5) $$u(x,t) = A' \exp\left[iw\left\{t - \frac{x}{v_M}\right\} - a_M x\right]$$

Also in the absence of coupling $(\epsilon = 0)$, we have

(2.6) $$V_M = |C^*| sec\left(\delta_{1/2}\right)$$

$$a_M = \frac{\omega}{|C^*|} sin\left(\delta_{1/2}\right) = \frac{\omega}{v_M} tan\left(\delta_{1/2}\right)$$

We now assume that these same definitions are valid when the material properties are functions of temperature T. The only point to be noted now is that the these velocity v_M and attenuation a_M will be functions of not only ω but also T. We further assume that C^*satisfies W.L.F. law.

(2.7) $$C^* = C^*(i\omega \, exp\{-\phi T\}),$$

where the 'Shift factor' $\phi(t)$ is given by

(2.8) $$\phi(T) = \frac{k(T - T_g)}{(\beta + T - T_g)}$$

Here $C^*(i\omega)$ is the value of C^* at $T = T_g$, k and β are constants.

For any polymer materials k and β are universal constants given by

(2.9) $k = 8.86, \beta = 101.6^0 C$

3. Solution

An approximate solution for u(x,t) can be obtained from (2.5) by making use of the zero-th order WKBJ type approximation (Bremmer [1], Singh & Singh [5]),given by

(3.1) $$u = u_0 \exp\left[iw\left(t - \int_0^x \frac{dx}{v}\right) - \int_0^x a dx\right]$$

where for convenience we have written

(3.2) $A' = u_0, v_M = v, a_M = a$

The expression (3.1) reduces to (2.5) where v and a are constant. The approximation (3.1) is valid for slowly varying temperature fields.

A precise inequality for the validity of (3.1) for thermorheologically simple solid obeying (2.7) and (2.8) is

(3.3) $$\frac{k\beta}{(\beta+T_0-T_g)^2}\left|\frac{a}{dx}\right| < \frac{2nw}{v}$$

In the derivation of (3.3) we have with the help of the approximation (Hunter[3]).

(3.4) $$\frac{d(\log E_1)}{d(\log w)} \approx \frac{2\tan\delta_1}{\pi}$$

where

(3.5) $\rho C^{*2} = E = E_1 + iE_2$

If the rod is traced to oscillate at $x = 0$, then we may take the boundary condition to be

(3.6) $u = u_0 \sin \omega t,$ at $x = 0$

which prescribes the amplitude u_0 and frequency ω. Solution (3.1) satisfying (3.6) should be taken as

$$u = u_0 \sin\left[\omega\left\{t - \int_0^x \frac{dx}{v}\right\}\right] \exp\left[-\int_0^x a dx\right]$$

4. Discussion

The sudation obtained here is actually a steady-state solution which obtains some time after the intial of the forced oscillations specified by (3.1). However, generally many mechanical cycles might be required to generate significant temperature changes, therefore the effects of the neglected intial transients longitudinal force on the temperature field will be small.

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