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Analysis of Shear Deformation in Rectangular and Wide Flange Sections

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ABSTRACT

Shear deformations are, generally, not considered in structural analysis of beams and frames. But shear deformations in members with low clear span-to-member depth ratio will be higher than normally expected, thus adversely affecting the stiffness of these members. Inclusion of shear deformation in analysis requires the values of shear modulus (modulus of rigidity, G) and the shear area of the member. The shear area of the member is a cross sectional property and is defined as the area of the section which is effective in resisting shear deformation. This value is always less than the gross area of the section and is also referred to as the form factor. The form factor is the ratio of the gross area of the section to its shear area. There are a number of expressions available in the literature for the form factors of rectangular and wide flange sections. However, preliminary analysis revealed a high variation in the values given by them. The variation was attributed to the different assumptions made, regarding the stress distribution and section behavior. This necessitated the use of three dimensional finite element analysis of rectangular and wide flange sections to resolve the issue.

1. Introduction

Structural design is based on the forces developed in the members due to the applied loads. The member forces are developed primarily due to deformations in the structures caused by these loads. Thus it is very important to accurately determine the deformations for the design to be adequate. Flexural and axial deformations in structures can be determined to a high level of accuracy. Deformations in multi-story structures are affected by shear deformations in the members. However, shear deformations in structures are, typically, a very low percentage of the total deformation, allowing them to be neglected in most cases. A good amount of research has been done on modeling and predicting the value of shear deformations, but the issue is still far from being resolved. This is because the determination of shear deformation requires the calculation of a quantity referred to as 'Shear Area' of the member cross section. 'Shear Area' is generally understood to be the effective area of the section participating in the shear deformation and as such is a nebulous value ranging from the gross cross-sectional area to the area of the web for a wide flange section. For structures involving members having low clear span to member depth ratio, shear deformations could be responsible for as high as 20% of the total drift (Charney, 1990). This necessitates a better understanding of shear deformations. The traditional method of including the effects of shear deformation is also not accurate. Consider a cantilever beam of length L. The deflection of the tip, Δ, under a point load P including the shear deformation is given by the Equation 1.1.

$$\Delta = \frac{PL^3}{3EI} + \frac{PL}{GA_v} \quad (1.1)$$

where E is the modulus of elasticity, G is the shear modulus, I is the moment of inertia of the section about the bending axis and Av is the shear area of the section. The first term in Equation 1.1 represents the flexural deformation and the shear deformation is denoted by the second term. It is clear that the flexural term will dominate the deflection for increasing values of member length if the other quantities are held constant. Figure 1. shows the contribution of flexural and shear deformations for a W24x335 section for different span to depth ratios. The shear area, Av, was taken as the web thickness times the depth between the centers of the flanges for this exercise.

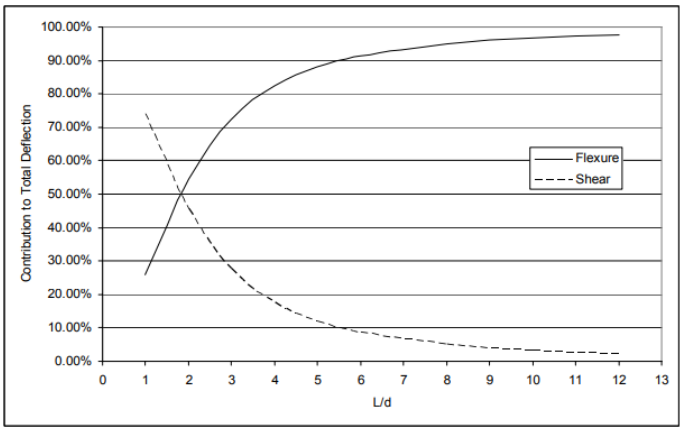

Figure 1: Flexure and shear contribution to the total deflection of a cantilevered W24x335

2. Literature review

There are many closed form solutions in the literature for calculation of shear area of rectangular and wide flange beams. Newlin and Trayer (1924) (NT) present a formula for calculation of shear area based on the principle of virtual work. The expression can be divided into flange and web components of the form factor. Cowper (1966) has developed expressions for form factors for a variety of sections. The Cowper expression is a strain energy solution of the three-dimensional theory of elasticity equation using shear stress functions given in Love (1944) for the various cross sections. Schramm et al. (1994) develop a tensor of 'shear deformation coefficients' using the energy solution of the two-dimensional theory of elasticity equation.

The non-diagonal terms are zero when the coefficients are computed for the principal axes of the section. Timoshenko and Goodier (1970) introduced compatibility conditions for shear stress distributions in beams, resulting in the shear stress distribution along the width of a section to be a function of Poisson's ratio and the shape of the cross section. This affects the calculations of form factor, and Renton (1991) presents a closed form solution for the form factor of rectangular sections incorporating the compatibility conditions of Timoshenko and Goodier (1970). This is also an energy solution of the elasticity equation. Timoshenko (1940) defines form factor as the ratio of maximum stress in the section at the neutral axis to the average stress across the section.

Thus, the form factor for rectangular sections is 1.5 according to Timoshenko (1940). Pilkey (2003) develops a finite element analysis based solution similar to Schramm et al. (1994) and provides a computer program for calculation of shear deformation coefficients of any arbitrary cross section. The values for rectangular sections are nearly identical to those of Renton (1991). Young and Budynas (2002) have presented the simplified expression found in Newlin and Trayer (1924) and McGuire et al (2002) have presented the expression in Timoshenko (1940) as the shear area of the wide flange sections. The following sections will discuss in detail some of the solutions mentioned above.

Rectangular Sections

The form factor of the section depends on the shear stress distribution assumed along the section. The stresses obtained from simple beam theory are different from those obtained from the theory of elasticity. The rectangular section considered for discussion in this thesis is shown in Figure 2.

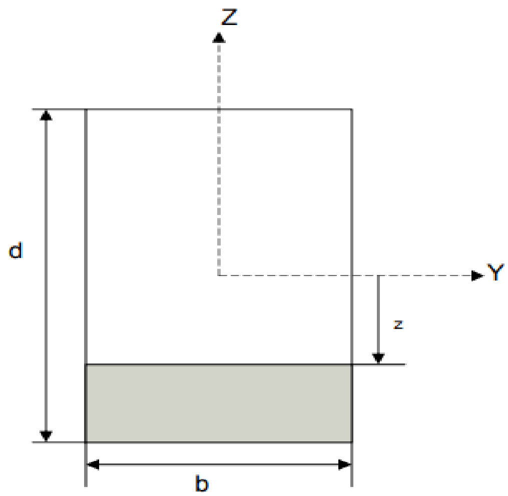

Figure 2: Typical rectangular section

3. Shear deformation for rectangular sections

The software package SAP 2000 v7.44 (CSI, 2002) was used for the analysis. Linear static analysis was performed using SAP2000. The cantilever beams were modeled using the eight-node brick element available in SAP2000. The brick element has three translational degrees of freedom per node, resulting in twenty-four degrees of freedom per element. The brick element was chosen over a shell element because it allows for more elements through the width and depth of the section. All six components of the stress tensor are calculated without any assumption regarding behavior of the cross section. A special program was written for generating the input files for SAP2000. Only half of the cross-section of the beams was modeled to increase computational efficiency and permit higher mesh density across the model. Beams were oriented such that the beam length extended in the X direction, the cross-section strong axis was along the Y direction and the weak axis was along the Z direction.

Early in the testing program, it was discovered that a true fixed boundary condition, where every node was restrained in the X, Y and Z directions at the support, caused tension field action in some beams at certain lengths. This phenomenon is shown in the Figure 3. The beam in the Figure 3 is a 10 ft long W36x798 with a 10,000 kip load applied at the tip of the cantilever. The lightly colored areas denote compression and the dark colored areas denote tension. The stress contours should have shown compression in the bottom half of the beam along the entire length and tension in the top of the beam.

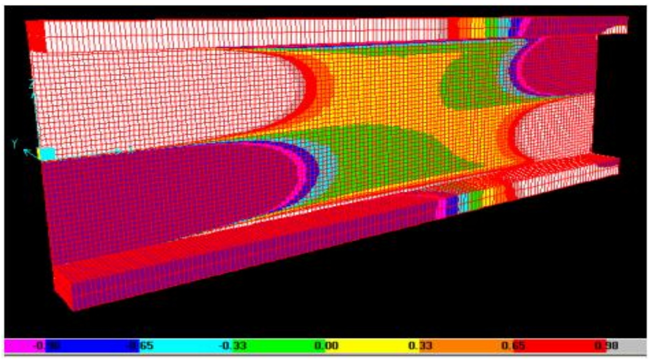

Figure 3: Tension field action due to fixed boundary condition

To combat this problem, two steps were taken. First, the fixed boundary conditions were modified as previously mentioned. Second, the load at the tip was applied differently and equal and opposite loads were added at the support. The loads were modified to more closely resemble the shear stress distribution in a rectangular section. For a rectangular section, the shear stress diagram is parabolic, reaching a maximum at the neutral axis. Since the applied loads could only be nodal, they were scaled to resemble the shear stress distribution. Also, the equal but opposite loads, with the same distribution, were applied at the fixed boundary condition. This was done to create an ideal shear stress distribution at the support as at the free end. As stated earlier, the new restraints coupled with the applied support forces at the fixed boundary condition aided in creating an ideal stress distribution.

Before discussing the results for shear area from the finite element analyses, it is interesting to discuss the τ_{zx} shear stresses reported for sections with various width to depth ratios for different values of Poisson's ratio. The stress function, φ , provided in Timoshenko and Goodier (1970), is a function of the width and the depth of the section. It is in the form of an infinite series and is used to determine the correction to be applied to the stresses obtained by elementary beam theory. It is developed for any given section by solving the equilibrium conditions and compatibility conditions on the boundary of the section. The correction to the stresses is provided for τzx stresses and also for τyx stresses. But since the τyx stresses are zero from the elementary beam theory, the corrections are the net resultant τyx stresses in the section. Figure 4 shows the rectangular section used for determining the stress function. The corrective stresses are applied accordingly since the nomenclature and orientation of the rectangular section considered are different from those adopted for this paper.

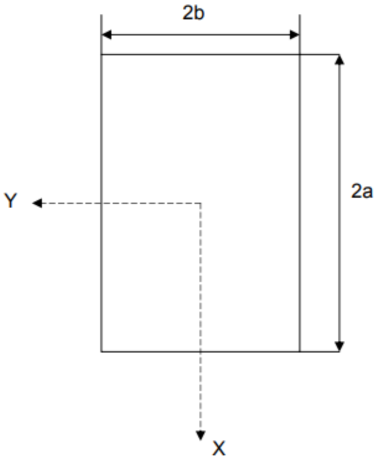

Figure 4: Rectangular cross section used for stress function

4. Shear deformation for wide flange sections

The wide flange sections were modeled about the strong axis using eight-node solid elements in SAP2000 v7.44 (CSI, 2002). Only half of the cross-section of the beams was modeled to increase computational efficiency and permit higher mesh density across the model. Beams were oriented such that the beam length extended in the X direction, the crosssection strong axis was along the Y direction and the weak axis was along the Z direction. The applicable boundary conditions are as follows:

- Restraints were placed in the Y direction at the plane where the beams were cut in half. This set of restraints ensured that the half model behaved like the full model.
- Restraints were placed at the support end in the X direction at all nodes. Only the node along the horizontal neutral axis of the cross-section lying in the plane of symmetry of the model is restrained in the Y direction. All the nodes on the horizontal neutral axis of the cross-section are restrained in the Z direction.

This set of restraints ensured that the cross-section was free to expand and contract due to Poisson's effects, but behaved like a fixed connection for the loads applied. It is important to note that the fixed end restraints just described had to be coupled with applied support forces to accurately create the fixed condition. The above procedure was adopted due to the presence of tension field action in some beams as explained in Chapter 3. The models of wide flange sections developed did not include the fillets found in the rolled sections. The fillets serve the purpose of reducing the stress concentration at the region of the web-flange interface. It will be established that neglecting the fillets in the model does not affect the calculation in an appreciable manner. The half models of the cantilever beams were generated by a special auxiliary program. The program was configured to create the input files for SAP2000 using the input data provided.

5. Results of Analytical Expressions

For all sections and for all weights there appear to be significant differences between the analytical expressions, and the relative applicability of the empirical expressions appears to vary with both depth and weight. Figure 5: provides the values of shear area using different expressions for some of the W14, W24 and W36 sections. The largest dispersion among results produced by the various expressions occurs for the heavier sections, particularly for the heaviest W14s. It is also noted that there appears to be consistent agreement between the results computed from Cowper's expression and those obtained from the empirical equation when the effective depth is taken as the depth between the centers of the flanges. Table 1 provides the values of form factors due to different expressions for some sections.

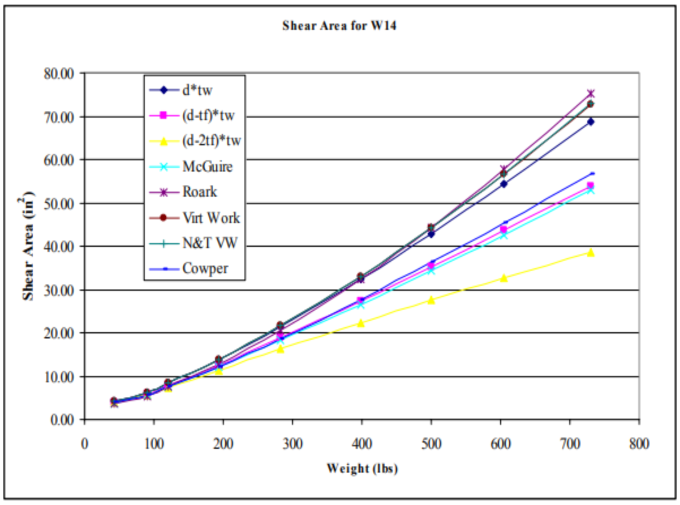

Figure 5: Values of shear area for various sections using the different expressions

Table 1: Variation of Form Factor values given by different expressions for some wide flange section

Section	Form Factor							
	N&T Virtual Work	N&T Simplified (Roark)	Timoshenko (McGuire)	Cowper (Pilkey, 1994)	Cowper Trend Line	Effective depth, d-2tf	Effective depth, d-tf	Effective depth, d
W14x730	2.9350	2.8470	4.0490	3.80	3.82	5.551	3.993	3.118
W24x250	2.7210	3.0090	3.0950	2.93	2.97	3.13	2.888	2.68
W36x135	1.8880	2.1760	2.1420	1.96	1.88	1.929	1.885	1.843

6. Conclusions

For rectangular sections, the form factor consists of two terms, one representing simple beam theory and the other representing the effect of warping of the section. The portion of the form factor based on simple beam theory is exactly 1.2 regardless of the material and the width to depth ratio of the section. The term associated with warping depends on the Poisson's ratio, and should, according to the theory of elasticity, increase with increasing width to depth ratios. Cowper's expression for the form factor of rectangular sections produces 1.2 when Poisson's ratio is zero, and decreases slightly for larger values of Poisson's ratio. The sectional width to depth ratio does not enter into Cowper's expression. Both of these trends are contrary to the solution, which uses shear stresses predicted by Timoshenko and Goodier's theory of elasticity. Renton's solution for the form factor for rectangular sections is based on Timoshenko and Goodier's theory of elasticity formulation. Renton's form factor is exactly 1.2 when Poisson's ratio is zero, and converges to 1.2 for sections that have width to depth ratios less than unity. Renton's expression for form factor produces values significantly greater than 1.2 when the section width is much greater than the depth. Pilkey's finite element analysis program produces form factors similar to those predicted by Renton.

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