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Mathematical psychology: An Approach to Psychological Research

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ABSTRACT

Mathematical psychology is an approach to psychological research that is based on mathematical modeling of perceptual, thought, cognitive and motor processes, and on the establishment of law-like rules that relate quantifiable stimulus characteristics with quantifiable behaviour. The mathematical approach is used with the goal of deriving hypotheses that are more exact and thus yield stricter empirical validations. Quantifiable behaviour is in practice often constituted by task performance. As quantification of behaviour is fundamental in this endeavour, the theory of measurement is a central topic in mathematical psychology. Mathematical psychology is therefore closely related to psychometrics. However, where psychometrics is concerned with individual differences (or population structure) in mostly static variables, mathematical psychology focuses on process models of perceptual, cognitive and motor processes as inferred from the 'average individual'. Furthermore, where psychometrics investigates the stochastic dependence structure between variables as observed in the population, mathematical psychology almost exclusively focuses on the modeling of data obtained from experimental paradigms and is therefore even more closely related to experimental psychology/cognitive psychology/psychonomics. Like computational neuroscience and econometrics, mathematical psychology theory often uses statistical optimality as a guiding principle, assuming that the human brain has evolved to solve problems in an optimized way. Central themes from cognitive psychology; limited vs.unlimited processing capacity, serial vs. parallel processing, etc., and their implications, are central in rigorous analysis in mathematical psychology.

1. Introduction

Mathematical psychology is not, per se, a distinct branch of psychology. Indeed, mathematical psychologists can be found in any area of psychology. Rather, mathematical psychology characterizes the approach that mathematical psychologists take in their substantive domains. Mathematical psychologists are concerned primarily with developing theories and models of behavior that permit quantitative prediction of behavioral change under varying experimental conditions. There are as many mathematical approaches within psychology as there are substantive psychological domains. As with most theorists of any variety, the mathematical psychologist will typically start by considering the psychological phenomena and underlying structures or processes that she wishes to model. A mathematical model or theory (and we do not distinguish between them here) is a set of mathematical structures, including a set of *linkage* statements. These statements relate variables, equations, and so on with components of the psychological process of interest and possibly also aspects of the stimuli or environment. Regardless of the domain, then, the first step in a mathematical approach is to quantify the variables, both independent and dependent, measured to study a psychological process. Quantification permits variables to be represented as parameters in a mathematical equation or statistical expression, the goal and defining feature of the mathematical psychology enterprise.

Mathematical psychologists, then, construct mathematical and statistical models of the processes they study. Some domains, such as vision, learning and memory, and judgment and decision making, which frequently measure easily quantifiable performance variables like accuracy and response time, exhibit a greater penetration of mathematical reasoning and a higher proportion of mathematical psychologists than other domains. Processes such as the behavior of individual neurons, information flow through visual pathways, evidence accumulation in decision making, and language production or development have all been subjected to a great deal of mathematical modeling. However, even problems like the dynamics of mental illness, problems falling in the domains of social or clinical psychology, have benefited from a mathematical modeling approach (e.g., see the special issue on modeling in clinical science in the *Journal of Mathematical Psychology* [Townsend & Neufeld, 2010]).

The power of the mathematical approach arises when unrealized implications of particular model structures become obvious after the mathematical representation of the model has been written down. By contrast, although verbal models might possess logical structure, the inability to interpret concepts in a mathematical fashion means that we cannot derive their logical implications. The ability to make such derivations for mathematical representations leads to better testability of theories, improved experimental designs targeting specific model predictions, and better data analyses—such analyses frequently being rooted in the statistical properties of the model variables.

Mathematical modeling is the foundation of many of the physical sciences. In comparison to these, psychology is often described as a “young” science; as Laming (1973) described several decades ago, psychologists are still often focused on the questions of what is happening rather than why it is happening. Mathematical psychologists, pointing to the role that mathematics has played in the advancement of the physical sciences, have argued that advancement in psychology (and other social sciences) will depend on the extent to which mathematical theorizing is applied to psychological issues. A testament to this argument is the fact that although not all important psychological models are mathematical, a great many of them are.

Psychology differs from a physical science in more than its age, and the use of mathematical models will not, on its own, carry psychology forward. First, the systems studied by psychologists are far more complex than comparable systems in the physical sciences; and second, relationships between psychological variables are obscured by intrinsic variability in these complex systems. Thus, progress in psychology is tied to progress in statistics as well as technological developments that improve our ability to measure behavior. Even the best mathematical tools may not improve our understanding of some quirk of human behavior if we are unable to measure that behavior or discriminate between changes in that behavior and random fluctuations— fluctuations in either our measurements or the cognitive system we are studying.

2. Modern Mathematical Psychology

If one sampled a mathematical psychologist at random, one would find that she could be roughly categorized along four (nonorthogonal) dimensions. First of all, we might determine whether her modelling is strictly axiomatic or more loosely formulated. Next, we could determine whether she takes primarily a deterministic or a stochastic modelling approach. Then, we could ask whether her approach is primarily analytic or computational. Finally, her work may be primarily empirical or theoretical. At the risk of oversimplification, an axiomatic approach is one in which the modeler writes down some primary definitions and then statements (axioms) about what should be true. For example, the modeler may specify mathematical definitions on the basis of the desire to represent situations in which people are presented with stimulus pairs and that their task is to choose the stimulus in the pair with the greatest perceived magnitude. An axiom might then be that, when presented with two tones (the stimulus pair), people should be able to identify correctly the one that is louder with probability greater than or equal to 0.5. These axioms, then, permit the association of mathematical variables and formulas to psychological concepts. Given a set of axioms, the modeler can go on to make logical inferences about what people should do under different conditions.

Axiomatic theorems do not usually address issues of intrinsic randomness—they tend to be deterministic. Given fixed-model parameters and a fixed stimulus, the model produces one and only one result. A stochastic model, by contrast, might produce very different results even when the parameters and the stimulus are fixed. Models of cognitive processing are frequently stochastic. Sequential sampling models, such as those reviewed by Ratcliff and Smith (2004), are a perfect example of the stochastic approach. Predictions about behavior are often focused on how dependent variables are distributed, and how the parameters of those distributions change with changes in experimental procedures. An analytical approach is one in which dependent variables Y can be written as analytical expression involving independent variables X, or Y = g(X) for a function g that does not require any messy numerical calculations (like taking a limit or integrating). The general linear model employed in regression is one example of an analytic expression. The expressions providing finishing time distributions for serial and parallel processing systems (e.g., Townsend, 1972, 1976; also see the section Model Testing, Evaluation, and Comparisons) are other examples.

In contrast, a nonanalytic expression does not allow one to write Y = g(X) and generate predictions for Y algebraically; instead, a computer must be used to simulate the model or solve for Y. Often, the more complex the issue being addressed, the more likely it is that a computational approach will be necessary. Techniques for model comparison (Pitt, Myung, & Zhang, 2002), Bayesian model fitting (Lee, 2008), and models devoted to particularly intractable problems like text comprehension or language processing (e.g., Dennis & Kintsch, 2007) often require a computational approach.

Finally, many mathematical psychologists are also empiricists: They collect data to test their models. However, there is a subset of mathematical psychologists who rarely or never collect data; their work is primarily theoretical. When theoretical work suggests a certain empirical approach, they either collaborate with empiricists or, if it is available, they reanalyze already published data. These mathematical psychologists make theoretical contributions that suggest new mathematical representations of different psychological problems, or methodological contributions that provide new techniques of analysis. They are rather akin to theoretical physicists, some of whom had remarkable insights about the nature of things but were notoriously inept in the laboratory.

3. Foundational Measurement

Work in foundational measurement has followed the tradition established by Fechner (Falmagne, 1986). It is axiomatic, analytic, deterministic and, for the most part, theoretical. Its goal is to find measurement systems capable of quantifying psychological experience—to measure such experience. In the physical world, we measure objects frequently. We weigh ourselves, we compute distance, we mark time. Such physical quantities are based in extensive measurement, which requires the existence of a ratio scale (one with a true zero). We are so accustomed to making measurements of this sort that it seems natural to extend this kind of logic to psychological problems. However, the axioms of extensive measurement may not be justified for the measurement of psychological experience (cf. Narens, 1996).

Foundational measurement represents the first and oldest approach to applying mathematical reasoning to psychological problems. In many ways, foundational measurement set the tone for mathematical work in psychology, especially in psychophysics and decision making. The pioneering research of Patrick Suppes and R. Duncan Luce is especially notable.

Suppes, although officially a philosopher, was perhaps the first, along with Dana Scott, to put a mathematical foundation under the psychological scales proposed by Stevens (1961; Scott & Suppes, 1958; Suppes & Zinnes, 1963). Luce brought the mathematics developed for foundational measurement to bear on problems both in psychophysics and decision making, leading to some of the field's most impressive contributions extending from the 1950s until the present day (Luce, 1959, 2004; Narens & Luce, 1986; Steingrimsson & Luce, 2005a, 2005b, 2006, 2007).

Psychophysics is amenable to a measurement approach because the physical quantity of interest is usually easy to measure (e.g., frequency of a tone) and there is a corresponding continuum of psychological experience (e.g., pitch). A fairly large body of beautiful mathematics has been developed to represent the psychological experience of magnitude in detection and discrimination tasks (e.g., Colonius & Dzhafarov, 2006; Falmagne, 1985; Krantz, Luce, Suppes, & Tversky, 1971; Luce, Krantz, Suppes, & Tversky, 1990; Suppes, Krantz, Luce, & Tversky, 1989). For decision making, the goal of foundational measurement has been to derive scales of preference for objects on the basis of the frequency with which people choose one object over another. An axiomatic approach provides a basis for predicting what people should prefer in various circumstances. Violations of these predicted preferences point to incorrect axioms, which in turn leads to a greater understanding of how people make decisions. Tversky and Kahneman's work (e.g., Tversky & Kahneman, 1974, 1981) demonstrated above all that perfectly sensible axioms, such as those underlying expected utility theory, do not apply in many decision-making environments. Their work led to Kahneman's Nobel prize in Economics in 2002. Work in foundational measurement is generally deterministic, meaning that it deals primarily with the algebraic properties of different measurement systems. This fact means that, although mathematically quite elegant, measurement theories are often quite removed from empirical treatments and,

indeed, may be difficult or impossible to empirically evaluate because the variability of real data obscure and distort the relationships predicted by the theories (Luce, 2005; Narens & Luce, 1993). Although there have been several promising inroads to formulating stochastic approaches to foundational measurement over the past decade or so (Falmagne, Reggenwetter, & Grofman, 1997; Heyer & Niederée, 1989; Myung, Karabatsos, & Iverson, 2005), as yet there is no completely satisfactory solution.

4. Cognitive Modeling

Mathematical approaches to modeling cognitive processes are now fairly well ingrained in mainstream cognitive psychology. These approaches are equally balanced between analytic and computational models, but they are primarily stochastic and almost always empirical. It will not be possible for us to give a comprehensive treatment of every area in cognitive psychology for which mathematical modeling is important because this task would require many books. We focus on memory, categorization, choice response time, and neural modeling. Memory. Nowhere else in experimental psychology has mathematical work had a greater impact than in the development of models for memory.

Mathematical models of recognition and recall now set the standard for theoretical developments in this area and have driven empirical research before them. Memory models no longer follow the early examples set by statistical learning theory and models of information processing. It became obvious in the 1960s and early 1970s that the complexity of the process to be modeled was not adequately captured by linearly decomposing it into a sequence of subtasks(e.g., Sternberg, 1966). This led to the development of connectionist models (see below) and machine-learning-inspired models that incorporate learning, problem solving, and language comprehension (e.g., Dennis, 2005; Jilk, Lebiere, O'Reilly, & Anderson, 2008; Kintsch, McNamara, Dennis, & Landauer, 2007). Signal-detection theory still plays a very important role in most memory models. Older strength theories (Atkinson & Juola, 1973; Murdock, 1965; Parks, 1966) relied on the signal-detection framework as the basis for the old–new judgment. Newer global memory models—such as those proposed by Murdock (1982), Hintzman (1988), and Gillund and Shiffrin (1984), and even more recent models such as retrieving effectively from memory (Shiffrin & Steyvers, 1997)—develop encoding, storage, and retrieval architectures explaining how memory traces are established, maintained, and decay over time as well as how different memory traces become associated with each other and to the context in which they were experienced. Each of these models requires, however, an evaluation of memory strength for a recognition decision, and this evaluation is assumed to be performed within a signal-detection framework.

Although global memory models go some way toward explaining how memory strength contributes to recognition performance, many researchers have explored the contributions of other memory processes, often lumped together under the term *recall.* In this sense, recall is the ability to remember specific details of the remembered item, and this ability requires conscious effort. In contrast, recognition is based only on perceived strength, which happens effortlessly. Some memory work is focused on separating these different cognitive contributions to recognition decisions (e.g., Wixted, 2007). The receiver operating characteristic curve from signal detection is used to try and separate the signal detection recognition component from the recall component. Dual-process memory theories thus combine the signal-detection approach with a less quantitatively specified recall component.

Another theoretical avenue to multiprocess memory models are the multinomial processing-tree models explored by Batchelder and Riefer (1999). This general approach provides a way to explore many different structures producing categorical measurements of behavior. The multinomial processing tree model considers how different components of a task depend on each other (e.g., if recall fails, evaluate familiarity) but does not explain the mechanisms by which each component operates. So whereas signal-detection theory might explain the probability that a subject calls an item old, the multinomial approach only assumes that such a probability exists. The approach allows for a consideration of different latent structures and comparisons between different model architectures. It has been applied to a wide range of problems, most recently in the evaluation of cognitive deficits (e.g., Batchelder & Riefer, 2007). It lends itself well to Bayesian analysis and is closely linked to measurement problems in psychometrics (Batchelder, 2010).

5. Categorization

Categorization tasks ask observers to classify stimuli according to their types. These types may be quite concrete (e.g., chairs, dogs, diseases) or they may be very abstract. As in memory research, several influential mathematical models of categorization have set a standard for explanations of categorization behavior, and much of the empirical work in categorization over the past few decades has been driven by these models.

The first class of these models assumes that subjects construct a mental representation of different categories and that categorization decisions are made

on the basis of the psychological distances (often referred to as similarities) between a stimulus and other objects (exemplars) in the mental space (Nosofsky, 1988; Nosofsky & Palmeri, 1997). These models take much inspiration from early work in multidimensional scaling (Torgerson, 1958), which was used to derive scales that could measure multidimensional stimuli and place them in relation to each other.

The second class of these models assumes that categories of stimuli can be represented as probability distributions in multidimensional space (Ashby,1992; Ashby & Gott, 1988). Categorization judgments are made on the basis of a stimulus's location in that space relative to multidimensional discriminant functions (lines, planes, hyperplanes) that divide the space into categories. These models are called decision-bound models, and they are closely related to signal-detection models. They preserve the ideas of discriminability, bias, optimality, and so forth from signal detection, but the interest is more on how different stimulus dimensions are perceived and how those perceptions influence the placement of decision bounds.

6. Current Issues in Mathematical Modeling

As mathematical psychology continues to mature, with the inevitable growing pains that process engenders, there has been some navel-gazing about where the discipline is headed (Luce, 1999, 2005; Townsend, 2008). In the heady 1950s and 1960s, mathematical psychology seemed the road toward a physical science of psychology, but perhaps the road did not go to the places the field's founders anticipated it would. If true, there might be several reasons for this, one being that (of course) one's children never grow up to become what one thought they would. Mathematical psychology prospers, even though it hasn't quite followed in its parents' footsteps. Mathematical psychology is currently tackling two major issues, and both are focused primarily on methodology: How to distinguish between different models of the same process, and constructing Bayesian methods for the analysis of behavioral data. We discuss each of these before closing the paper.

7. Model Testing, Evaluation, and Comparisons

One very important area in mathematical psychology addresses the problem of how to discriminate between different models. This is a long-standing problem in any field that constructs mathematical and statistical models, including statistics, where this issue is dealt with by considering issues of goodness of fit, variance accounted for, information criteria, Bayes factors, and so forth. In addition, the possibility that models based on very different psychological principles or mechanisms might be mathematically similar or even identical, the challenge of *model mimicking*, can generate a formidable threat to the uncovering of psychological laws. These and other important topics are outlined in this section.

Mathematical psychologists have recently focused on the issue of model complexity. That is, one model may fit data better than another not because it is a better model but only because it is more complex. Complexity is not just a question of how many parameters a model has. Two models may have the same number of parameters yet one of them (the more complex one) may be able to accommodate a wider range of data patterns than the other. Dealing with this issue borrows ideas from computer science and has its roots in information theory. Computer scientists have developed numerical techniques for quantifying complexity, opening the way for a different perspective on model selection. Pitt, Myung, and colleagues (Pitt, Kim, Navarro, & Myung, 2006; Pitt, Myung, Montenegro, & Pooley, 2008) are applying these techniques to a number of different problems, including the optimization of experimental designs for model testing and explorations of model parameter spaces. Another method for model testing and selection is the powerful state-trace analysis methodology invented by Bamber (1979) and recently made popular by Dunn (2008). This technique is applied to problems for which the goal is to determine how many processes are contributing to the performance of a task (see the discussion of dual-process memory models). Many empirical pursuits try to answer the question of "how many processes" by looking for dissociations in patterns of data. That is, situations in which one experimental variable moves a dependent variable in the opposite direction (or not at all) of another variable. This finding is sometimes called selective influence, and it is used to argue that one variable affects one process whereas another variable affects a different process independent from the first. State trace analysis is a simple technique based on minimal assumptions. In particular, no particular probability distributions, other mathematical functions, or parameters are required. On the basis of this technique, Dunn and colleagues have argued that, in many situations, dissociations do not provide strong evidence for multiple processes (e.g., Dunn, 2004, 2008; Newell & Dunn, 2008). Another approach to model testing uses the strong inference philosophy described by Platt (1964). The fundamental idea requires the scientist to set up a series of two or more juxtaposed hypotheses, rather than the more typical "there is a (predicted) effect" versus "there is no effect." For example, we might first test whether a psychological phenomenon takes place within short-term versus long-term memory and then follow that with a test of whether the coding system in that memory is verbal or spatial. Or, we might formulate two or more entire classes of models that obey contrasting fundamental principles. The scientist first tests among these models and, in a second stage of research, begins to test among more specific models within the winning class.

Research on serial versus parallel processing of elements in visual and memory search illustrates the challenges of model mimicking (e.g., Townsend, 1972, 1974) as well as the opportunity for implementation of strong inference (e.g., Townsend, 1984). For instance, parallel and serial models can, for some popular experimental designs, produce exactly the same predictions and thus be totally indiscriminable (e.g., Townsend, 1972). However, Townsend and Wenger (2004) presented mathematical formulations for large classes of parallel and serial models, formulations that highlight empirically distinguishable aspects of the different structures. They then use these class differences as assays to test the models.

The strategies we mentioned earlier for identification of even more complex architectures (Schweickert,1978; Schweickert & Townsend, 1989) also adhere to this strategy. With these assays, juxtaposed models can be refined to be more and more specific so that, for example, if the assays suggest that processing is parallel, then we might go on to test, say, a diffusion process (e.g., Ratcliff, 1978) versus a counting mechanism (e.g., Smith & Van Zandt, 2000). The issue of how to select among different mathematical models of a process will never be considered “solved” any more than the perfect statistical procedure for all circumstances will be discovered. As models change over the years, techniques for testing and selecting them will necessarily evolve.

8. Conclusion

Modern mathematical psychology is a critical component of modern experimental psychology. From its earliest inception, mathematical psychology has made important contributions to our understanding of learning, memory, perception, and choice behavior; mathematical models continue to guide research in these areas as well as language acquisition and comprehension, problem solving, categorization, and judgment. Although modest in number, mathematical psychologists appear as leaders in many psychological disciplines, especially in cognition and neuroscience. They have been elected to the most esteemed societies in experimental psychology as well as the elite National Academy of Sciences. Several mathematical psychologists (Herbert Simon, Patrick Suppes, William K. Estes, and R. Duncan Luce) have received the highest scientific honor in the United States, that of receiving the National Medal of Science.As experimental psychology matures, it is likely that our current definition for what constitutes mathematical psychology will change. Eventually, we hope, experimental psychologists will all use mathematical reasoning and develop mathematical models, and thus everyone will be mathematical psychologists under the definition we have provided in this paper. However, just as there remain specifically mathematical subdisciplines in the physical and life sciences (e.g., physics, chemistry, and biology), we anticipate that mathematical psychology will endure as a unique endeavor among the different subdisciplines that make up the science of psychology.

Mathematical psychologists are active in many fields of psychology, especially in psychophysics, sensation and perception, problem solving, decision-making, learning, memory, and language, collectively known as cognitive psychology, and the quantitative analysis of behaviour but also, e.g., in clinical psychology, social psychology, and psychology of music.

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