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Somewhat Preopen Functions

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ABSTRACT

A weakened form of open function termed somewhat open function has been defined in this paper. Some basic properties of this function has been investigated.

1. Introduction

In 1971 K.R.Gentry [3] introduced a weakened form of continuity termed somewhat continuity and a weakened open function named Somewhat open function. In 1982, Mashhour et al. [5] defined preopen sets and precontinuous function. In this paper somewhat pre open function is defined with the aid of preopen sets. Composition of somewhat open functions and how these functions behave in subspaces have been discussed in detail. Finally some basic properties of this function have been investigated.

Section 2 of this paper contains some known definitions and results that have been frequently used in this paper.

2. Preliminaries

Throughout the paper (X, τ) or X always denotes a non trivial topological space. The closure and interior of $A \subset X$ are respectively denoted by Cl (A) and Int (A). The restriction of a function $f : X \to Y$ to $A \subset X$ has been denoted by f/A throughout the paper. The following definitions will be required for the presentation of the paper.

Definition 2.1. A subset A of X is called a preopen set [5] briefly p.o.(resp. semi preopen[1], an α-set [6]) if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$) .

The family of all preopen (resp. semi preopen,α-sets) sets of X is denoted by PO(X) (resp.SPO(X),α(X)).The complement of a p.o. set is called preclosed.

Definition 2.2. [5] The preclosure of $A \subset X$ is denoted by pcl (A) and is defined by pcl (A) = ∩ {B : B is preclosed and $B \supset A$}.

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly α continuous iff [7] briefly w.α.c. iff for each $x \in X$ and each open set V containing f (x) in Y there exists an α set U containing x in X such that $f[U] \subset \mathrm{Cl}(V)$.

Lemma 2.2. [4]. Let $A \subset Y \subset X$ and $Y \in PO(X)$, then $A \in PO(X)$ iff $A \in PO(Y)$.

Lemma 2.3. [8]. In a space X if $A \in PO(X)$, $B \in \alpha(X)$, then $A \cap B \in PO(X)$.

Definition 2.4. [3] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat open if for each $U \in \tau$ with $U \neq \phi$ there exists a $V \in \sigma$ with $V \neq \phi$ such that $V \subset f[U]$.

Definition 2.5. [2] Let X be a set with two topologies τ_1 and τ_2. Then τ_1 is termed weakly pre equivalent to τ_2 provided if $U \in PO(X, \tau_1)$ and $U \neq \phi$, then there is a $V \in PO(X, \tau_2)$ such that $V \neq \phi$ and $V \subset U$ and if $U \in PO(X, \tau_2)$ and $U \neq \phi$ then there exists a $V \in PO(X, \tau_1)$ with $V \neq \phi$ and $V \subset U$.

Definition 2.6. [2] A subset A of a space X is said to be predense if pcl (A) = X.

3. Results and Discussions

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat preopen (briefly s.p.o.) provided that if $U \in PO(X, \tau)$ and $U \neq \phi$, then there is a $V \in PO(Y, \sigma)$ such that $V \neq \phi$ and $V \subset f[U]$.

Remark 3.1. Obviously every somewhat open function is s.p.o. but not conversely as is clear from the following example.

Example 3.1. Let X ={a,b,c}, τ = the discrete topology on X and σ={ϕ, X}.Then the identity function $f : (X, \tau) \to (X, \sigma)$ is s.p.o. but not somewhat open.

Theorem 3.1. If $f_1 : X \to Y$ and $f_2 : Y \to Z$ are s.p.o. functions, then $f_2 0 f_1 : X \to Z$ is s.p.o..

Proof : Let $U \in PO(X)$ and $U \neq \phi$. Since f_1 and f_2 are s.p.o. there exist a non-empty sets $V \in PO(Y)$ with $V \subset f_1[U]$ and a non-empty set $W \in PO(Z)$ such that $W \subset f_2[V]$ respectively. This leads to $W \subset f_2[V] \subset f_2 0 f_1[U]$. Hence $f_2 0 f_1$ is s.p.o..

s.p.o. IN SUBSPACES

Example 3.2. Let X = {a, b, c, d} = Y be endowed with the topology $\tau_X = \{\phi, X, \{c, d\}\} = \tau_Y$. Then the identity mapping f : (X,

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$\tau_X) \to (Y, \tau_Y)$ is s.p.o. Taking A = {a}, it is easy to see that f/A : A → Y is not s.p.o..

Remark 3.2. The above example shows that the s.p.o. of a function does not ensure the s.p.o. of the function in any subspace.

On the otherhand, it has been found that a function is s.p.o. in any preopen subspace if it is given to be so in the whole space. In fact the following holds.

Theorem 3.2. If f : X → Y is s.p.o. and A ∈ PO (X), then f/A : A → Y is s.p.o..

Proof : Let U ∈PO (A) and U ≠ ϕ.Now U ⊂ A ⊂ X, A∈ PO (X), U ∈PO (A) imply that U ∈ PO (X). Since f is s.p.o. there exists a V ∈ PO (Y) such that V ≠ ϕ and V⊂ f [U]. Hence f/A is s.p.o..

Remark 3.3. In the preceding example the failure of Theorem 3.2 is due to the fact that A ∉ PO (X).

Theorem 3.3. For a function f : X → Y, if A ∈ α (X) is predense and f/A : A → Y is s.p.o., then f is s.p.o..

Proof : Let U ∈ PO (X) and U ≠ ϕ. A being an α-set, U ∩ A ∈ PO (X). Again since A is predense in X, U ∩ A ≠ ϕ. Let U ∩ A = G.Then G ∈ PO (X).Now since every α-set is a p.o. set, A ∈ PO (X). Thus G ⊂ A ⊂ X, G ∈ PO (X), A ∈ PO (X). This implies that G ∈ PO (A). S.p.o. of f/A guarantees the existence of a H ∈ PO (Y) such that H ≠ ϕ, and H ⊂ f [G]. Therefore, H ⊂ f [U ∩ A] ⊂ g [U ∩ A] ⊂ g [U]. Hence g is s.p.o..

Remark 3.4. The condition “A is predense” cannot be dropped from the hypothesis of the above theorem as shown by the following Example.

Example 3.3. Let X = {a, b, c} = Y be the set with the topologies τ_X = {ϕ, X, {a}, {b, c}}and τ_Y = {ϕ, Y, {a}}respectively. Let A = {a}, then A ∈ α (X), which is not predense. But the identity map f/A : A → Yis s.p.o. On the otherhand, the identity map f : X → Y is not s.p.o..

Theorem 3.4. Let f : X → Y be a mapping and X = $X_1 \cup X_2$ where X_1, X_2 ∈ α (X). If both the restrictions $f/X_i : X_i \to Y$, i = 1, 2 are s.p.o., then f is s.p.o..

Proof : Let U ∈ PO (X) and U ≠ ϕ. Since U ⊂ $X_1 \cup X_2$ at least one of U ∩ X_i (i = 1, 2) ≠ ϕ, otherwise U = ϕ and it is contrary to our assumption. To fix our ideas we suppose U ∩ X_1 ≠ ϕ.Because every α set is a p.o. set U ∩ X_1 ∈ PO (X). Again U ∩ X_1 ⊂ X_1 ⊂ X, produces that U ∩ X_1 ∈ PO (X_1). Since f/X_1 is s.p.o. there exists a V ∈ PO (Y) such that V ⊂ (f/X_1) (U ∩ X_1) ⊂ f [U] which, in its turn, yieldsV ⊂ f [U].This indicates that f is s.p.o. .

Basic properties of *spc* functions

Theorem 3.5. If f : X → Y is a function, then the following two conditions are equivalent

(i) f is s.p.o. ;

(ii) if M is a predense subset of Y, then f^{-1} [M] is a predense subset of X.

Proof. Let f be s.p.o. and M be a predense subset of Y. If possible f^{-1} [M] is not a predense subset of X. This then implies that there exists a non-empty set U ∈ PO (X) such that f^{-1} [M] ∩U = ϕ ⇒ f [f^{-1} [M]∩ U] = ϕ ⇒ M ∩ f [U] = ϕ.Now the s.p.o. of f and preopenness of the non-empty set U together imply the existence of a non-empty set V ∈ PO (Y, σ) such that V ⊂ f [U]. Then from above M ∩ V = ϕ, a contradiction to the hypothesis that M is a predense subset of Y. Thus f^{-1} [M] is a predense subset of X.

Definition 3.7. A function f : X → Y is said to be quasi-semi-preopen (briefly qsp-open) if f [U] ∈ SPO(Y) for every U ∈ PO (X).

Theorem 3.6. Let f : X → Y be a function. If f is weakly α contnuous and s.p.o. then f is qsp-open.

Proof : Let A ∈ PO (X) and y ∈ f [A].Then there exists a x ∈ A such that f (x) = y. Let V be an open containing y in Y. Since f is w.α.c. there exists a U ∈ α (X, x) such that f [U] ⊂ Cl (V).Clearly x ∈ A ∩ U.⇒ A ∩ U ≠ ϕ.Set W = A ∩ U. Then W ≠ ϕ and by W ∈ PO (X). Again since f is s.p.o. there exists a G ∈ PO (Y), G ≠ ϕ, such that G ⊂ f [W] ⇒ G ⊂ f [U ∩ A] ⊂ f [U] ∩ f [A] ⊂ Cl (V) ∩ f [A] ⊂ Cl (V) ∩ Cl (f [A]).From the forgoing it then follows that G ⊂ Cl (f [A])⇒ G ⊂ pint (Cl (f [A])).

⇒ G ⊂ f [A] ∩ Int (Cl (f [A])) ⇒ G ⊂ Int (Cl (f [A])).Also from above G ⊂ Cl (V).Thus Cl (V) ∩ Int (Cl (f [A])) ⊃ G ≠ ϕ

⇒ V ∩ Int (Cl (f [A])) ≠ ϕ⇒ f (x) ∈ Cl (Int Cl (f [A]))

⇒ y ∈ Cl (Int (Cl (f [A])⇒ f [A] ⊂ Cl (Int (Cl (f [A]))) ⇒ f [A] ∈ SPO (Y). Therefore f is qsp-open.

Theorem 3.7. Let f : (X, τ) → (Y, σ) be s.p.o., τ′ and σ′ are topologies for X and Y respectively such that τ′ is weakly pre equivalent to τ and σ′ is weakly pre equivalent to σ, then f : (X, τ′) → (Y, σ′) is s.p.o..

Proof : Let U′ ∈ PO (X, τ′) and U′ ≠ ϕ. Since τ′ is weakly pre equivalent to τ there exists a U ∈ PO (X, τ) such that U ≠ ϕ and U ⊂ U′.Since f is s.p.o. there exists a non-empty set V ∈ PO (Y, σ) such that V ⊂ f [U]. Again since σ′ is weakly pre equivalent toσ there exists aV′ ∈PO (Y, σ′) such that V′ ≠ ϕ and V′ ⊂ V. Then from above V′ ⊂ f [U] ⊂ f [U′]. Hence f is s.p.o..

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