Volume-04
Issue-05
May-2019

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com[UGC Listed Journal]

Charged Elastic Fluid Sphere In General Relativity

[1]Dr. Purushottam & [2]Dr. R.B.S. Yadav

[1]*Department of Mathematics, Govt. Women's Polytechnic, Patna, Bihar (India)*
[2]*P.G. Department of Mathematics, Magadh University, Bodh Gaya, Bihar (India)*

ARTICLE DETAILS

Article History
Published Online: 15 May 2019

Keywords
metric, density, potential, elasticity, variable.

ABSTRACT

The present paper provides solution of Einstein's field equations in charged elastic media in two different cases. In case- 1, the field equations have been solved by choosing a specific relation between metric potential γ and space charge density ρ_e where as in case-II, the same has been done by choosing a particular choice of field tensorF_{ij}. In both the cases we have assumed the density to be a variable quantity. Here we have also discussed the boundary conditions.

1. Introduction

Researches relating to charged elastic media in general relativity has not been studied much.For example, the general elastic analogue of Schwarzschild's interior solution is unknown, in contrast to the Talman-Oppenheimer-Volkoffintegrodifferential equation (Harrison et al. [2]). Roy and singh [6] found a general solution of an elastic sphere of constant density. They solved the equations under condition that the pressure is zero at the boundary of the sphere and the density is constant inside the boundary. As a matter of fact their investigation was an attempt to apply Rayner's theory [5] of elasticity in general relativity for the construction of realistic models. Further the same authors [7]studied the field equations for a charged elastic fluid sphere of constant density and made an additional assumption about the elastic charge density which considerably simplifies the problem. Again, Nduka [4] tried to show that if the spherical body is elastic, equilibrium configuration implies that an extra condition must be imposed on the system. Krori and Pal [3] have solved the field equations of Roy and Singh [7] by working an assumption of a similar character on the electric field and have found that charge density and electric field both vanish at the center. In both the above works [3,7] the density has been taken as constant. Latter on Singh and Yadav [8] and Brito et.al [1] have extended their work by taking the density to be a variable quantity.

In this communication we have studied the charged elastic fluid sphere in two different cases. In case- 1st, we have solved the field equations by taking a suitable relation between metric potential γ and space charge density ρ where as in case 2nd the same has been done by taking a particular choice of field tensor F_{ij} in both the cases we have assumed the density to be variable quantity. Here we have also discussed the boundary conditions.

2. The Field Equations and their Solutions:

We consider the case of a static, spherically symmetric distribution of charged fluid which forms a linear isotropic medium. The space-time representing the fluid is spherically symmetric. It's line element can therefore be put in the form.

$$ds^2 = e^{\gamma}dr^2 + r^2(d\theta^2 + sin^2\theta d\emptyset'^2) - e^{\delta}dt^2 \quad (1)$$

Where γ and δ are functions of r only. For the field equations and symbols we follow related to krori and Pal [3] and Roy and Singh [7].

For the metric (1), the relevant field equations are [7]

$$e^{-\gamma}\left(\frac{\delta'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2} = 4\pi\,(\vartheta+2\mu)(e^{-\gamma}-1)+4\pi\,\bar{\epsilon}\,e^{-(\gamma+\delta)}F_{14}^2 \quad (2)$$

$$e^{-\gamma}\left\{\frac{1}{2}\delta''-\frac{1}{4}\gamma'\delta'+\frac{1}{4}\delta'^2+\frac{1}{2r}(\delta'-\gamma')\right\} = 4\pi\vartheta(1-e^{\gamma})-4\pi\bar{\epsilon}e^{-(\gamma+\delta)}F_{14}^2 \quad (3)$$

$$e^{-\gamma}\left(\frac{\gamma'}{r}-\frac{1}{r^2}+\frac{e^{\gamma}}{r^2}\right) = 8\pi\rho+4\pi\bar{\epsilon}\,e^{-(\gamma+\delta)}F_{14}^2 \quad (4)$$

Also $J^1 = J^2 = J^3 = 0 \quad (5)$

From Maxwell's equations we have

$$\frac{d}{dr}\left(\bar{\epsilon}\,F_{14}r^2e^{-(\gamma+\delta)/2}\right) = \rho_e r^2 e^{\gamma/2} \quad (6)$$

Case 1 : Here we assume that

$$\rho_e = 3Ae^{-\gamma/2} \quad (7)$$

Where A is a constant

Then from (4) and (5) we get

$\bar{\epsilon}F_{14}e^{-(\gamma+\delta)/2}$ = Ar + $\frac{B}{r^2}$ (8)

B being constant. Since we require that F_{14} be regular every where we set B equal to zero.

Hence

$\bar{\epsilon}\, F_{14}e^{-(\gamma+\delta)/2}$ = Ar (9)

The left has side is the physical component of the tensor M_{ij} and as in classical electromagnetic theory it varies as r.

We further assume that

$\rho = \rho_0 + \rho_1 r^n$ (10)

where ρ_0, ρ_1 and n are positive constants.

Putting this values of ρ, f_{14} from (9) and (10) in eq^n (4) we have

$$e^{-\gamma} = 1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4}{5}\pi\frac{A^2}{\bar{\epsilon}}r^4 + \frac{\bar{B}}{r}.$$

However to avoid singularity we set $B = 0$ so that we have

$$e^{-\gamma} = 1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A^2}{\bar{\epsilon}}r^4. \quad (11)$$

Form equations (1) and (2) we have

$$\delta = \int\left[e^{\gamma}\left\{8\pi\left(\frac{\rho_0 r}{3} + \frac{\rho_1 r^{n+1}}{n+3}\right) + \frac{24\pi A^2 r^3}{5\bar{\epsilon}} + 4\pi r(\vartheta + 2\mu)(1 - e^{\gamma})\right\}\right] dr + c \quad (12)$$

where μ and ϑ are elastic constants and c is constant of integration.

Hence the spherically symmetric line element in charged elastic media is given by

$$ds^2 = \exp\left[\int\left\{8\pi\left(\frac{\rho_0 r}{3} + \frac{\rho_1 r^{n+1}}{n+3}\right) + \frac{24\pi A^2 r^3}{5\bar{\epsilon}} + 4\pi r(\vartheta + 2\mu)\left(8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) + \frac{4\pi A^2 r^4}{5\bar{\epsilon}}\right)\right\}\left(1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A^2 r^4}{5\bar{\epsilon}}\right) dr + c\right] dt^2 - \left[1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A^2 r^4}{5\bar{\epsilon}}\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\emptyset^2). \quad (13)$$

Outside the fluid sphere the line element is that due to a spherically symmetric charged body of total charge $4\pi e$, namely

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{4\pi e^2}{r^2}\right)dt^2 - \left(1 - \frac{2m}{r} + \frac{4\pi e^2}{r^2}\right)^{-1} dt^2 - r^2(d\theta^2 + \sin^2\theta d\emptyset^2). \quad (14)$$

The total charge $4\pi e$ *within* the fluid sphere of radius r_1 is given by

$4\pi\rho = \int_0^{r1}\int_0^{\pi}\int_0^{2\pi} 3Ar^2 \sin\theta dr\, d\theta\, d\emptyset.$ (15)

so that A=$\frac{e}{r_1^3}$. (16)

The internal fluid (13) must fit at the boundary with the external field for which we require that g_{ij} and τ_1^1 be continuous at r = r_1.

We thus have three conditions

m= $4\pi\left(\frac{\rho_{0r}{}^3}{3} + \frac{\rho_1 r^{n+3}}{r^2}\right) + \frac{2\pi A^2 r_1^3}{5\bar{\epsilon}}$(17)

$\vartheta(r_1) + 2\mu(r_1) = 0$ (18)

$\left[1 - 8\pi\left(\frac{\rho_{0r}{}^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) + \frac{4\pi A^2 r_1^4}{5\bar{\epsilon}}\right] = \left(e^{\delta}\right)_{r=r_1}$ (19)

Now since $e^{-\gamma}$ is to be positive for all values of r, we require that

$\left(r_1^2 - \frac{3}{16\pi\vartheta_0}\right)^2 + \frac{3\rho_1 r_1^{n+1}}{\rho_0(n+3)} < \left(\frac{3}{16\pi\rho_0}\right)^2 - \frac{3e^2}{10\rho_0\bar{\epsilon}}$ (20)

From (20) it follows that

e $< \left(\frac{15\bar{\epsilon}}{128\pi^2\rho}\right)^{1/2}$(21)

Case- 2 : Here in this case we assume that

$\bar{\epsilon}\, F_{14}e^{-(\gamma+\delta)/2} = A'r^2 + \frac{B'}{r^2}$ (22)

Where A' and B' are constants.

Then from (6) and (22) we get

$\rho_e = 4A're^{-\gamma/2}$ (23)

We choose the material density ρ as in case 1 ie. $\rho = \rho_0 + \rho_1 r^n$.

Putting this value of ρ and $\bar{F}_{14}$ from (22) in equation (4) we have

$e^{-\gamma} = 1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi}{\bar{\epsilon}}\left(\frac{A'^2 r^6}{7} + \frac{2A' B' r^2}{2} - \frac{2B'^2}{r^4}\right)$ (24)

Here we have taken integration constant to be zero.

Then from equation (24) and (2) we can find δ as in case- 1 and thus using values of γ and δ the spherically symmetric line element in charged elastic media can be written down.

However, by the choice of F_{14} as in (22), the solution obtained in this case is singular. So we set $B' = 0$, in equation (22) so that F_{12} becomes regular everywhere and the solution also becomes non-singular.

Thus when $B' = 0$, we get

$\bar{\epsilon}\, F_{14} e^{-(\gamma+\delta)/2} = A' r^2$ (25)

$$e^{-\gamma} = 1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A' r^6}{7\bar{\epsilon}} \quad (26)$$

$$\delta = \int\left[e^{\gamma}\left\{8\pi\left(\frac{\rho_0 r}{3} + \frac{\rho_1 r^{n+1}}{n+3}\right) + \left(\frac{32\pi A'^2 r^5}{7\bar{\epsilon}}\right) + 4\pi(\vartheta + 2\mu)(1 - e^{\gamma})\right\}\right] dr + c' \quad (27)$$

where c' is constant of integration.

Hence the spherically symmetric line element in charged elastic media in this case can be written as:

$$ds^2 = exp\left[\int.\left\{8\pi\left(\frac{\rho_0 r}{3} + \frac{\rho_1 r^{n+1}}{n+3}\right) + \left(\frac{32\pi A'^2 r^5}{7\bar{\epsilon}}\right) + 4\pi r(\vartheta + 2\mu)\left(8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) + \frac{4\pi A'^2 r^6}{7\bar{\epsilon}}\right)\right\}\left(1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A'^2 r^6}{7\bar{\epsilon}}\right) dr + c'\right] dt^2 - \left[1 - 8\pi\left(\frac{\rho_0 r^2}{3} + \frac{\rho_1 r^{n+2}}{n+3}\right) - \frac{4\pi A'^2 r^6}{7\bar{\epsilon}}\right]^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \quad (28)$$

The total charge $4\pi e$ within the fluid sphere of radius $r = r_1$ is given by

$$4\pi\rho = \int_0^{r_1}\int_0^{\pi}\int_0^{2\pi} 4\pi A' r^3 sin\theta dr d\theta d\emptyset \quad (29)$$

So that $A' = \frac{e}{r_1^4}$. (30)

Again as in case 1, the boundary condition yield

$$m = 4\pi\left(\frac{\rho_0 r_1^3}{3} + \frac{\rho_1 r^{n+3}}{n+3}\right) + \frac{2\pi A'^2 r_1^7}{7\bar{\epsilon}} \quad (31)$$

$$\left[1 - 8\pi\left(\frac{\rho_0 r_1^2}{3} + \frac{\rho_1 r_1^{n+2}}{n+3}\right) + \frac{4\pi A_1^2}{7\bar{\epsilon}} r_1^6\right] = (e^{\delta}) r = r_1 \quad (32)$$

The equation (18) remaining the same.

In case II for $e^{-\gamma}$ to be positive for all values of r, we need

$$\left(r_1^2 - \frac{3}{16\pi\rho_0}\right)^2 + \frac{3\rho_1 r^{n+1}}{\rho_0(n+3)} < \frac{9}{256\pi^2\rho_0^2} - \frac{3e^2}{14\rho_0\bar{\epsilon}}. \quad (33)$$

which leads to

$$e < \left[\frac{21\bar{\epsilon}}{2\rho}\right]^{1/2}\frac{1}{8\pi}. \quad (34)$$

3. Discussion & Conclusion:

In case I, equation (21) shows that for a given denity, the total charge has an upper bound. Also equation (20) puts a limit on the radius of fluid sphere. If we put $\rho_0 = 0$ in the results of this case, we get the results due to Roy and Singh [7]. Similarly in case II, equation (34) shows that for a given density, the total charge has an upper bound where as equation (33) puts a limit on the radius of fluid sphere.

Further our solutions presented in this paper are made non-singular &F_{12} also becomes regular everywhere by taking suitable choice of constant B (in case I) and B′ (in case II).

References

1. Brito, I. ,Carot, J. , Vaz. , EGLR. : Gen. Relativ. Gravit., 42, 2357, (2010)
2. Harrison, B. K., Thorne, K. S.,Wakeno, M. and Wheeler, T. A.: Gravitational theory and gravitational collapse, The university of Chicago press (1965)
3. Krori, K. D. and Pal, B. : J. Pure appl. Phys., 14, 852 (1976)
4. Naduka, A. : J. Phys., A8, 1982 (1975)
5. Rayner, C. B. : Proc. Soc. (London), A 272, 44 (1963)
6. Roy, S. R. and Singh, P. N. : J. Phys., A6, 1862 (1973)
7. Roy, S.R. and Singh, P. N. : J. Phys., A7, 1866 (1974)
8. Singh, T. and Yadav, R.B.S. : J. Pure Appl. Phys., 17, 181 (1979)