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Degree Based Reverse Connectivity Indices of Dendrimers

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ABSTRACT

In Medical Science, the connectivity indices are used in the analysis of drug molecular structures. These indices are helpful for medical scientists to find out the chemical and biological characteristics of drugs. In this paper, we compute the sum connectivity reverse index, product connectivity reverse index, atom bond connectivity reverse index and geometric-arithmetic reverse index of prophyrin, prophyl ether imine, poly ethylene amide amine dendrimers and zinc prophyrindendrimers and graphically compared these results.

1. Introduction

Let $G = (V(G), E(G))$ be a finite simple connected graph. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. Let $\Delta(G)$ denote the maximum degree among the vertices of a graph G. The reverse vertex degree of a vertex v in a graph G is defined as $c_v = \Delta(G) - d_G(v) + 1$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. We refer [7] for undefined terms and notations.

A chemical graph or molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Medical Graph Theory has an important effect on the development of Medical Sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Numerous topological indices have been considered in Chemistry and Medical Science see [6, 8].

Recently, the sum connectivity reverse index was introduced by Kulli in [9] and defined as

$$SC(G) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u + c_v}}\right].$$

The product connectivity reverse index was proposed in [10] and defined as

$$PC(G) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u \cdot c_v}}\right].$$

In [11], Kulli introduced atom bond connectivity reverse index of a graph G and it is defined as

$$ABCC(G) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u \cdot c_v}}.$$

The geometric-arithmetic reverse index was introduced in [12] and defined as

$$GAC(G) = \sum_{uv \in E(G)} \left[\frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}\right].$$

Dendrimers are extremely branched, star-shaped macromolecules with nanometer-scale dimensions. Generally dendrimers are large, complex and hyper branch with multiple functional groups on the surface. Dendrimers were first introduced in 1985 by D. A. Tomaliaet. al. [14].Dendrimers are specified by three components : a central core, an interior dendritic structure (the branches), and an exterior surface with functional surface groups. Dendrimers have a vast area of applications in all branches of chemistry, especially in host-guest reactions and self-assembely procedures.

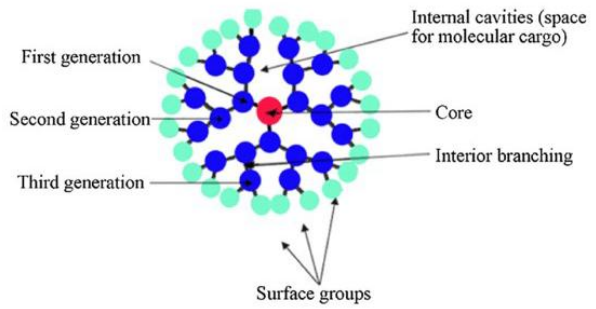

Figure 1: The Structure of Dendrimer.

Dendrimers are used in the formation of nanotubes, nanolatex, chemical sensors, micro/macro capsules, coloured glass, modified electrodes and photon funnels such as artificial antennas [3, 4, 5]. Because dendrimers are widely used in different applied fields, the study of dendrimers has a great deal of attention in both chemical and mathematical literature [1, 2, 13]. These dendrimers have more than forty families which carry unique properties. These particular properties make dendrimers worthy for various applications in medical and industrial technology. In this paper, we derive an expression for some degree based reverse connectivity indices such as sum connectivity reverse index, product connectivity reverse index, atom bond connectivity reverse index and geometric-arithmetic reverse index of prophyrin [1], prophyl ether imine [1], poly ethylene amide amine dendrimers [13] and zinc prophyrindendrimers [13] and graphically compared these results.

2. ProphyrinDendrimerD_nP_n

We consider the family of prophyrindendrimers. This family of dendrimers is denoted by D_nP_n, where n is the steps of growth in this type of dendrimers. The graph of D_nP_n is presented in Figure 2.

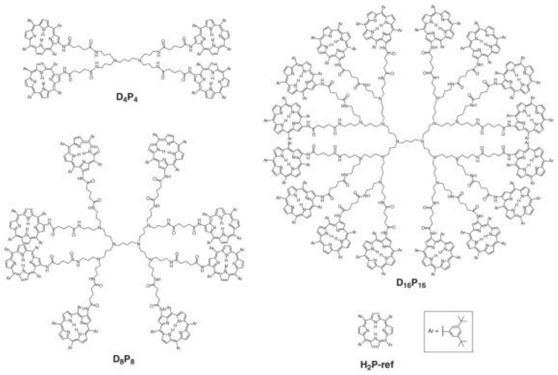

Figure 2: The graph of D_nP_n.

Let $G = D_nP_n$. By calculation, we obtain that G has $96n - 10$ vertices and $105n - 11$ edges. In G, there are six types of edges based on degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G): d_G(u) = 1, d_G(v) = 3\}, |E_1| = 2n.$

$E_2 = \{uv \in E(G): d_G(u) = 1, d_G(v) = 4\}, |E_2| = 24n.$

$E_3 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 2\}, |E_3| = 10n - 5.$

$E_4 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 3\}, |E_4| = 48n - 6.$

$E_5 = \{uv \in E(G): d_G(u) = 3, d_G(v) = 3\}, |E_5| = 13n.$

$E_6 = \{uv \in E(G): d_G(u) = 3, d_G(v) = 4\}, |E_6| = 8n.$

From Figure 2, we see that $\Delta(G) = 4$. Therefore $c_u = \Delta(G) - d_G(u) + 1$. Therefore $c_u = 5 - d_G(u)$. Thus there are six types of reverse edges as given in Table 1.

Table 1

Reverse edge partition of D_nP_n

$c_u, c_v: uv \in E(G)$	(4,2)	(4,1)	(3,3)	(3,2)	(2,2)	(2,1)
Number of edges	$2n$	$24n$	$10n - 5$	$48n - 6$	$13n$	$8n$

Theorem 2.1.1The sum connectivity reverse index of D_nP_n is

$$SC(D_nP_n) = \left(\frac{12}{\sqrt{6}} + \frac{72}{\sqrt{5}} + \frac{13}{2} + \frac{8}{\sqrt{3}}\right)n - \left(\frac{5}{\sqrt{6}} + \frac{6}{\sqrt{5}}\right).$$

Proof. By definition, we have

$$SC(D_nP_n) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u + c_v}}\right]$$

Thus by using Table1, we deduce

$$\begin{aligned} SC(D_nP_n) &= \frac{1}{\sqrt{4+2}}(2n) + \frac{1}{\sqrt{4+1}}(24n) + \frac{1}{\sqrt{3+3}}(10n-5) \\ &+ \frac{1}{\sqrt{(3+2)(48n-6)}} + \frac{1}{\sqrt{(2+2)(13n)}} + \frac{1}{\sqrt{(2+1)(8n)}} \\ &= \left(\frac{12}{\sqrt{6}} + \frac{72}{\sqrt{5}} + \frac{13}{2} + \frac{8}{\sqrt{3}}\right)n - \left(\frac{5}{\sqrt{6}} + \frac{6}{\sqrt{5}}\right). \end{aligned}$$

Theorem 2.2.2The product connectivity reverse index of D_nP_n is

$$PC(D_nP_n) = \left(\frac{9}{\sqrt{2}} + \frac{48}{\sqrt{6}} + \frac{131}{6}\right)n - \left(\frac{5}{3} + \frac{6}{\sqrt{6}}\right).$$

Proof. By definition, we have

$$PC(D_nP_n) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u \cdot c_v}}\right]$$

Thus by using Table1, we derive

$$\begin{aligned} PC(D_nP_n) &= \frac{1}{\sqrt{4 \cdot 2}}(2n) + \frac{1}{\sqrt{4 \cdot 1}}(24n) + \frac{1}{\sqrt{3 \cdot 3}}(10n - 5) \\ &+ \frac{1}{\sqrt{3 \cdot 2}}(48n - 6) + \frac{1}{\sqrt{2 \cdot 2}}(13n) + \frac{1}{\sqrt{2 \cdot 1}}(8n) \\ &= \left(\frac{9}{\sqrt{2}} + \frac{48}{\sqrt{6}} + \frac{131}{6}\right)n - \left(\frac{5}{3} + \frac{6}{\sqrt{6}}\right). \end{aligned}$$

Theorem 2.3.3The atom bond connectivity reverseindex of D_nP_n is

$$ABCC(D_nP_n) = \left(\frac{71}{\sqrt{2}} + 12\sqrt{3} + \frac{20}{3}\right)n - \left(\frac{10}{3} + \frac{6}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$ABCC(D_nP_n) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u \cdot c_v}}$$

Hence by using Table 1, we obtain

$$\begin{aligned} ABCC(D_nP_n) &= \left(\sqrt{\frac{4+2-2}{4 \cdot 2}}\right)2n + \left(\sqrt{\frac{4+1-2}{4 \cdot 1}}\right)24n \\ &+ \left(\sqrt{\frac{3+3-2}{3 \cdot 3}}\right)(10n - 5) + \left(\sqrt{\frac{3+2-2}{3 \cdot 2}}\right)(48n - 6) \\ &+ \left(\sqrt{\frac{2+2-2}{2 \cdot 2}}\right)(13n) + \left(\sqrt{\frac{2+1-2}{2 \cdot 1}}\right)(8n) \\ &= \left(\frac{71}{\sqrt{2}} + 12\sqrt{3} + \frac{20}{3}\right)n - \left(\frac{10}{3} + \frac{6}{\sqrt{2}}\right) \end{aligned}$$

Theorem 2.4.4The geometric-arithmetic connectivity reverse index of D_nP_n is

$$GAC(D_nP_n) = \left(\frac{4\sqrt{2}}{3} + \frac{16\sqrt{2}}{3} + \frac{211}{5} + \frac{96\sqrt{6}}{5}\right)n - \left(5 + \frac{12\sqrt{6}}{5}\right).$$

Proof. By definition, we have

$$GAC(D_nP_n) = \sum_{uv \in E(G)} \left[\frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}\right]$$

Therefore by using Table 1, we have

$$\begin{aligned} GAC(D_nP_n) &= \left(\frac{2\sqrt{4 \cdot 2}}{2+2}\right)2n + \left(\frac{2\sqrt{4 \cdot 1}}{4+1}\right)24n + \\ &\left(\frac{2\sqrt{3 \cdot 3}}{3+3}\right)(10n - 5) + \left(\frac{2\sqrt{3 \cdot 2}}{3+2}\right)(48n - 6) \\ &+ \left(\frac{2\sqrt{2 \cdot 2}}{2+2}\right)(13n) + \left(\frac{2\sqrt{2 \cdot 1}}{2+1}\right)(8n) \quad . \end{aligned}$$

$$= \left(\frac{4\sqrt{2}}{3}+\frac{16\sqrt{2}}{3}+\frac{211}{5}+\frac{96\sqrt{6}}{5}\right)n-\left(5+\frac{12\sqrt{6}}{5}\right)$$

Table 2

Topological index of D_nP_n

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
$SC(G)$	43.49	91.71	139.93	188.14	236.36	284.58	332.80	381.01	429.23	411.48
$PC(G)$	32.08	68.28	104.48	140.67	176.87	213.07	249.26	285.46	321.66	357.86
$ABCC(G)$	70.08	147.74	225.39	303.05	380.70	458.36	536.02	613.67	691.33	768.98
$GAC(G)$	87.78	186.48	285.10	383.75	482.41	581.07	679.73	778.39	877.05	975.70

3. Propyl Ether Imine Dendrimer PETIM.

We consider the family of propyl ether imine dendrimers which is denoted by PETIM. This dendrimer is shown in Figure 3.

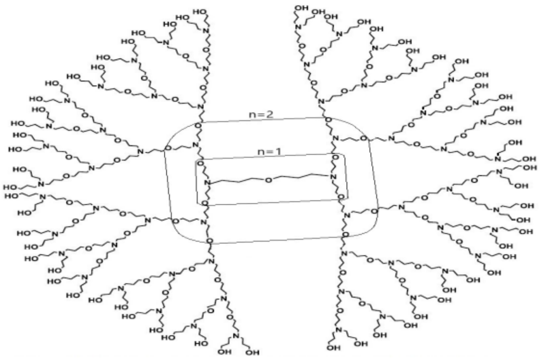

Figure 3: The graph of $PETIM$.

Let G be the graph of $PETIM$ dendrimer. By calculation, Ghas$24\times 2^n-23$ vertices and $24\times 2^n-24$ edges. In G, there are three types of edges based on degree of end vertices of each edge as follows:

$$E_1=\{uv\in E(G): d_G(u)=1, d_G(v)=2\}, |E_1|=2\times 2^n.$$
$$E_2=\{uv\in E(G): d_G(u)=2, d_G(v)=2\}, |E_2| = 16\times 2^n-18.$$
$$E_3=\{uv\in E(G): d_G(u)=2, d_G(v)=3\}, |E_3|=6\times 2^n-6.$$

Clearly$\Delta(G)=3$. Thus$c_u=\Delta(G)-d_G(u)+1=4-d_G(u)$. Thus there are three types of reverse edges as given in Table 3.

Table 3

Reverse edge partition of $PETIM$

$c_u, c_v: uv \in E(G)$	(3,2)	(2,2)	(2,1)
Number of edges	2×2^n	$16\times 2^n-18$	$6\times 2^n-6$

Theorem 3.1.5The sum connectivity reverse index of PETIM dendrimer is

$$SC(PETIM)=\left(\frac{2}{\sqrt{5}}+8+2\sqrt{3}\right)2^n-\left(9+2\sqrt{3}\right).$$

Proof. By definition, we have

$$SC(PETIM)=\sum_{uv\in E(G)}\left[\frac{1}{\sqrt{c_u+c_v}}\right]$$

Therefore by using Table 3, we have

$$SC(PETIM) = \frac{1}{\sqrt{3+2}}(2\times 2^n)+\frac{1}{\sqrt{2+2}}(16\times 2^n-18) +\frac{1}{\sqrt{2+1}}(6\times 2^n-6)$$

$$= \left(\frac{2}{\sqrt{5}}+8+2\sqrt{3}\right)2^n-\left(9+2\sqrt{3}\right).$$

Theorem 3.2.6The product connectivity reverse index of PETIM dendrimer is

$$PC(PETIM)=\left(\frac{2}{\sqrt{6}}+\frac{6}{\sqrt{2}}+8\right)2^n-\left(9+\frac{6}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$PC(PETIM)=\sum_{uv\in E(G)}\left[\frac{1}{\sqrt{c_u\cdot c_v}}\right]$$

Thus by using Table 3, we derive

$$PC(PETIM) = \frac{1}{\sqrt{3\cdot 2}}(2\times 2^n)+\frac{1}{\sqrt{2\cdot 2}}(16\times 2^n-18) +\frac{1}{\sqrt{2\cdot 1}}(6\times 2^n-6)$$

$$= \left(\frac{2}{\sqrt{6}}+\frac{6}{\sqrt{2}}+8\right)2^n-\left(9+\frac{6}{\sqrt{2}}\right).$$

Theorem 3.3.7The atom bond connectivity reverse index of PETIM dendrimer is

$$ABCC(PETIM)=12\sqrt{2}\times 2^n-12\sqrt{2}.$$

Proof. By definition, we have

$$ABCC(PETIM)=\sum_{uv\in E(G)}\sqrt{\frac{c_u+c_v-2}{c_u\cdot c_v}}$$

Thus by using Table 3, we deduce

$$ABCC(PETIM)=\left(\sqrt{\frac{3+2-2}{3\cdot 2}}\right)(2\times 2^n) +\left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)(16\times 2^n-18)+\left(\sqrt{\frac{2+1-2}{2\cdot 1}}\right)(6\times 2^n-6)$$

$$= \quad 12\sqrt{2}\times 2^n - 12\sqrt{2}.$$

Theorem 3.4.8The geometric-arithmetic connectivity reverse index of PETIM dendrimer is

$$GAC(PETIM) = \left(\frac{4\sqrt{6}}{5} + 16 + 4\sqrt{2}\right)2^n - (18 + 4\sqrt{2}).$$

Proof. By definition, we have

$$GAC(PETIM) = \sum_{uv\in E(G)} \left[\frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}\right]$$

Therefore by using Table 3, we derive

$$GAC(PETIM) \quad = \quad \left(\frac{2\sqrt{3\cdot 2}}{3+2}\right)(2\times 2^n)$$

$$+\left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)(16\times 2^n - 18) + \left(\frac{2\sqrt{2\cdot 1}}{2+1}\right)(6\times 2^n - 6)$$

$$= \quad \left(\frac{4\sqrt{6}}{5} + 16 + 4\sqrt{2}\right)2^n - (18 + 4\sqrt{2}).$$

Table 4

Topological index of $PETIM$

	$SC(G)$	$PC(G)$	$ABCC(G)$	$GAC(G)$
$n=1$	12.25	12.88	$12\sqrt{2}$	23.58
$n=2$	36.97	38.99	$36\sqrt{2}$	70.81
$n=3$	86.40	91.23	$84\sqrt{2}$	165.27
$n=4$	185.27	195.70	254.56	354.21
$n=5$	383.01	404.65	526.09	732.07
$n=6$	778.48	822.54	1069.15	1487.79
$n=7$	1569.43	1658.33	2155.26	2999.25
$n=8$	3151.32	3329.89	4327.49	6022.15
$n=9$	6315.10	6673.04	8671.96	12067.96
$n=10$	12642.67	13359.31	17360.89	24159.58

4. Poly Ethylene Amide Amine Dendrimer PETAA.

We consider the family of poly ethylene amine amide dendrimers. This family of dendrimers is denoted by $PETAA$. The graph of $PETAA$ dendrimer is depicted in Figure 4.

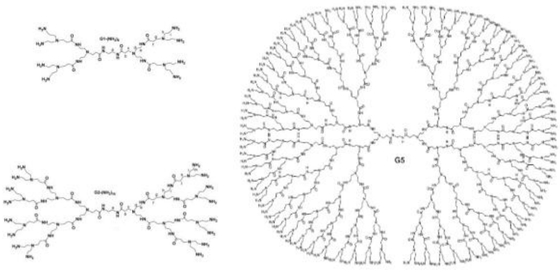

Figure 4: The graph of $PETAA$.

Let G be the graph of $PETAA$ dendrimer. By algebraic method, we obtain that G has $44\times 2^n - 18$ vertices and $44\times 2^n - 19$ edges. In $PETAA$ dendrimer, there are four types of edges based on degree of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G): d_G(u) = 1, d_G(v) = 2\}, |E_1| = 4\times 2^n.$$
$$E_2 = \{uv \in E(G): d_G(u) = 1, d_G(v) = 3\}, |E_2| = 4\times 2^n - 2.$$
$$E_3 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 2\}, |E_3| = 16\times 2^n - 8.$$
$$E_4 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 3\}, |E_4| = 20\times 2^n - 9.$$

Clearly, $\Delta(G) = 3$.
Therefore $c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u)$. Thus there are four types of reverse edges as given in Table 5.

Table 5

Reverse edge partition of $PETAA$

$c_u, c_v: uv \in E(G)$	(3,2)	(3,1)	(2,2)	(2,1)
Number of edges	4×2^n	$4\times 2^n - 2$	$16\times 2^n - 8$	$20\times 2^n - 9$

Theorem 4.1.9The sum connectivity reverse index of PETAA dendrimer is

$$SC(PETAA) = \left(\frac{4}{\sqrt{5}} + 10 + \frac{20}{\sqrt{3}}\right)2^n - (5 + 3\sqrt{3}).$$

Proof. By definition, we have

$$SC(PETAA) = \sum_{uv\in E(G)} \left[\frac{1}{\sqrt{c_u + c_v}}\right]$$

Hence by using Table 5, we derive

$$SC(PETAA) \quad = \quad \frac{1}{\sqrt{3+2}}(4\times 2^n) + \frac{1}{\sqrt{3+1}}(4\times 2^n - 2)$$

$$+\frac{1}{\sqrt{2+2}}(16\times 2^n - 8) + \frac{1}{\sqrt{2+1}}(20\times 2^n - 9)$$

$$= \quad \left(\frac{4}{\sqrt{5}} + 10 + \frac{20}{\sqrt{3}}\right)2^n - (5 + 3\sqrt{3}).$$

Theorem 4.2.10The product connectivity reverse index of PETAA dendrimer is

$$PC(PETAA) = \left(\frac{4}{\sqrt{6}} + \frac{4}{\sqrt{3}} + 8 + \frac{20}{\sqrt{2}}\right)2^n - \left(4 + \frac{2}{\sqrt{3}} + \frac{9}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$PC(PETAA) = \sum_{uv\in E(G)} \left[\frac{1}{\sqrt{c_u \cdot c_v}}\right]$$

Thus by using Table 5, we derive

$$PC(PETAA) = \frac{1}{\sqrt{3\cdot 2}}(4\times 2^n) + \frac{1}{\sqrt{3\cdot 1}}(4\times 2^n - 2) + \frac{1}{\sqrt{2\cdot 2}}(16\times 2^n - 8) + \frac{1}{\sqrt{2\cdot 1}}(20\times 2^n - 9)$$

$$= \left(\frac{4}{\sqrt{6}} + \frac{4}{\sqrt{3}} + 8 + \frac{20}{\sqrt{2}}\right)2^n - \left(4 + \frac{2}{\sqrt{3}} + \frac{9}{\sqrt{2}}\right).$$

Theorem 4.3.11The atom bond connectivity reverse index of PETAA dendrimer is

$$ABCC(PETAA) = \left(\frac{40}{\sqrt{2}} + \frac{4\sqrt{2}}{\sqrt{3}}\right)2^n - \left(\frac{2\sqrt{2}}{\sqrt{3}} + \frac{17}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$ABCC(PETAA) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u \cdot c_v}}$$

Thus by using Table 5, we derive

$$ABCC(PETAA) = \left(\sqrt{\frac{3+2-2}{3\cdot 2}}\right)(4\times 2^n) + \left(\sqrt{\frac{3+1-2}{3\cdot 1}}\right)(4\times 2^n - 2) + \left(\sqrt{\frac{2+1-2}{2\cdot 1}}\right)(20\times 2^n - 9)$$

$$= \left(\frac{40}{\sqrt{2}} + \frac{4\sqrt{2}}{\sqrt{3}}\right)2^n - \left(\frac{2\sqrt{2}}{\sqrt{3}} + \frac{17}{\sqrt{2}}\right).$$

Theorem 4.4.12The geometric-arithmetic connectivity reverse index of $PETIM$ dendrimer is

$$GAC(PETAA) = \left(\frac{8\sqrt{6}}{5} + 16 + 2\sqrt{3} + \frac{40\sqrt{2}}{3}\right)2^n - (\sqrt{3} + 8 + 6\sqrt{2}).$$

Proof. By definition, we have

$$GAC(PETAA) = \sum_{uv\in E(G)} \left[\frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}\right]$$

Therefore by using Table 5, we have

$$GAC(PETAA) = \left(\frac{2\sqrt{3\cdot 2}}{3+2}\right)(4\times 2^n) + \left(\frac{2\sqrt{3\cdot 1}}{3+1}\right)(4\times 2^n - 2) + \left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)(16\times 2^n - 8) + \left(\frac{2\sqrt{2\cdot 1}}{2+1}\right)(20\times 2^n - 9)$$

$$= \left(\frac{8\sqrt{6}}{5} + 16 + 2\sqrt{3} + \frac{40\sqrt{2}}{3}\right)2^n - (\sqrt{3} + 8 + 6\sqrt{2}).$$

Table 6

Topological index of $PETAA$

	$SC(G)$	$PC(G)$	$ABCC(G)$	$GAC(G)$
$n=1$	36.48	40.65	49.47	66.26
$n=2$	83.15	92.82	112.55	150.74
$n=3$	176.49	197.16	238.75	319.69
$n=4$	363.18	405.83	491.15	657.61
$n=5$	736.55	823.19	995.95	1333.45
$n=6$	1483.29	1657.89	2005.56	2685.11
$n=7$	2976.79	3327.30	4024.78	5388.43
$n=8$	5963.78	6666.12	8063.21	10795.09
$n=9$	11937.76	13343.76	16140.08	21608.39
$n=10$	23885.72	26699.04	32293.81	43234.99

5. Zinc ProphyrinDendrimerDPZ_n.

We consider the family of zinc prophyrindendrimers. This family is denoted by DPZ_n, where n is the steps of growth in this type of dendrimers. The graph of DPZ_n is presented in Figure 5.

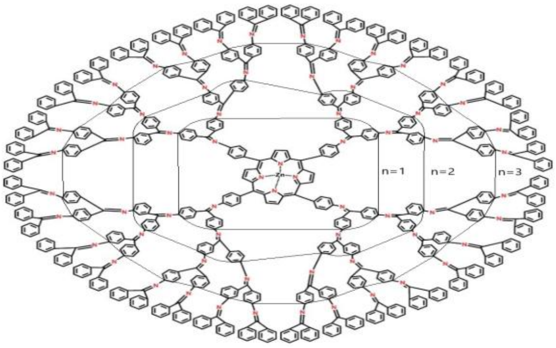

Figure 5: The graph of DPZ_n.

Let G be the graph of DPZ_n dendrimer. By calculation, we obtain that G has $56\times 2^n - 7$ vertices and $64\times 2^n - 4$ edges. In DPZ_n dendrimer, there are four types of edges based on degree of end vertices of each edge. The edge partition of DPZ_n is given below:

$E_1 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 2\}, |E_1| = 16\times 2^n - 4.$

$E_2 = \{uv \in E(G): d_G(u) = 2, d_G(v) = 3\}, |E_2| = 40\times 2^n - 16.$

$E_3 = \{uv \in E(G): d_G(u) = 3, d_G(v) = 3\}, |E_3| = 8\times 2^n + 12.$

$E_4 = \{uv \in E(G): d_G(u) = 3, d_G(v) = 4\}, |E_4| = 4.$

Clearly, we have $\Delta(G) = 4$.

Therefore $c_u = \Delta(G) - d_G(u) + 1 = 5 - d_G(u)$. Hence there are four types of reverse edges as given in Table 7.

Table 7

Reverse edge partition of DPZ_n

$c_u, c_v: uv \in E(G)$	(3,3)	(3,2)	(2,2)	(2,1)
Number of edges	$16 \times 2^n - 4$	$40 \times 2^n - 16$	$8 \times 2^n + 12$	4

Theorem 5.1.13The sum connectivity reverse index of DPZ_n dendrimer is

$$SC(DPZ_n) = \left(\frac{16}{\sqrt{6}} + 4 + \frac{40}{\sqrt{5}}\right)2^n - \left(\frac{4}{\sqrt{6}} + \frac{16}{\sqrt{5}} - 6 - \frac{4}{\sqrt{3}}\right).$$

Proof. By definition, we have

$$SC(DPZ_n) = \sum_{uv\in E(G)} \left[\frac{1}{\sqrt{c_u + c_v}}\right]$$

Hence by using Table 7, we obtain

$$SC(DPZ_n) = \frac{1}{\sqrt{3+3}}(16 \times 2^n - 4) + \frac{1}{\sqrt{3+2}}(40 \times 2^n - 16) + \frac{1}{\sqrt{2+2}}(8 \times 2^n + 12) + \frac{1}{\sqrt{2+1}}(4)$$

$$= \left(\frac{16}{\sqrt{6}} + 4 + \frac{40}{\sqrt{5}}\right)2^n - \left(\frac{4}{\sqrt{6}} + \frac{16}{\sqrt{5}} - 6 - \frac{4}{\sqrt{3}}\right).$$

Theorem 5.2.14The product connectivity reverse index of DPZ_n dendrimer is

$$PC(DPZ_n) = \left(\frac{16}{3} + \frac{40}{\sqrt{6}} + 4\right)2^n - \left(\frac{4}{3} + \frac{16}{\sqrt{6}} - 6 - \frac{4}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$PC(DPZ_n) = \sum_{uv\in E(G)} \left[\frac{1}{\sqrt{c_u \cdot c_v}}\right]$$

Therefore by using Table 7, we deduce

$$PC(DPZ_n) = \frac{1}{\sqrt{3\cdot 3}}(16 \times 2^n - 4) + \frac{1}{\sqrt{3\cdot 2}}(40 \times 2^n - 16) + \frac{1}{\sqrt{2\cdot 2}}(8 \times 2^n + 12) + \frac{1}{\sqrt{2\cdot 1}}(4)$$

$$= \left(\frac{16}{3} + \frac{40}{\sqrt{6}} + 4\right)2^n - \left(\frac{4}{3} + \frac{16}{\sqrt{6}} - 6 - \frac{4}{\sqrt{2}}\right).$$

Theorem 5.3.15The atom bond connectivity reverse index of DPZ_n dendrimer is

$$ABCC(DPZ_n) = \left(\frac{32}{3} + \frac{48}{\sqrt{2}}\right)2^n - \frac{8}{3}.$$

Proof. By definition, we have

$$ABCC(DPZ_n) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u \cdot c_v}}$$

Thus by using Table 7, we derive

$$ABCC(DPZ_n) = \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)(16 \times 2^n - 4) + \left(\sqrt{\frac{3+2-2}{3\cdot 2}}\right)(40 \times 2^n - 16) + \left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)(8 \times 2^n + 12) + \left(\sqrt{\frac{2+1-2}{2\cdot 1}}\right)(4)$$

$$= \left(\frac{32}{3} + \frac{48}{\sqrt{2}}\right)2^n - \frac{8}{3}.$$

Theorem 5.4.16The geometric-arithmetic connectivity reverse index of DPZ_n dendrimer is

$$GAC(DPZ_n) = (24 + 16\sqrt{6})2^n - \left(\frac{32\sqrt{6}}{5} - 8 - \frac{8\sqrt{2}}{3}\right).$$

Proof. By definition, we have

$$GAC(DPZ_n) = \sum_{uv\in E(G)} \left[\frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}\right]$$

Hence by using Table 7, we obtain

$$GAC(DPZ_n) = \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)(16 \times 2^n - 4) + \left(\frac{2\sqrt{3\cdot 2}}{3+2}\right)(40 \times 2^n - 16) + \left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)(8 \times 2^n + 12) + \left(\frac{2\sqrt{2\cdot 1}}{2+1}\right)(4)$$

$$= (24 + 16\sqrt{6})2^n - \left(\frac{32\sqrt{6}}{5} - 8 - \frac{8\sqrt{2}}{3}\right).$$

Table 8

Topological index of DPZ_n

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
$SC(G)$	56.36	113.20	226.89	454.25	908.98	1818.43	3637.17	7275.17	14550.82	29102.13

$PC(G)$	$ABCC(G)$	$GAC(G)$
48.29	86.55	122.48
99.62	175.76	248.86
202.27	354.19	501.63
407.58	711.06	1007.16
818.19	1424.78	2018.23
1639.41	2852.23	4040.37
3281.86	5707.13	8084.65
6566.76	11416.93	16173.20
13136.55	22836.52	32350.31
26276.15	45675.71	64704.53

6. Graphical Comparison and Conclusion

In this article, we dealt with four dendrimers families and studied their connectivity indices. We determined expressions for sum connectivity reverse index $SC(G)$, product connectivity reverse index $PC(G)$, atom bond connectivity reverse index $ABCC(G)$ and geometric-arithmetic connectivity reverse index $GAC(G)$ for these dendrimers families such as Prophyrin dendrimers D_nP_n, Zinc Prophyrin dendrimers DPZ_n, Propyl Ether Imine Dendrimer $PETIM$ and Poly(Ethylene Amide Amine)dendrimer $PETAA$ and hence obtained numerical value for degree based topological indices particularly from $n = 1$ to $n = 10$ from derived results. Thus we also graphically compare these results from $n = 1$to $n = 10$. Figures 6-9, depict the topological index of sum connectivity reverse index $SC(G)$, product connectivity reverse index $PC(G)$, atom bond connectivity reverse index $ABCC(G)$ and geometric-arithmetic connectivity reverse index $GAC(G)$. In Figure 6, we see that Zinc Prophyrin dendrimers DPZ_n get the maximum value and the Prophyrin dendrimer D_nP_n get the minimum value of sum connectivity reverse index $SC(G)$. The same result is followed in Figures 7-9 with respect to the product connectivity reverse index $PC(G)$, atom bond connectivity reverse index $ABCC(G)$ and geometric--arithmetic connectivity reverse index $GAC(G)$ respectively.

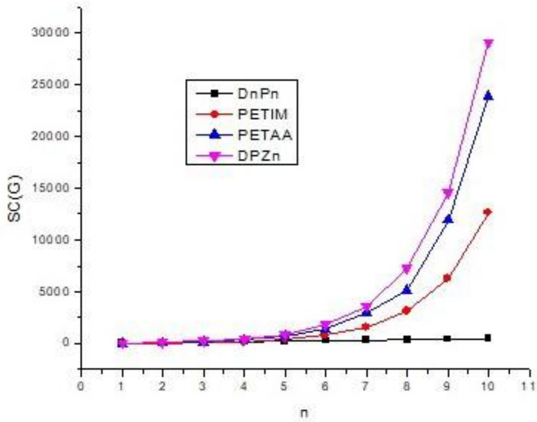

Figure 6: Topological index of $SC(G)$.

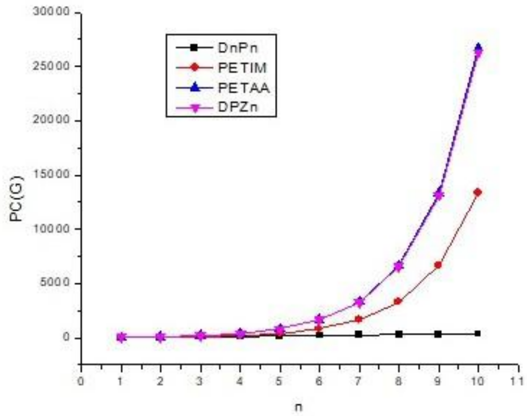

Figure 7: Topological index of $PC(G)$.

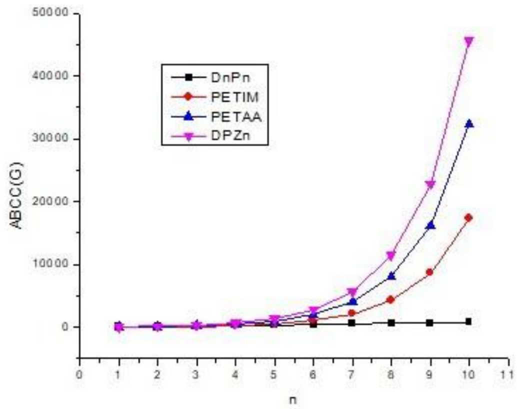

Figure 8: Topological index of $ABCC(G)$.

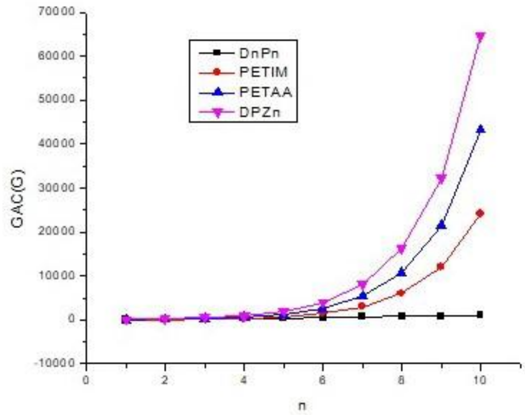

Figure 9: Topological index of $GAC(G)$.

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