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New Approach to Determine the Highest Common Factor (H.C.F.) of Quadratic Expressions

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ABSTRACT

The aim of this paper is to introduce a new approach to determine the Highest Common Factor of quadratic expressions. This is an approach to determine H.C.F. with the help of diagrams.

1. Introduction

Recently in 2017, R B Nelsen introduced "Proof without words: Diophantus of Alexandria's Sum of Squares Identity". Being inspired from his paper in this paper I shall introduce a new approach to determine the Highest Common Factor of quadratic expressions. I shall introduce diagrams to determine the H.C.F. of quadratic expressions.

2. Preliminaries

Definition: H.C.F. of Quadratic Expressions is the largest of the common factors of the Quadratic Expressions.
Examples: 1. To determine the H.C.F. of $a^2+b^2-c^2+2ab$ and $a^2-b^2-c^2+2bc$ we have to factorize the quadratic expressions.
Thus $a^2+b^2-c^2+2ab$ can be written as
$a^2+b^2-c^2+2ab=(a+b+c)(a+b-c)$.
Similarly $a^2-b^2-c^2+2bc$ can be written as
$a^2-b^2-c^2+2bc=(a+b-c)(a-b+c)$.
Hence the H.C.F. is $(a+b-c)$.

2. To determine the H.C.F. of $a^2-b^2-c^2+2bc$, $b^2-c^2-a^2+2ac$ and $c^2-a^2-b^2+2ab$ we have to factorize each of the quadratic expressions.
Thus $a^2-b^2-c^2+2bc= (a+b-c)(a+c-b)$
$b^2-c^2-a^2+2ac= (b+c-a)(a+b-c)$
$c^2-a^2-b^2+2ab= (a+c-b)(b+c-a)$.
Hence the H.C.F. is 1.

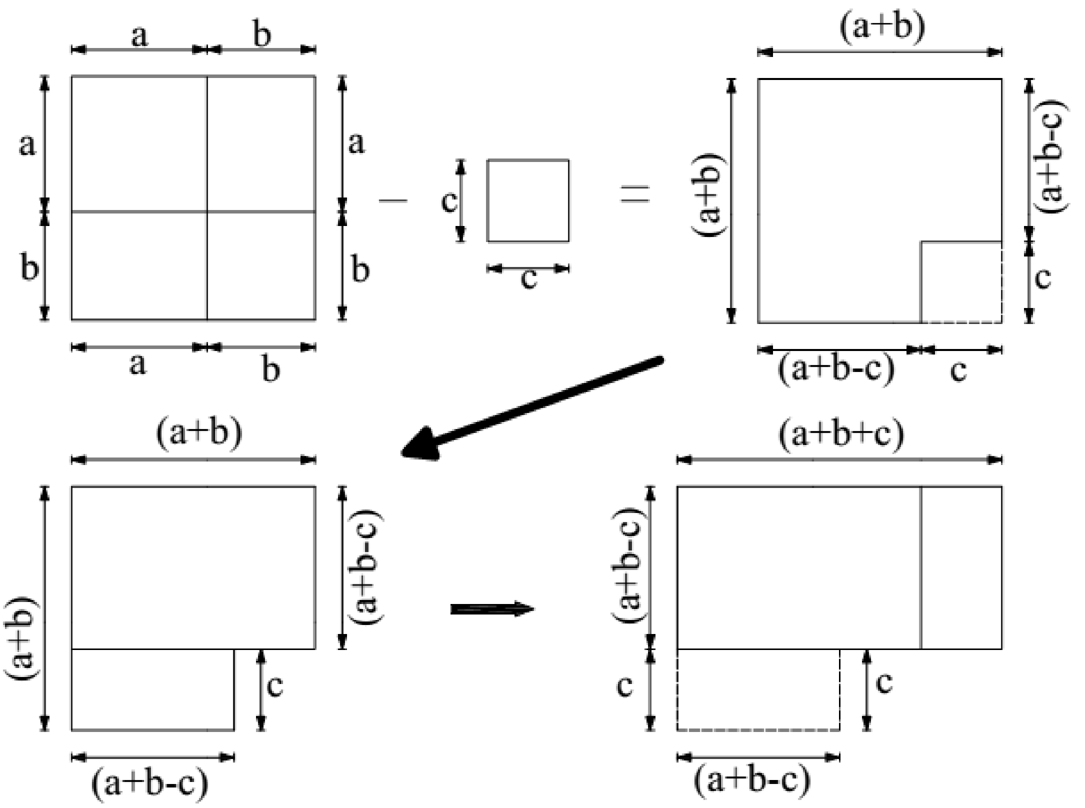
3. New Approach

To determine the H.C.F. of these kind of quadratic expressions we consider a,b,c as the sides of a triangle where a>b>c and a,b,c are natural numbers.

We know that the area of a rectangle is its (length x breadth) and the area of a square is its $(\text{length})^2$.

Now first of all we are trying to draw the squares and rectangles with the given sides of the triangle. We consider the height of each square and rectangle as 1 unit. Thus each square and rectangle has become actually a rectangular parallelepiped with a height of 1 unit. Our target is to construct a rectangular parallelepiped with a height of 1 unit for each quadratic expressions and find the lengths, breadths and heights. Height will match for every case (as height = 1 unit). If length or breadth will match then that will be the H.C.F. as that will be greater than or equal to 1. Otherwise 1 will be the H.C.F.

Now we draw diagrams to determine the H.C.F. of previous two problems as follows: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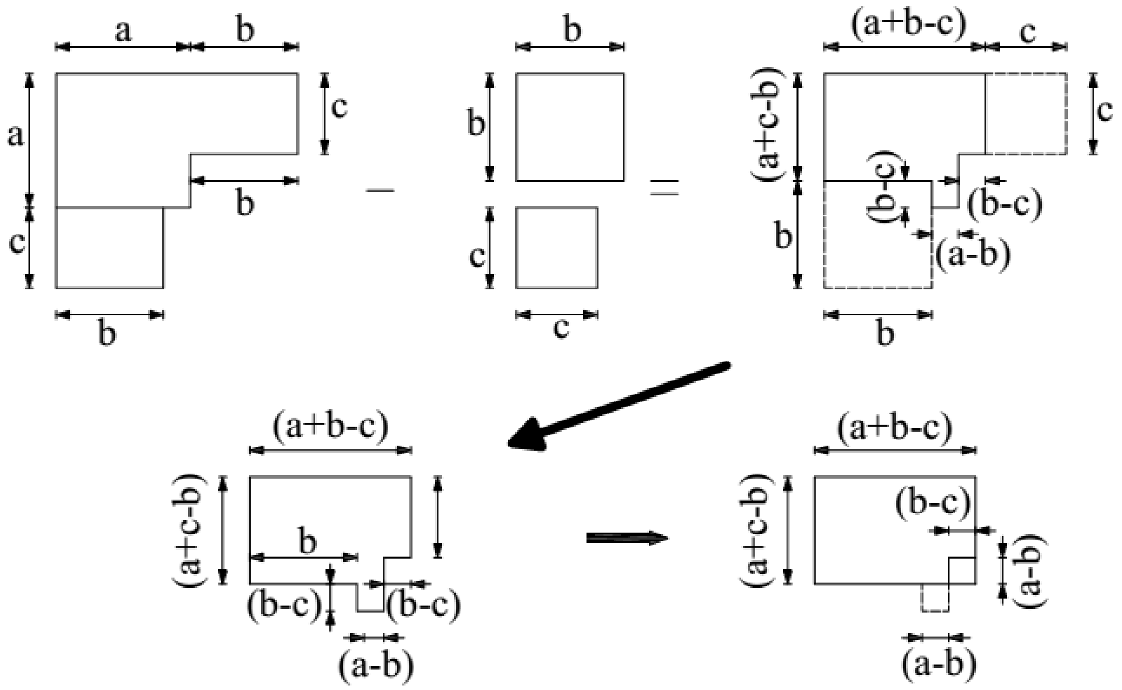
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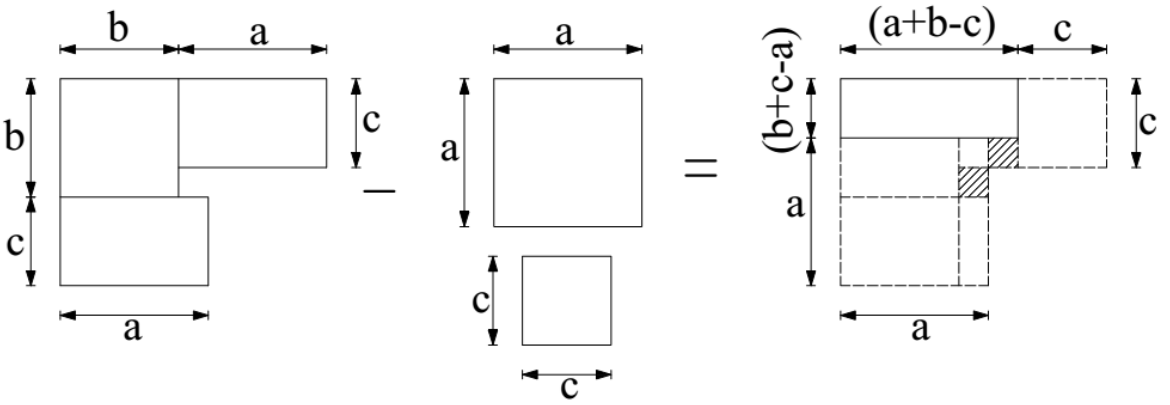

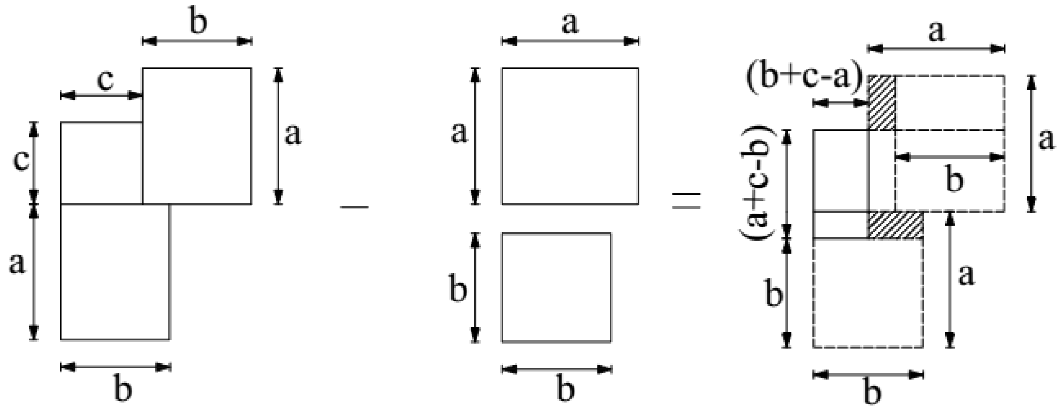

Here we observe that the H.C.F. are same as previous.

Hence we can conclude that the H.C.F. of those kind of quadratic expressions can be obtained with the help of such diagrams.

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