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# Comprehensive Study of Reliability Theory and its basic concepts

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**ABSTRACT**

In this paper, the author has discussed that an object is meant to be reliable if it works upto expectation in times of need. However, for the development of a proper theory out of this intuitive meaning, one need to have a more precise definition that would be capable of utilizing our scientific knowledge to characterize, quantify and measure reliability. A commonly used definition of reliability is that it is the probability that an equipment or device will perform the functions intended of it under conditions specified for its operation for a given period of time. At the point when the gadget does not do its proposed function, we may state that it has failed. The idea of failure can be all the more intricately depicted, obviously. Failure can be any episode or condition that causes a mechanical plant, made item, procedure, material or administration to corrupt or wind up inadmissible for performing acceptably, securely, dependably and cost-viably.
The fundamental goal of reliability theory is to comprehend the example in which failures happen for various components under various working conditions.

## 1. Introduction

Rapid strides in technology during the last few decades have given rise to new products, devices and services that aim to make human life more comfortable. These include a wide range of products, from consumer goods for daily use to sophisticated devices for inter-planetary missions. From the producer to the customer, everyone is interested in ensuring that the product undergoes failure-free operation at least for a specified period of time. Reliability theory is a body of concepts, practices and methodology that attempts to meet this objective. Blischke and Murthy (2000) [1] have distinguished five primary regions through which the undertakings in dependability can be practiced, and these are reliability displaying, dependability examination, reliability building, dependability science and reliability administration. Of these, reliability demonstrating manages the distinguishing proof of system that produces perceptions on the failure times in a particular report. Different ideas that portray the failure instrument and their properties can reveal some insight into the examples in the perceptions in this manner empowering the assurance of the law that oversees the lifetimes of the gadget [2].

Numerous outcomes in likelihood theory, insights and stochastic procedures become the principle contributions to this association. A reasonable numerical model, alongside the genuine information, would then be able to be utilized to draw deductions about the reliability. This is the primary subject in reliability investigation. Reliability building goes for creating progressively dependable items through testing, planning and development of items. Some key properties of materials that may cause failure and the impacts of the assembling procedure are considered in dependability science. Accentuation in reliability administration is on the outcomes of questionable items, for example, cost, loss of generosity and fix, and to guarantee nature of the item through administration of structure, task and production. Of these distinctive regions, this book concentrates just on reliability displaying and examination. The pertinent numerical theory is portrayed in detail with various models. In the dialogs held up until this point, accentuation has been made on the reliability of items and the job of dependability theory in evaluating it [3].

The time interim of failure free activity of a gadget is frequently named as the lifetime of the gadget. Be that as it may, lifetime can be seen inside an increasingly broad structure. On the off chance that the length spent by a component or item in a given state is taken to be the lifetime, at that point the theory of dependability can promptly discover applications in life tables, nuptiality contemplates, time of administration before leaving a specific occupation in labor arranging, etc. Reliability ideas have likewise discovered numerous helpful applications in the development of pay dispersions, examination of human settlements, under-detailed wages, and bibliometry. An overview of some imperative applications is displayed. In this manner, an exchange of the fundamental ideas and philosophy in reliability theory expect significance in a plenty of zones of logical action [4].

In a large portion of the exchanges on reliability theory, the lifetime is treated as consistent. The displaying and investigation viewpoints use disseminations and procedures that normally speak to a non-negative nonstop arbitrary variable. Relatively, substantially less writing is accessible when the lifetime is discrete. Be that as it may, there are some convincing motivations to consider failure times as discrete irregular factors taking on non-negative whole number qualities. At the point when a bit of hardware works in cycles and the perception is the quantity of cycles finished before failure, the lifetime is plainly discrete. So additionally is the situation when the gadget is checked just in finished units of time, similar to what number of failures have happened toward the consummation of 60 minutes, two hours, etc. The absence of precision of estimating gadgets may likewise produce discrete lives. There are events to lean toward tallies over clock time notwithstanding when the last is accessible. In weapons reliability, the quantity of rounds shot could really compare to the age at failure. The equivalent is the situation with lifetimes of vehicle tires wherein the quantity of kilometers keep running before it ends up out of utilization is liked to the quantity of days before failure. These sorts of issues give a solid force to examining reliability in discrete time. The ideas in nonstop and discrete occasions are the equivalent, yet the definitions and elucidations may vary between the two [5].

To determine dependability properties similar to the ceaseless case, infrequently persistent disseminations are discretized. In any case, it isn't essential in such cases that the distributional properties are not the equivalent nor the discretization procedure will dependably result in important discrete models. Thus, there are calculated and scientific issues in creating discrete dependability theory. These focuses will be thought at fitting spots in the resulting areas. The development of dependability theory for discrete lifetimes requires contributions from different controls, dominatingly devices from arithmetic, measurements and likelihood theory [6].

## 2. Important aspects of reliability theory

The reliability of a device is the probability that the device performs its intended function for a given period of time under conditions specified for its operation. When the device does not perform its function satisfactorily, we say that it has failed. When the random variable X represents the lifetime of a device, the observation on X is realized as the time of failure. The primary concern in reliability theory is then to understand the pattern in which failures occur for different devices and under varying operating environments. This is frequently done by dissecting the watched failure times or ages at failure with the assistance of a model that palatably speaks to the prevalent highlights of the information. One direct strategy is to discover a likelihood circulation that gives a sensible fit to the perceptions. Once in a while, there may exist more than one dispersion that can pass a fitting decency of-fit test. Regardless, it is progressively attractive to discover a likelihood demonstrate that shows certain physical properties of the failure system. In dependability theory, some fundamental ideas that assistance in the investigation of failure designs have been created. The goal of this segment is to characterize such ideas and talk about their properties and between connections [7].

Two important aspects that necessitate the study of these concepts [8] are:

(a) various functions considered in this context determine the life distribution uniquely, so that the knowledge of their functional form is equivalent to that of the distribution itself, and

(b) it should be easier to deal with these functions than the distribution function or probability density function of the corresponding distributions [9].

## 3. Basic concepts of reliability theory

### Hazard Rate Function

Let X be a discrete random variable assuming values in N = (0, 1,...) with probability mass function f (x) and survival function $S(x) = P(X \geq x)$. We will think of X as the random lifetime of a device that can fail only at times (ages) in N. The hazard rate function of X is defined as [10].

$$h(x) = f(x) / S(x) \quad (1)$$

at points x for which $S(x) > 0$. Treated as a function of x, the hazard rate is also called failure rate, instantaneous death rate, force of mortality and intensity function in other disciplines such as survival analysis, actuarial science, demography, extreme value theory and bio-sciences. Although in the continuous case, the concept of hazard rate dates back to historical studies in human mortality, its discrete version came up much later in the works of Barlow and Proschan (1965), Cox (1972) and Kalbfleisch and Prentice (2002), to mention a few [11].

When X has a finite support (0, 1,...,n), $n < \infty$, then $h(n) = 1$. As a convention we take $h(x) = 1$ for $x>n$. The hazard function h(x) is interpreted as the conditional probability of the failure of the device at age x, given that it did not fail before age x. Thus, $0 \leq h(x) \leq 1$. The interpretation and boundedness of the discrete hazard rate is thus different from that of the continuous case. We see from (1) that h(x) is determined from f (x) or S(x). The converse that the hazard rate function determines the distribution of X uniquely is also true. To see this [12], we note that

$$h(x) = S(x) - S(x + 1) / S(x) ,$$

or

$$S(x + 1) / S(x) = 1 - h(x).$$

So,

$$S(x) = S(x + 1) / 1 - h(x) , \quad x \geq 1, \quad 1, \quad x = 0. \quad (2)$$

Eq. (2) reveals also that h(x) can be used as a tool to model the life distribution. When a functional form for h(x) is assumed as model for a given data set, one has to ensure that the assumed form conforms to the hazard rate function of a distribution [13].

### Mean Residual Life

The analysis of the lifetime of a device after it has attained age x is of special relevance in reliability and survival analysis. Thus, if X is the original lifetime with survival function $S(x) = P(X \geq x)$, the corresponding residual lifetime after age x is the random variable $X_x = (X - x | X > x)$. From the definition of conditional probability, one can arrive at the distribution of $X_x$ as

$$S_x(t) = S(x + t + 1) / S(x + 1) , \quad t = 0, 1, .... \quad (3)$$

The mean, variance, partial moments, coefficient of variation and percentiles of the distribution in (3) have been discussed extensively in the literature in the continuous case [14].

The mean residual life function may exist when the hazard function does not exist and vice-versa, as will be seen in some of the examples that are considered later. While h(x) is a local measure of failure patterns for any x, the mean residual life function depends upon the life history of devices at all ages beyond x and hence the latter is more informative. At the same time, m(x) as a summary measure is highly sensitive to a single long-term survivor in a data set, which is not desirable.

**4. Variance residual life function**

The variance of the residual life (X − x|X > x) = Xx is studied in reliability theory in various contexts. Primarily, its role is to define ageing concepts that are weaker than some ageing criteria based on the hazard rate and the mean residual life. Secondly, variance of residual life has the same role as the usual variance when estimators of mean residual life are discussed. It is also required in the study of coefficient of variation of residual life. Assuming that E(X2) < ∞, we define the variance residual life function as [15] :

$$\sigma^2(x) = E[(X-x)^2|X>x] - m^2(x), \quad (4)$$

where m(x) is the mean residual life function defined. Alternatively,

$$\sigma^2(x) = E(X^2|X>x) - E^2(X|X>x). \quad (5)$$

**5. Upper partial moments**

A concept that is closely related to moments of residual life is that of partial moments, which can also be interpreted as moments of a different kind of residual life. The rth upper partial moment of X about a point x is defined as [16]

$$\alpha r(x) = E((X-x)r+),\ r = 0, 1, ...;\ x = 0, 1, 2, ..., \quad (6)$$

where, (X−x)+ = max(X−x, 0). In the case of discrete models, it is sometimes more convenient to work with factorial partial moments defined as

$$\alpha(r)(x) = E((X-x)+(r)),\ X > x + r - 1 \quad (7)$$

where

$$t(r) = t(t-1)...(t-r+1) \quad (8)$$

Any one partial moment sequence, particularly (α1(x)), x = 0, 1, 2,... , determines all other partial moments. Since only the first two partial moments are of importance in reliability analysis we concentrate on their properties [17].

**6. Reversed hazard rate**

The reversed hazard rate of X is defined as

$$\lambda(x) = P(X = x|X \le x) = f(x) / F(x). \quad (9)$$

Thus, λ(x) in the discrete case is interpreted as the conditional probability that a device fails at age x, given that its lifetime is at most x. Being a conditional probability, 0 ≤ λ(x) ≤ 1. Keilson and Sumita (1982), who first defined the reversed hazard rate in continuous time, called it the dual failure function by the property that X has reversed hazard rate λ(x), −∞ ≤ a ≤ x ≤ b < ∞ if and only if the random variable −X has a hazard rate λ(−x) on (−b,−a).

Finkelstein (2002) observed that, in reliability, one often works with nonnegative random variables and therefore the above duality is not applicable. Further, the upper point of support is generally infinite. Thus, the properties of reversed hazard rate of non-negative random variables with infinite support cannot be formally obtained from those of the hazard rates. This makes a study of λ(x) becomes necessary in its own right [18].

**7. Reversed variance residual life**

Just as the mean of the reversed residual life xX, the variance of xX is also an important function reliability analysis, called the reversed variance residual life or variance inactivity time, and is denoted by v(x).

In algebraic manipulations, different expressions for v(x) have been employed [19].

**8. Odds function**

Along with the traditional reliability functions presented so far, there has been some interest in discovering the potential of odds function and log odds function in reliability analysis.

The motivation for the consideration of these two functions are [20] :

(i) they are easy to compute and interpret

(ii) the estimation of these functions is relatively simpler, and

(iii) the behaviour of other reliability functions can be ascertained through them.

The concept of odds ratio originated from gambling wherein the odds of an event A against another event B is defined as the ratio P(A)/P(B). In reliability theory, we can take the event A as survival of age x and B as failure by age x. Then, the odds ratio of the events become a function of x. The odds ratio for surviving age x is defined as

$$\omega(x) = P(X > x) / P(X \le x) = S(x+1) / F(x),\ x = -1, 0, 1, ..., \quad (10)$$

and it is called the odds function for survival.

Similarly, the odds function for failure by age x is

$$\omega(x)^{-} = F(x) / S(x+1). \quad (11)$$

**9. Conclusion**

Technology has made tremendous progress in devising new products, devices and systems to the service of mankind for which the tools of reliability are indispensable to ensure their failure-free operation. During the early phase of the development of reliability theory and methods, discussions were mainly confined to continuous lifetimes with only occasional reference to discrete models. Though the trend on emphasis on continuous models still continues, the role of discrete models in lifetime studies is receiving much recognition in the last two decades. The fact that there are natural cases in which lifetimes emerge as a counting variable and also that there are situations in which discrete lifetimes are more appropriate than clock time when the latter is available, have made this shift appealing. Inspite of the growing interest in this field, a reference book exclusively for study and research in discrete reliability analysis does not appear to be available.

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