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Commonly Used Parameters Types and Life Distributions Used to Model Reliability Data

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ABSTRACT

In this paper, the author has studied that the analysis of failure time data over the years has given birth to a number of parametric models. These models were found suitable for representing a wide range of situation and particularly in problems related to the modeling of various aging or failure process. Among univariate models, a few particular distributions occupy a central role because of their demonstrated usefulness. The choice of a failure-time model is largely a skill. In most experiments, the measurements are assumed to be drawn from certain distribution. The choice of the distributions depends on past experience with the process, mathematical expediency and to some extent faith. However, in some cases, the relationship between the failure-mechanism and the failure-time function may be used in making a choice.

1. Introduction

We use the term *life distributions* to describe the collection of statistical probability distributions that we use in reliability engineering and life data analysis. A statistical distribution is fully described by its *pdf* (or probability density function). In the past segments, we utilized the meaning of the pdf to indicate how all different capacities most ordinarily utilized in dependability building and life information examination can be determined; to be specific, the reliability capacity, failure rate work, mean time capacity and middle life work, and so on. These can be resolved legitimately from the pdf definition, or f(t) . Distinctive circulations exist, for example, the ordinary, exponential, and so forth., and every single one of them has a predefined type of f(t). These conveyance definitions can be found in numerous references. Actually, whole messages have been committed to characterizing groups of factual disseminations. These appropriations were defined by analysts, mathematicians and architects to scientifically display or speak to certain conduct. For instance, the Weibull appropriation was figured by Waloddi Weibull, and along these lines it bears his name. A few disseminations will in general better speak to life information and are regularly called lifetime distributions. One of the least difficult and most ordinarily utilized dispersions (and frequently wrongly abused because of its effortlessness) is the exponential circulation. The pdf of the exponential dispersion is scientifically characterized as [1]:

$$f(t) = \lambda e^{-\lambda t}$$

In this definition, note that t is our random variable, which represents time, and the Greek letter λ (lambda) represents what is commonly referred to as the *parameter* of the distribution. Depending on the value of λ, $f(t)$ will be scaled differently. For any distribution, the parameter or parameters of the distribution are estimated from the data. For example, the well-known normal (or Gaussian) distribution is given by [2]:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

μ, the mean, and σ, the standard deviation, are its parameters. Both of these parameters are estimated from the data (i.e., the mean and standard deviation of the data). Once these parameters have been estimated, our function $f(t)$ is fully defined and we can obtain any value [3] for $f(t)$ given any value of t.

Given the mathematical representation of a distribution, we can also derive all of the functions needed for life data analysis, which again will depend only on the value of t after the value of the distribution parameter or parameters have been estimated from data. For example, we know that the exponential distribution *pdf* is given by:

$$f(t) = \lambda e^{-\lambda t}$$

Thus, the exponential reliability function can be derived as:

$$\begin{aligned} R(t) &= 1 - \int_0^t \lambda e^{-\lambda s} ds \\ &= 1 - [1 - e^{-\lambda \cdot t}] \\ &= e^{-\lambda \cdot t} \end{aligned}$$

The exponential failure rate function is:

$$\begin{aligned} \lambda(t) &= \frac{f(t)}{R(t)} \\ &= \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} \\ &= \lambda \end{aligned}$$

The exponential mean-time-to-failure (MTTF) is given [4] by:

$$\mu = \int_0^\infty t \cdot f(t)dt$$
$$= \int_0^\infty t \cdot \lambda \cdot e^{-\lambda t}dt$$
$$= \frac{1}{\lambda}$$

This exact same methodology can be applied to any distribution given its *pdf*, with various degrees of difficulty depending on the complexity of $f(t)$.

2. Review of literature

In recent years reliability has been formulated as the science of predicting, estimating or optimizing the probability or survival, the mean life or more generally the life distribution of components or systems. To study and take care of issues that emerge in reliability hypothesis, the learning of techniques for probability hypothesis and numerical insights is important. At present, architects and researchers as well as government pioneers are worried about expanding the dependability of a framework [5].

So as to acquire distinctive parameters of intrigue like dependability (survival) work, nature of risk rate, mean time to framework failure, accessibility, and so forth called reliability attributes, the examination zone can be comprehensively grouped into the accompanying two classifications:

(1) In concentrates like Dhillon and Singh (1980), Govil (1983), and Balagurusamy (1984), the creators created stochastic models under the different presumption which best fit to the designing framework utilized in everyday viable life. They got the dependability qualities and net expected benefit amid a limited interim of time utilizing the outstanding procedures, for example, regenerative point method, semi-markov process and strengthening variable system [6].

(2) On the other hand, considers like Epstein and Sobel (1952), incorporate account of lifetime information on people and after that different derivation procedures are utilized to evaluate different reliability attributes of the framework. The dependability attributes are broke down in regard of variety in the parameters engaged with lifetime distributions and fix time appropriations [7].

The writing on dependability examination incorporates, in a wide sense, the investigation of a positive esteemed irregular variable speaking to time to failure of a physical or organic framework and its investigation is picking up significance for research specialists in the field of industry and building. Clearly, the nature of issues in such investigation is very differed. In this class, tests are directed to record life time information and these are utilized for examining the existence wonder of different frameworks (human made or natural framework) as far as dependability or survival work, expanding or diminishing danger rate, mean time to survival and so forth. At the end of the day, the recorded life time information are utilized to draw derivations on the reliability attributes of the framework or subunit to see its value in achieving an expecting task and accordingly, we can properly call it as "Inferential Reliability investigation". A huge writing on this perspective is accessible in Lawless (1982), Sinha (1986) [8].

Life testing tests are exorbitant and tedious wonder and in this way it ought to be perceived that the parameters describing dependability attributes in an actual existence time disseminations will undoubtedly pursue some irregular varieties because of ecological changes. Hence, it is a factor which ought to be considered with the exploratory information for investigating the reliability attributes of the framework. Thomas Bayes (1763) presented Bayesian induction in his popular research paper entitled, "An exposition towards taking care of an issue in the Doctrine of Chance". Further, for essential hypothesis and establishments one can likewise allude to Jeffreys (1961) and Savage (1962). Lindley (1965) and Box and Tiao (1973) have promoted and given this methodology a one of a kind imperative spot in the field of measurements. They built up a writing dependent on Bayes' methodology. Today a tremendous writing on Bayesian investigation of life testing issues as far as some standard content is accessible. A couple of them are Savage (1962), Bhattacharya (1967), Martz and Waller (1982), Sinha (1986) and Gelman et al. (1995) displayed the Bayesian investigation of the framework reliability utilizing numerous earlier appropriations. A few, priors with their inborn measurable properties are likewise given in the examination by Raiffa and Schlaifer (1961). Concentrates like Sharma et al. (1993, 1994, 1995) are likewise exertion a similar way. Apostolakis (1990) evaluated the writing on Bayesian hypothesis to survey the probabilistic security of different building framework. Yet, in numerous handy circumstance it might happen that the operational try different things with the total framework is constrained, non-existent or over the top expensive to figure it out. Additionally, we regularly need to anticipate the reliability of complete framework at the beginning time of planning. In such manner, Kaplan et al. (1989) learned about the expectation of reliability of complete framework accepting that the operational involvement with the total framework is constrained, non-existent or over the top expensive to acknowledge by utilizing the data accessible on boxes or subunits of the framework [9].

The investigation broke down the conduct of different probability bends, which inturn might be utilized to express our level of certainty about the total framework reliability and the manner by which Bayes' hypothesis refreshes earlier probability bends to represent different confirmations. Like dependability, accessibility is additionally a proportion of adequacy of a framework for long haul execution. Dark and Lewis (1967) and Masters and Lewis (1987) have acquired certainty interim for enduring state accessibility subsequent to utilizing failure and fix data on particular circulations [10].

Be that as it may, this methodology was not viewed as tasteful and an adjusted methodology was talked about in later. In this altered methodology certainty interim for accessibility was created by getting synchronous interims for MTBF and MTTR. Be that as it may, when failure and fix data are recorded over an expansive interim of time, it might be sensible to expect irregular varieties in the parameters of failure time and fix time appropriations. These parametric irregular varieties may result because of natural effect on working states of the framework or part. Late commitments in these ways are by Sharma and Krishna (1994, 1995a,b), Sharma and Bhutani (1994a, b) and Sharma et al. (2004) [11].

Lining hypothesis is worried about the measurable portrayal of the conduct of lines with discoveries. For instance, the probability circulation of the number in the line from which the mean and difference of line length can be found. In lining hypothesis, the specialists must gauge the current framework to influence a target appraisal of its qualities and must to decide how changes might be made to the framework and what impact of different sorts of changes in the framework's attributes would be there. Probability mass capacity (p.m.f.) got in a relentless state circumstance is the premise of breaking down different line frameworks in regard of their attributes [12].

3. Parameter types

Distributions can have any number of parameters. Do note that as the number of parameters increases, so does the amount of data required for a proper fit. In general, the lifetime distributions used for reliability and life data analysis are usually limited to a maximum of three parameters. These three parameters are usually known as the *scale parameter*, the *shape parameter* and the *location parameter [13]*.

Scale Parameter The scale parameter is the most common type of parameter. All distributions in this reference have a scale parameter. In the case of one-parameter distributions, the sole parameter is the scale parameter. The scale parameter defines where the bulk of the distribution lies, or how stretched out the distribution is. In the case of the normal distribution, the scale parameter is the standard deviation.

Shape Parameter The shape parameter, as the name implies, helps define the shape of a distribution. Some distributions, such as the exponential or normal, do not have a shape parameter since they have a predefined shape that does not change. In the case of the normal distribution, the shape is always the familiar bell shape. The effect of the shape parameter on a distribution is reflected in the shapes of the *pdf*, the reliability function and the failure rate function.

Location Parameter The location parameter is used to shift a distribution in one direction or another. The location parameter, usually denoted as γ, defines the location of the origin of a distribution and can be either positive or negative. In terms of lifetime distributions, the location parameter represents a time shift [14].

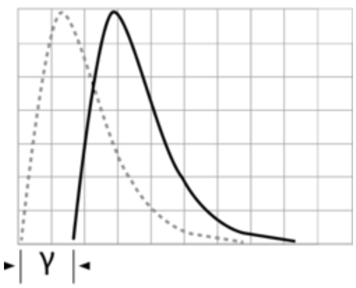

Fig. 1: Location Parameter graph

This means that the inclusion of a location parameter for a distribution whose domain is normally $[0, \infty]$ will change the domain to $[\gamma, \infty]$, where γ can either be positive or negative. This can have some profound effects in terms of reliability. For a positive location parameter, this indicates that the reliability for that particular distribution is always 100% up to that point. In other words, a failure cannot occur before this time γ. Many engineers feel uncomfortable in saying that a failure will absolutely not happen before any given time. On the other hand, the argument can be made that almost all life distributions have a location parameter, although many of them may be negligibly small. Similarly, many people are uncomfortable with the concept of a negative location parameter, which states that failures theoretically occur before time zero. Realistically, the calculation of a negative location parameter is indicative of quiescent failures (failures that occur before a product is used for the first time) or of problems with the manufacturing, packaging or shipping process. More attention will be given to the concept of the location parameter in subsequent discussions of the exponential and Weibull distributions, which are the lifetime distributions that most frequently employ the location parameter [15].

4. Most commonly used lifetime distributions

There are many different lifetime distributions that can be used to model reliability data. Leemis [22] presents a good overview of many of these distributions. In this reference, we will concentrate on the most commonly used and most widely applicable distributions for life data analysis, as outlined in the following sections [16].

The Exponential Distribution

The exponential distribution is commonly used for components or systems exhibiting a *constant failure rate*. Due to its simplicity, it has been widely employed, even in cases where it doesn't apply. In its most general case, the 2-parameter exponential distribution is defined by:

$$f(t) = \lambda e^{-\lambda(t-\gamma)}$$

Where λ is the constant failure rate in failures per unit of measurement (e.g., failures per hour, per cycle, etc.) and γ is the location parameter. In addition, $\lambda = \frac{1}{m}$, where m is the mean time between failures (or to failure).

If the location parameter, γ, is assumed to be zero, then the distribution becomes the 1-parameter exponential or:

$$f(t) = \lambda e^{-\lambda t}$$

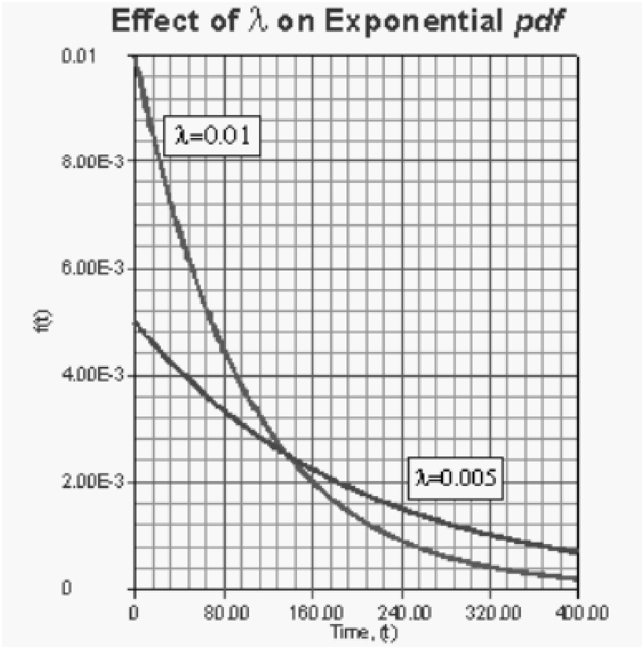

Fig. 2: The Exponential Distribution effect

For a detailed discussion of this distribution, see The Exponential Distribution.

The Weibull Distribution

The Weibull distribution is a general purpose reliability distribution used to model material strength, times-to-failure of electronic and mechanical components, equipment or systems. In its most general case, the 3-parameter Weibull *pdf* is defined by [17]:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$

where β = shape parameter, η = scale parameter and γ = location parameter.

If the location parameter, γ, is assumed to be zero, then the distribution becomes the 2-parameter Weibull or:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

Bayesian-Weibull Analysis

Another approach is the Weibull-Bayesian analysis method, which assumes that the analyst has some prior knowledge about the distribution of the shape parameter of the Weibull distribution (beta). There are many practical applications for this model, particularly when dealing with small sample sizes and/or when some prior knowledge for the shape parameter is available. For example, when a test is performed, there is often a good understanding about the behavior of the failure mode under investigation, primarily through historical data or physics-of-failure.

Note that this is not the same as the so called "WeiBayes model," which is really a one-parameter Weibull distribution that assumes a fixed value (constant) for the shape parameter and solves for the scale parameter. The Bayesian-Weibull feature in Weibull++ is actually a true Bayesian model and offers an alternative to the one-parameter Weibull by including the variation and uncertainty that is present in the prior estimation of the shape parameter.

This analysis method and its characteristics are presented in detail in Bayesian-Weibull Analysis.

The Normal Distribution

The normal distribution is commonly used for general reliability analysis, times-to-failure of simple electronic and mechanical components, equipment or systems. The *pdf* of the normal distribution is given by:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

where μ is the mean of the normal times to failure and σ is the standard deviation of the times to failure.

The normal distribution and its characteristics are presented in The Normal Distribution.

The Lognormal Distribution

The lognormal distribution is commonly used for general reliability analysis, cycles-to-failure in fatigue, material strengths and loading variables in probabilistic design. When the natural logarithms of the times-to-failure are normally distributed, then we say that the data follow the lognormal distribution.

The *pdf* of the lognormal distribution is given by:

$$f(t) = \frac{1}{t\sigma'\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t'-\mu'}{\sigma'}\right)^2}$$

$$f(t) \geq 0, t > 0, \sigma' > 0$$

$$t' = \ln(t)$$

where μ' is the mean of the natural logarithms of the times-to-failure and σ' is the standard deviation of the natural logarithms of the times to failure.

5. Conclusion

In reliability studies, because failure times are defined only for T > 0, some families of distributions are commonly used in place of the normal distribution (truncated at t = 0). In addition to the distributions mentioned above, which are more frequently used in life data analysis, the following distributions also have a variety of applications and can be found in many statistical references. They are included in Weibull++, as well as discussed in this reference. These functions are the basis for other important reliability functions, including the reliability function, the failure rate function, and the mean life.

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