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An EOQ model when quantity received uncertain under permissible delay in payment with planned shortages and equivalent holding cost and shortages cost

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ABSTRACT

In this article, an attempt is made to derive new inventory model with equivalent holding and shortages cost, under the planned expected shortages of quantity and delay in payment for inventory when quantity received is uncertain. The sensitivity analysis to study interdependence of parameters on the decision variables and objective function for derived model is explain by hypothetical numerical illustration.

Introduction

In the traditional inventory EOQ model, quantity received uncertain means there is no any equality between quantity received and the quantity requisitioned. There are deferent situations and strategies in inventory system. An EOQ model dealing mainly with holding cost and shortages cost. This model depends on planned shortages and dealing with the equivalent holding and shortages cost. Also it is regular practice in business and market that the wholesaler tolerates assured fixed credit policy period for manage the balance sheet of profit and no any interest charges are payable if the balance sheet is settled within the decided period.

Silver (1976) has settled an EOQ model when the quantity received is uncertain and is a random variable with stated mean and variance. Kalro and Gohil (1982) have protracted the above model by allowing shortages. Noori and Keller (1986) developed a stochastic model when quantity received is uncertain. Ghare and Schrader (1963) developed an EOQ model for exponentially decaying inventories. The model has been generalized by Covert and Philip (1973) and the by Philip (1974) by using weibulldistribution to define time to deterioration of an inventory. Goyal (1985) has established an EOQ model when supplier allows fixed credit period for settling the accounts. Mandal and Phaujdar (1989) generalized the work of Goyal (1985) by taking into consideration a variety of accurate situations. Kingsman (1997) planned the relationship between the inventory cost and payment rules. Chang (2002) discussed an inventory model with time dependent deterioration rate considering the effect of permissible delay in payment. Rao and Rao (2015) analyzed an inventory model for deterioration item with permissible delay in payment. Ghosh and Chaudhuri (2004) study an order level inventory model with shortages. Eoq for planned shortages it seen in very few books(gupta 2008, Narsimha 2010, Sharma 2010, and Vora 2011). Some authors use expressions of back ordering while many authors go with the planned shortages to derive model. Here prefer the period of planned shortages for the EOQ model when stock extents maximum planned shortages. Equivalent holding cost really makes things easier the traditional inventory model (Kharde. Vikhe, Patil, 2011a, 2011b) . We derive the model equivalent holding cost when quantity received uncertain and it's just parallel using eoq formulas.

In this paper, we analyze an inventory EOQ model for planned shortages when the quantity received uncertain and the delay in payment is permissible. During the time account is not settled, it is expected that the cost of unit sold is deposited in an interest manner accounts and the profit margin is used to meet the further operational expenses of the inventory system. However the cause happen shortages of a stock planned that the quantity order and received no equality. The supplier must have to pay for the goods purchase as early as possible. Also routine business that the supplier allow certain fixed time for settling account and there is no interest charges if account settle within the fixed period of time.

Notations:

Q = the order quantity
Q^*= Economic order quantity
C_3= Replenishment cost per order
Ic= The interest charges per rupee investment in stock per time unit.
Ie= the interest that can be earned per rupee per time unit

T*= permissible delay in payment in time unit
T (Q)= Total annual inventory
d = uniform demand
R = Inventory fix rate
LT = Lead Time
A = Re-order level
F_b= Factor for equivalent holding cost for back and shortages cost
EC_1C_2 = Equivalent Holding and shortages cost when Back-order are permitted under permissible delay in payment.
M= Maximum Inventory level units
S= Maximum shortages level or maximum back-order quantity unit
D= Annual demand units.
N= Number of order per year.
TEC (Q) = Total expected cost per time unit
Q-S= Shortages before quantity added.

Assumptions

1. The demand Rate of D units per time unit is known and constant.
2. Lead time is fixed
3. Holding cost (C_1) per unit per unit time is constant and does not change for differ order quantity
4. Shortages (C_2) per unit per time is known and constant
5. Time horizon is infinite
6. Ordering cost (C_3) per order is known and constant.
7. Purchase price of the item is constant and does not change with delay in payment. Number of discount is applicable.
8. The replenishment for order quantity is done when shortages level reaches planned shortages level in one lot.
9. The supplier gives a listed credit period of T* time unit.
10. Stock out are acceptable and shortages or back ordering cost per unit is known and constant.
11. The quantity requisitioned does not necessary match with the quantity received. It Q does not he quantity requisitioned and Y is the quantity received then Y is a random Variable with E(Y) = bQ and V (Y) = $\sigma_0^2 + (\sigma_1^2 + b^2) Q^2$ when b > 0 is the bias factor and σ_0^2, $\sigma_1^2 > 0$ are constant

Mathematical model: An EOQ model under permissible delay in payment

Let Q (t) denotes the on hand inventory during planned shortages at time t of a cycle and Y is expected uncertain quantity. Where $T=T(Y)=\frac{Y}{R}$ is cycle time. Then the differential equation that defines the instantons states of method is

$$\frac{d(Q(t))}{dt} = -R \quad o \le t \le T$$

$\therefore$ Q (t) = R (T-t) Here, $[RT = Y \therefore T = \frac{Y}{R}]$

Q (t) = Y – RT

Holding cost:C_1

Inventory demand rate : d when time : t, Q = order quantity;
So, Maximum inventory: M = Q – S = d.t

Average inventory during t_1 = $\frac{d.t}{2} = \frac{Q-S}{2}$

M =d.t; Q = d.T

$$\frac{M}{Q} = \frac{dt}{dT} = \frac{t}{T}; \; N = \frac{1}{T}; \; NT = 1 \quad (1)$$

MT = Q (t)

Inventory Quantity is

Q (t) = $\int_0^T$ R (T – t) d t OR

Q (t) = $\int_0^T$ R (T – t) d t

= Y^2 – Rt

Put the values of Q (t)

MT = Y^2 – Rt

R t = Y^2 - MT

= Y^2 – (Q-S) T

$$t = \frac{Y^2 - (Q-S)T}{R}$$

Holding cost during time t

C_1 (Q) = (Avg. inventory) (Inventory Cost during t)

$$C_1 (Q) = \frac{(Q-S)}{2} C_1 t$$

Annual holding cost:

$$C_1 (Q) = \left(\frac{Holding\ \cos t\ per\ Cycle}{Cycle\ per\ year} \right)$$

$$= \left[\frac{Q-S}{2} \right] [C_1 t] N$$

$$C_1 (Q) = \left[\frac{(C_1[Y^2 - (Q-S)]}{2R} \right] \quad - \quad (1)$$

Shortages cost (C_2)

When time = t and shortages rate: d Maximum shortages = S = d t

Average shortages during = $\frac{d\,t}{2} = \frac{S}{2}$ when time: t

S = d t &Q = d.T.

$$\frac{S}{Q} = \frac{d\,t}{d\,T} = \frac{t}{T}\ S = \frac{Q\ (t)}{T}$$

ST = Q (t) = Y – RT

$$t = \frac{Y^2 - ST}{R}$$

$$C_2 (Q) = (\text{average shortages}) \left(\frac{shortages\ \cos t}{unit\ time\ t} \right) = \frac{S}{2}.C_2\, t$$

Annual shortages cost:

$$C_2 (Q) = \left(\frac{shortages\ \cos t}{cycle} \right) (cycleperyear)$$

$$C_2 (Q) = \frac{S}{2} (C_2 .t).N$$

$$C_2 (Q) = \frac{C_2.(Y^2 - S)}{2R} \text{ -------(2)}$$

Ordering cost = $C_3(Q) = \frac{annual\ demand}{unit\ order\ Quantity} = \frac{D}{Q}$

Annual ordering cost = $C_3 (Q) = \left(\frac{order}{year} \right) (ordering \cos t) = \frac{D}{Q}. C_3$ ------ (3)

We analyze one cycle two cases may arise:

Case: I Y > RT* or T > T*

In this situation the account is to be settled earlier close of cycle.

Interest earned during (0, T*) is

Ie C.R $\int_0^{T^*}$ t. dt= $\dfrac{IeCRT^*}{2}$ (4)

Interest charges during (T^*, T) is

CICR $\int_{T^*}^{T}$ (T-t) dt= $\dfrac{CIcR}{2R}(y-RT^*)^2$ −(5)

The net (annual) cost of the structure is given by

$$K_1(Y/Q)=\frac{C_1[Y^2-(Q-S)]}{2R}+\frac{CIc\,(Y-RT^*)^2}{2R}+\frac{C_z(y^2-S)}{2R}+\frac{D}{Q}C_3-\frac{CIeRT^{*2}}{2}\quad -\ (6)$$

$$=\frac{[C_1+CIc][Y^2-(Q-S)]}{2R}-CIc\,T^*Y+\frac{C_z(Y^2-S)}{2R}+\frac{D}{Q}C_3+\frac{C[Ic-Ie]RT^{*2}}{2}\ -(7)$$

$$=\frac{Y^2}{2R}[(C_1+CIc)-(Q-S)+.C_2(1-S)]-CIc\,T^*Y+\frac{D}{Q}C_3+C\,[Ic-Ie]RT^{*2}\ -(8)$$

Therefore we proceeds equivalent Holding and shortages cost

E $C_1.C_2$= C1 (Q) + C_2 (Q)

$$=\frac{C_1(Y^2-(Q-S))}{2R}+\frac{C_2(Y^2-S)}{2R}$$

$$=Y^2\left[\frac{C_1(1-(Q-S))}{2R}+\frac{C_2(1-S)}{2R}\right]$$

Taking expatiation an equations,

$$E\ (EC_1C_2)=[\sigma_0^{\ 2}+(\sigma_1^{\ 2}+b^2)Q^2]\left[\frac{C_1(1-(Q-S))}{2R}+\frac{C_2(1-S)}{2R}\right]$$

So, estimated total cost of the system during the random cycle time with equivalent holding shortages cost is –

$E[K_1CY/Q]+EC_1C_2$=

$$\frac{[C_1+CIc-(Q-S)+.C_2(1-S)]}{2R}[\sigma_0^{\ 2}+(\sigma_1^{\ 2}+b^2)Q^2]-CIc\,T^*bQ+C\,[Ic-Ie]RT^{*2}+\frac{D}{Q}C_3$$

$$+[\sigma_0^{\ 2}+(\sigma_1^{\ 2}+b^2)Q^2]\left[\frac{C_1(1-(Q-S))}{2R}+\frac{C_2(1-S)}{2R}\right]$$

Now, term (Q-S) and (1-S) multiply with (-1/2) and (1/2) corresponding and it remains (Q-S)

$$=(Q-S)\ (\sigma_0^{\ 2}+(\sigma_1^{\ 2}+b^2)Q^2)\ \frac{[C_1+CIc+.C_2]}{2R}-CIc\,T^*bQ+C\,[Ic-Ie]RT^{*2}+C_3D\ \_\_\_\_\_\_\_\_\_\_(9)$$

Also, estimated duration of a cycle is …

E (T(y)] = E [Y/R] = $\dfrac{bQ}{R}$ ---- (10)

Following silver [1976] the average estimated cost of the system per time unit is

$$=TEC(Q)=\frac{E[K(Y/Q)]}{E[T(Y)]}$$

$$=(Q-S)\frac{[C_1+CIc+C_2]\,\sigma_0^{\ 2}}{2bQ}+\frac{[C_1+CIc+.C_2]\,(\sigma_1^{\ 2}+b^2)Q}{2b}-CIc\,T^*R+\frac{CR^2T^{*2}}{2bQ}[Ic-Ie]+\frac{C_3DR}{bQ}\quad(11)$$

$$=(Q-S)\frac{1}{2bQ}[4C_3DR+\{C_1+CIc+.C_2\}\ \sigma_0^{\ 2}+C[Ic-Ie]R^2T^{*2}-CIcT^*R+\frac{1}{2b}$$

$$[C_1+CIc+.C_2](\sigma_1^{\ 2}+b^2)Q-\ (12)$$

For optimum value of (Q), $\dfrac{dECPUT(Q)}{\partial Q}=0\ Gives$

$$Q^* = (Q-S) - \frac{[C_1 + CIc + .C_2]\sigma_0^2}{2bQ} - \frac{C_3 DR}{bQ^2} - \frac{CR^2T^{*2}\,(Ic - Ie)}{2bQ^2} + \frac{(C_1 + CIc + C_2)(\sigma_1^2 + b^2)}{2b} = o$$

$$\therefore \frac{1}{Q^2}[(Q-S)(C_1 + CIc + .C_2]\,\sigma_0^2 + 4C_3DR + CR^2T^{*2}[Ic - Ie] = (C_1 + CIc + C_2](\sigma_1^2 + b^2)$$

$$\therefore Q^* = \sqrt{\frac{4C_3DR + (Q-S)[C_1 + CIc + .C_2]\sigma_0^2 + C[Ic - Ie]R^2T^{*2}}{[C_1 + CIc + .C_2]\,(\sigma_1^2 + b^2)}} \quad - \ (13)$$

At that time the economic cost per unit time is

TEC(Q*)= $\frac{1}{b}$ [(Q-S) [C_1+CIc + .C_2] σ_0^2 + 4DC_3R + CR^2 T^{*2} [Ic-Ie] [C_1+CIc+C_2] (σ_1^2+b^2)]1/2 - CIcRT*2

To minimize the TEC (Q*),partial derivatives with the variable Q and S should be zero. Taking first derivative with respect to S and equate it to zero.

$$\frac{\partial(TEC(Q^*))}{\partial S} = \frac{1}{b}[(Q-S)[C_1 + CIc + .C_2]\,\sigma_o^2 + 4C_3DR + CR^2T*^2[Ic - Ie][C_1 + CIc + C_2]\,(\sigma_1^2 + b^2)]1/2 - C\,Ic\,RT*^2$$

$$= 0 - [(Q-S)(C_1 + CIc) + C_2]$$

$$Q(C_1 + CIc) = SC_1 + SCIc + C_2 \therefore \frac{Q}{S} = \frac{C_1 + CIc + C_2}{C_1 + CIc}$$

This is the optimum condition for S. hence, S is replaced by optimum S*.We have equation (1) and (2)

$$\frac{Q}{S^*} = \frac{C_1 + CIc + C_2}{C_1 + CIc} - (1) \ and \ \frac{S^*}{Q} = \frac{C_1 + CIc}{C_1 + CIc + C_2} - (2)$$

Here, RHS term is defined as settle factor for back order or shortages with interest charges

$$F_b = \frac{C_2}{C_1 + CIc + C_2}(3)$$

Factors are depends upon C_1& C_2, subtracting LHs and RHs from 1.

$$1 - F_b = 1 - \frac{C_2}{C_1 + CIc + C_2} = \frac{C_1 + CIc}{C_1 + CIc + C_2} = \frac{S^*}{Q} \therefore S^* = (1 - Fb)Q - - - - (4)$$

Following are some particular cases:-

If T* = 0 then Ie = o and Q* = $\left[\frac{4C_3DR + [C_1 + CIc + .C_2]\sigma_o^2}{[C_1 + CIc + .C_2]\,(\sigma_1^2 + b^2)}\right]^{1/2} \quad - \quad (14)$

$$Q^* = \left[\frac{(Q-S)[C_1 + CIc + C_2]\sigma_o^2 + 4C_3DR}{[C_1 + CIc + .C_2]\,(\sigma_1^2 + b^2)}\right]^{1/2} \quad - \quad (15)$$

TEC (Q*) = $\frac{1}{b}$ [(Q-S) [C_1+CIc + .C_2] σ_0^2 + 4C_3R} {C_1 $\frac{1}{b}\left[\frac{(Q-S)4C_3DR}{[C_1 + CIc + .C_2]} + \sigma_0^2\right]^{1/2}$ CIc + SC_2} {σ_1^2 + b^2}] ½ -----

(16)

Here, if σ_1^2= 0 then Q* = $\frac{1}{b}\left[\frac{(Q-S)4C_3DR}{[C_1 + CIc + .C_2]} + \sigma_0^2\right]^{1/2} \quad - \quad (17)$

And,

TEC(Q*) = [(Q-S) [C_1+CIc +.C_2] 4C_3R + [C_1+CIc +.C_2]^2 σ_0^2] ½

= [(Q-S) [C_1+CIc +.C_2] 4C_3R + [C_1+CIc +.C_2]^2 σ_0^2] ½ ------ (18)

And,

TEC (Q*) = $\frac{1}{b}$ [(Q-S) [C_1+CIc +.C_2] (σ_1^2 + b^2) + 4 DC_3R] ½ ----- (19)

If σ_o^2 = 0 then Q* = $\left[\frac{(Q-S)4C_3DR}{[C_1 + CIc + .C_2](\sigma_1^2 + b^2)}\right]^{1/2} \quad - \quad (20)$

Equations (17) (18) (19) (20) are as per those given by silver [1976] for a corresponding model without the permissible delay in payment.

It b= 1 and $\sigma_o^2 = \sigma_1^2$ = o then there is no uncertain in supply and in that case.

$$Q^* = \left[\frac{4C_3DR + C[Ic - Ie]R^2T^{*2}}{[C_1 + CIc + .C_2]}\right]^{1/2} \quad - \quad (21)$$

And,

TEC(Q*)= [{4DC$_3$R + C [Ic-Ic] R^2 T*^{2} {[C$_1$+CIc +.C$_2$]}] ^{½} -- CIcRT^{*2} ---- (22)

Equations (21) & (22) are as per those given by goyal [1985]

Here we can see that when we added Equivalent Holding & shortages cost there is no effect on C_3. Which is Replenishment cost andIf there is no equivalent cost then all formulation must be same as original paper.

Case: II - Y < RT

In this case no any interest charges are payable the quantity reserved in stock. The total variable cost of the system in this case is given by,

Interest earned during (0, T) is

CIe R $\int_0^T tdt$ + $\left[CIeRT(T^* - T)\right] = CIe\left[T^*y - \frac{y^2}{2R}\right]$

$$K_2\,(Y/Q) = \frac{[C_1 + CIe - (Q-S) + .C_2(1-S)]Y^2}{2R} - CIeT^*Y + \frac{D}{Q}C_3 \quad - \quad (23)$$

Using (1) expected total cost of the system during the random cycle time is

$$E\,[K_2\,(Y/Q)] = (Q-S)\frac{[C_1 + CIe + .C_2]}{2R}\,(\sigma_0^2 + (\sigma_1^2 + b^2)\,Q^2) + \frac{D}{Q}C_3 + CIeT^*\,bQ \; - \; (24)$$

we taking equivalent Holding & shortages cost with Replenishment cost

E C_1C_2 = C_1 (Q) + C_2 (Q)

$$= \frac{C_1(Y^2 - (Q-S))}{2R} + \frac{C_2\,Y^2(1-S)}{2R}$$

$$= Y^2\,\left[\frac{C_1(1-(Q-S))}{2R} + \frac{C_2(1-S)}{2R}\right]$$

Take expatiation,

$$E\,[E\,C_1C_2] = [\sigma_0^2 + (\sigma_1^2 + b^2)\,Q^2]\,\left[\frac{C_1(1-(Q-S))}{2R} + \frac{C_2(1-S)}{2R}\right]$$

So, estimated total cost of the system during the random cycle time with equivalent Holding and shortages cost is

$$E\,[K_2\,(Y/Q)] + E\,C_1C_2 = (Q-S)\frac{[C_1 + CIe + .C_2]}{2R}\,[\sigma_0^2 + (\sigma_1^2 + b^2)\,Q^2]$$

$$+ C_3 + CIeT^*bQ + [(\frac{C_1(1-(Q-S))}{2R} + \frac{C_2(1-S)}{2R}].[\sigma_0^2 + (\sigma_1^2 + b^2)\,Q^2] + \frac{D}{Q}\,C_3]$$

$$= (Q-S)[\sigma_0^2 + (\sigma_1^2 + b^2)\,Q]\,\frac{[C_1 + CIe + .C_2]}{2R} + DC_3 + CIe\,T^*bQ \; - (25)$$

The average estimated cost per time unit is

$$TEC\,(Q) = \frac{1}{2bQ}\,[(Q-S)[C_1 + CIe + C_2]\,[\sigma_0^2 + 2\,C_3DR] - CIeRT^* + \frac{[C_1 + CIe + C_2]\,[\sigma_1^2 + b^2]Q}{2b} - \; (26)$$

For optimum value of Q ; $\frac{\partial E\,(PUT(Q))}{\partial Q} = 0\ Gives$

$$Q^* = \left[\frac{2(Q - S^*)[C_1 + CIe + C_2]\,[\sigma_0^2 + 2\,C_3DR}{[C_1 + CIe + C_2]\,(\sigma_1^2 + b^2)}\right]^{1/2}$$

And, optimum cost per unit time is

TEC(Q*)= $\frac{1}{b}[(Q-S^*)[C_1+CIe+C_2]\ [\sigma_0^2+2\ C_3DR]\ [C_1+CIe+C_2]\ [\sigma_1^2+b^2)]^{1/2}-CIeRT^*-(27)$

Some particular cases:-

If T* = 0 then Ie = 0

$$\therefore Q^*=\left[\frac{C_1+C_2+2\ C_3DR}{[C_1+C_2]\ (\sigma_1^2+b^2)}\right]1/_2\quad-(28)$$

And,

$$TEC\,(Q^*)=\frac{1}{b}[(Q-S^*)^2\{[C_1+C_2]\ \sigma_0^2+2\ C_3DR]\ \{[C_1+C_2]\ (\sigma_1^2+b^2)\}]1/_2$$

$$=\frac{1}{b}[(Q-S^*)\{[C_1+C_2]\ \sigma_0^2+2\ C_3DR]\ (\sigma_1^2+b^2)]1/_2\ -(29)$$

If σ_1^2 = 0 then,

$$Q^*=\frac{1}{b}\left[\frac{(Q-S^*)+2\ C_3DR}{[C_1+C_2]}+\sigma_0^2\right]^{1/2}-(30)$$

TEC (Q*) = [(Q-S*) [2 [C_1 + C_2 + DC_3R]] + [C_1^2 + C_2^2] σ_0^2] ½ --- (31)

It σ_0^2 = 0 then,

$$Q^*=\left[\frac{2\ C_3DR}{[C_1+C_2](\sigma_1^2+b^2)}\right]^{1/2}-(32)$$

TEC (Q*) = $\frac{1}{b}$ [2 [C_1+C_2+DC_3R] $(\sigma_1^2+b^2)]^{1/2}-(33)$

If b = 1, σ_1^2 = 1 σ_0^2 = 0 then in this case

$$Q^*=\left[\frac{2\ C_3DR}{[C_1+C_2+CIe]}\right]^{1/2}-(34)$$

TEC (Q*) = [2 [C_1 + CIe + C_2] DC_3R] $1/_2$ - CIeR T*--- (35)

Equations (34) &(35) one the same as those given by Goyal [1982] and If there is no equivalent cost then all formula must be as original paper.

Now,total cost with interest charges due to effect of planned shortages under condition of permissible delay in payment

T (Q) = C_1 (Q) + C_2 (Q)+C_3(Q)

$$=\frac{D}{Q}.C_3+\frac{C_1+CIc(Y^2-(Q-S))}{2R}+\frac{C_2(Y^2-S)}{2R}$$

$$\frac{\partial(T(Q))}{\partial S}=\frac{\partial}{\partial S}\left[\frac{D}{Q}.C_3+\frac{C_1+CIc(Y^2-(Q-S))}{2R}+\frac{C_2(Y^2-S)}{2R}\right]$$

$$=0-\frac{2(Y-(Q-S))C_1+CIc}{2R}+\frac{2(Y-S).C_2}{2R}$$

Equating it to zero

$$0=0-\frac{2(Y-(Q-S)).C_1+CIc}{2R}+\frac{2(Y-S).C_2}{2R}$$

$$=-Q(C_1+CIc)+S(C_1+CIc)+SC_2$$

$$Q(C_1+CIc)=SC_1+SC_2+SCIc$$

$$\frac{Q}{S}=\frac{C_1+CIc+C_2}{C_1+CIc}$$

This is the optimum condition for S. hence, S is replaced by optimum S*. now we have equation (1) and (2)

$$\frac{Q}{S^*}=\frac{C_1+CIc+C_2}{C_1+CIc}-(1)\ and\ \frac{S^*}{Q}=\frac{C_1+CIc}{C_1+CIc+C_2}-(2)$$

Here, RHS term is defined as settle factor for back order or shortages with interest charges

$$F_b = \frac{C_2}{C_1 + CIc + C_2} (3)$$

Factorsare depends upon C_1& C_2, subtracting LHs and RHs from 1.

$$1 - F_b = 1 - \frac{C_2}{C_1 + CIc + C_2} = \frac{C_1 + CIc}{C_1 + CIc + C_2} - (4)$$

Equations (2) & (4) $1 - F_b = \frac{C_1 + CIc}{C_1 + CIc + C_2} = \frac{S^*}{Q}$

∴ $S^* = (1 - F_b) Q$

$$Fb = \frac{Q - S^*}{Q} = \frac{M}{Q}$$

∴ $M = F_b Q$ ---- (6)

Also Equivalent Holding and shortages cost with interest charges

$EC_1C_2 = C_1 (Q) + C_2 (Q)$

$$= \left[\frac{(C_1 + CIc)(Y^2(-(Q - S))}{2R} + \frac{C_2(Y^2(-S)}{2R} \right]$$

Sub case put the values of S = S* = (1 – Fb) Q.

$$EC_1C_2 = \frac{(C_1 + CIc)(Y^2(-(Q - S))}{2R} + \frac{C_2(Y^2(-S)}{2R}$$

$$EC_1C_2 = \frac{1}{2R} [Y^2(Q - (1 - Fb) Q)^2(C_1 + CIc) + C_2 (Y^2 - (1 - Fb) Q)^2]$$

$$= \frac{1}{2R} Q^2Y^2 [(1 - (1 - Fb)^2(C_1 + CIc) + C_2 Q^2 (1 - Fb)^2]$$

If Y=1 then,

$$= \frac{Q^2}{2R} [Fb^2(C_1 + CIc) + C_2 (1 - Fb)^2]$$

Putting the values of Fb & (1 – Fb)

$$= \frac{Q^2}{2R}\left[\left(\frac{C_2}{C_1 + CIc + C_2}\right)^2 (C_1 + CIc) + C_2\left(\frac{C_1 + CIc}{C_1 + CIc + C_2}\right)^2\right]$$

$$= \frac{Q^2}{2R}\left[\frac{(C_1 + CIc)C_2}{C_1 + CIc + C_2}\right]$$

$$= \frac{Q^2}{2R} C_1 + CIc\ Fb$$

If Fb =1 .

$$= \frac{Q^2}{2R} C_1 + CIc$$

SENSITIVITY ANALYSIS BASED ON HYPOTHETICAL PROBLEM :

Consider an inventory system following parameters

Unit cost C = Rs. 10 per unit

Demand rate R = Rs. 10000 units per year

Replenishment cost C_3 = Rs. 300 per order

Inventory holding cost C_1 = Rs. 2 per unit per year

Shortages cost C_2 = Rs. 4 per unit per year

Interest charges payable = Rs. 0.18 per Rs. per year

Interest that can be earned = Rs. 0.09 per Rs. per year

Optimum values of lot size Q = Q* and the corresponding total minimum cost have been determine for different combination of b, σ_0^2, $\sigma_1^2$and T*

TABLE[1]	Variation in Q and TEC[Q] with b and σ_1^2 and σ_o^2 = 1, T* = 0.095				
σ_1^2\b	0.75	0.80	0.85	0.90	0.95
5	791.62	786.16	780.47	774.58	768.48
	9263.6	8571.96	7964.29	7426.55	6947.66
10	576.57	574.47	572.25	569.93	567.51
	12754.4	12339.27	11980.11	11293.77	10681.48
15	475	473.82	472.57	471.27	469.9
	15884.9	14858.49	13312.81	12850.88	12436.81
20	373.2	373.34	372.54	372.81	371.16
	17990.93	16662.8	15649.04	15030.28	13954.8
25	300.21	301.34	303.54	302.82	302.67
	19528.1	18267.66	16975.24	16815.24	15027.21

TABLE [2]	Variation in Q and TEC[Q] with T* and σ_1^2 and b=0.75, σ_o^2 =2				
σ_1^2 \T*	0.011	0.032	0.053	0.074	0.095
	770.47	772.98	777.89	785.11	794.51
5	5469.11	5615.65	6154.01	6872.84	7678.86
	559.12	560.94	564.51	569.74	576.57
10	5563.97	6221.64	7378.34	8713.23	10123.74
	460.63	462.11	465.07	469.38	475
15	5658.29	6751.52	8391.54	10198.81	12072.88
	400.73	402.03	404.58	408.34	413.23
20	5745.31	7228.34	9275.49	11479.11	13743.06
	359.4	360.57	362.86	366.23	370.62
25	5832.24	7665.41	10069.81	12621.2	15227.98

TABLE[3]	Variation in Q abd TEC[Q] with b and T* and σ_o^2 = 2 and σ_1^2 =5				
b\T*	0.011	0.032	0.053	0.074	0.095
	709.67	711.65	716.17	722.85	731.5
0.75	5488.34	5744.64	6422.8	7283.06	8228.11
	705.2	707.13	711.99	718.59	727.2
0.80	5306.06	5544.28	6193.28	7018.07	7925.07
	700.85	703.13	707.6	714.16	722.72
0.85	5140.25	5362.73	5985.51	6779.43	7652.64
	696.34	698.61	703.04	709.57	718.06
0.90	4988.59	5197.36	5797.23	6563.47	7406.56
	691.66	693.91	698.32	704.8	713.23
0.95	4849.16	5045.87	5625.34	6366.76	7182.8

TABLE[4]	Variation in Q and TEC[Q] with σ_o^2 , σ_1^2 , T* and b.					
	b	0.75	0.80	0.85	0.90	0.95
σ_o^2\ σ_1^2	T*	0.011	0.032	0.053	0.074	0.095
		782.93	774.04	766.95	761.83	758.83
1 5		4788.63	5171.05	5909.78	6643.95	7574.73
		572.05	565.46	560.28	556.62	554.58
2 10		5027.09	6097.66	7580.44	9104.14	10577.51
		472.46	466.97	462.69	459.71	458.07
3 15		5133.33	6670.23	8607.66	10521.51	12332.85
4 20		411.54	406.76	403.02	400.43	400.04

		5236.48	7173.11	9504.92	11745.73	13841.85
5 25		369.4	365.08	361.74	359.42	358.18
		5336.86	7658.2	10311.79	12839.2	15185.97

TABLE[5]	Variation in Q and TEC[Q] with σ_0^2 , σ_1^2 , T* and b.					
	b	0.95	0.90	0.85	0.80	0.75
$\sigma_0^2 \backslash \sigma_1^2$	T*	0.011	0.032	0.053	0.074	0.095
		747.96	756.34	766.95	779.7	794.51
1 5		4965.39	5402.17	6341.86	7640.46	9263.59
		550.34	554.48	560.28	577.6	576.57
2 10		5051.68	5980.88	7580.14	9624.48	12086.69
		455.68	458.5	462.69	468.22	475
3 15		5135.36	6487.81	8607.29	11228.34	14337.34
		397.46	399.64	403.02	407.58	413.23
4 20		5216.66	6944.48	950.48	12611.55	16265.89
		357.04	358.84	361.74	365.68	370.62
5 25		5295.77	7999.38	10311.28	13845.94	17980.52

Interpretation:

1) From the following table detect the following things
2) When σ_1^2,(Q) and TEC[Q] both reduces as b rises. While for any value of b, (Q) reduction and TEC[Q] rises as σ_1^2 rises
3) When σ_1^2, (Q) and TEC[Q] rises with T*whereas for any given T*, (Q) reduces and TEC[Q] rises as σ_1^2 rises.
4) When b, (Q) and TEC[Q] both rises with T*, Whereas for any given value of T*, (Q) reduces and TEC[Q] rises as b rises.
5) For any given values of $\sigma_0 2$ and $\sigma_1 2$, (Q) reduces and TEC [Q] rises as b and T* rises, whereas for any given value of b and T*, (Q) reduces and TEC [Q] increases $\sigma_1 2$ with and $\sigma_0 2$
6) For any given values of $\sigma_0 2$ and $\sigma_1 2$, (Q) and TEC[Q] both rises as b reduces and T* rises, where for any given values of b and T*, (Q) and TEC[Q] rises with $\sigma_0 2$ and $\sigma_1 2$.

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